Azərbaycan Milli Elmlər Akademiyası
Riyaziyyat və Mexanika İnstitutu

Riyaziyyat və mexanikanın müasir problemləri
Azərbaycan Milli Elmlər Akademiyasının
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60-illik yubileyinə həsr olunmuş beynəlxalq konfrans

Modern problems of mathematics and mechanics

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Institute of Mathematics and Mechanics of the
Azerbaijan National Academy of Sciences

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INVITED SPEAKERS

INSTITUTE OF MATHEMATICS AND MECHANICS OF THE NATIONAL ACADEMY OF SCIENCES OF AZERBAIJAN IN 60 YEARS

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The fundamentals of modern mathematics and systematic scientific studies are connected with establishment of Baku State University in the Muslim East, in 1919 and creation of Physics-Mathematics faculty in its structure in 1920. The mathematical unit engaged in purely mathematical researches was created in April, 1942 in the Azerbaijan branch of the USSR Academy of Sciences in the sector of Physics. **Zahid Khalilov** who defended his candidate's dissertation in 1940 and came from Tiflis to Baku, was its only research associate. In 1943, with the establishment of independent mathematical sector under Azerbaijan branch of the USSR Academy of Sciences more targeted research was launched to conduct research in mathematics. In October, 1945 the Institute of Physics and Sector of Mathematics were combined and the Institute of **Physics and Mathematics** was created. The first scientific journal of the Institute of Physics and Mathematics was "Eserler" journal published in 1940.

After 14 years, by the decision № 319 of the Council of Ministers of the Azerbaijan SSR dated from April 27, 1959, independent **Institute of Mathematics and Mechanics** was created on the base of the Mathematics sector of the Institute of Physics and Mathematics of AS of Azerbaijan SSR. The **first**

director of the Institute was appointed academician **Zahid Khalilov**.

The role of academicians **Zahid Khalilov, Ashraf Huseynov, Ibrahim Ibrahimov** and corr. member of the Academy, the great teacher **Maksud Javadov** in the further development of mathematics as a science and secular formation of mathematical education is undeniable.

Zahid Khalilov defended his doctoral dissertation in 1946 and became the first Azerbaijani doctor of mathematical sciences. Acad. Zahid Khalilov was one of the Soviet scientists feeling the importance of functional analysis, functional methods in theory of differential and integral equations.

His book “Fundamentals of functional analysis” published in 1949 in Baku in Russian was the first book in this field. He was also one of the first scientists creating abstract theory of linear singular integral equations in Banach spaces and a member of the editorial board of the All-Union journal “Functional Analysis and its applications”. It should be noted that the first All-Union Conference on functional analysis was held just in Baku in 1959.

In 1955 Zahid Khalilov was elected academician of Azerbaijan SSR Academy of Sciences, in 1957-1959 vice-president of the Academy, in 1959-1962 worked as acad.-Secretary of the department of “Physical-mathematical and Technical Sciences”, in 1962-1967 President of the Academy. From 1967 Zahid Khalilov returns again to the Institute of Mathematics and Mechanics and becomes its director until the end of his life.

At the time of its establishment, the institute had **six departments, one laboratory and Computing center.**

1. **“Functional Analysis”** department under **Zahid Khalilov’s** guidance conducted scientific research works on general issues of functional analysis, on study of special problems of linear and nonlinear operators theory and their application to mathematical physics.
2. **“Functions Theory”** guided by acad. **Ibrahim Ibrahimov** conducted research works on some problems of modern constructive theory of functions and their application to approximate analysis and theory of differential equations.
3. **“Differential Equations**” department under prof. **Hashim Agayev’s** leadership conducted research work on application of theory of differential functions and theory of generalized functions to mathematical physics problems.
4. **“Integral Equations”** department under the guidance of ass.prof. **Shamil Vekilov** studied the problems of application of theory of integral equations and potential theory.
5. **“Approximate Analysis”** department under the leadership of ass. prof. **Ali Jafarov** conducted scientific-research works on improvement and development of approximate methods of modern mathematics, finding new approximate calculation methods and application of these methods to the solution of different mathematical problems.

6. **“Elasticity Theory”** department guided by corr. member of ANAS **Yusif Amenzadeh** researched theoretical and practical study of torsion and bending problems of elasticity theory, study of materials properties by statistic methods.

7. **“Dynamical strength”** laboratory was guided by prof. **Kerim Ker**imov and conducted research works on studying theoretical and practical problems of strength of materials and installations operating under the action of high dynamic effect.

8. **“Computing Center”** under the guidance of prof. **Said Aleskerov** carried out researches on development of approximate and numerical methods of computational mathematics and mathematical analysis of problems related to economy.

In 1960, the Computing Center was separated from the Institute and operated as independent Computing Center of the Academy. In 1965, **the Institute of Cybernetics** (now Institute of Control Systems) was established on the basis of this Center.

In 1959, when the Institute of Mathematics and Mechanics was established, its **main scientific directions** were: Functional Analysis and its different applications, Functions theory, Differential and Integral Equations, Algebra and Topology, Computational Mathematics and Mathematical Cybernetics. That time the staff of the Institute consisted of **121** employees (**101** research associates): **one academician** (Zahid Khalilov), **one corr. member** (Ibrahim Ibrahimov), **four doctors of sciences** (Zahid Khalilov, Ibrahim Ibrahimov, Yusif Amenzadeh, Yaroslav Lopatinsky) and **twelve condidates of**

sciences (Shamil Vekilov, Ali Jafarov, Hashim Agayev, Kerim Kerimov, Said Aleskerov, Mais Javadov, Sasun Yakubov, Mabud Ismailov, Boris Ponayati, Rashid Mammadov, Faramaz Maksudov, Yahya Mammadov).

After being elected Zahid Khalilov acad.-Secretary to the department of "Physical-mathematical and Technical Sciences on October 1959, creator of functions theory school in Azerbaijan, outstanding mathematician, acad. **Ibrahim Ibis oglu Ibrahimov** headed the Institute. From 1959 he was a creator and the first head of "Functions Theory" department.

Ibrahim Ibrahimov defended his candidate's dissertation in 1939 under the guidance of famous mathematician A.O.Gelfond and was **the first Azerbaijani candidate of sciences.** In 1947 he defended his **doctoral** dissertation in Moscow. Ibrahim Ibrahimov was a known scientist accepted by world mathematicians. Extensive research on Theory of approximation of real and complex variable functions, completeness of the system of functions was conducted under his guidance and a big part of them was included into the monographs written by Ibrahim Ibrahimov and published in Moscow. The papers written by him in collaboration with great mathematicians on our time Keldysh and Gelfond had left a great trace in world mathematics. In the International Conference dedicated to the memory of acad. S.V.Bernstein in Moscow Ibrahim Ibrahimov was entrusted to write a report about one of the creators of constructive theory of functions and this was one of the indicators of his prestige. In 1959 he was elected a corr.-member, in 1968 academician of the Academy.

In 1962, in Baku the II All-Union Conference on “Constructive Theory of Functions” was held at his own initiative.

In 1963-1967 institute was headed by prof. **Hashim Nizam oglu Agayev**. In 1948-1953 he worked as a senior research associate in the “Mathematics” department of the Institute of Physics and Mathematics of Azerb. SSR Academy of Sciences, in 1948-1950 was at the head of “Algebra” chair of Correspondence Pedagogical Institute. In 1953-1954 he was sent on a professional trip to Bulgaria and headed the “Mathematical Analysis” chair in Sofia University. At that period he compiled a dictionary of mathematical terms for Turkish department of the University.

In 1959-1963, Hashim Agayev was deputy-director of IMM for scientific affairs. Hashim Hagayev was a highly qualified known scientist in the field of partial differential equations. For many years, under his leadership a seminar devoted to philosophical issues of natural science, successively worked. The great role in elaboration of scientific basis of formation of mathematical terminology in Azerbaijani language and also some works devoted to history of development of mathematical science in our Republic belongs to Hashim Agayev. In 1971-1980 he was Rector of Azerbaijan State Pedagogical University.

Many prominent **Soviet scientists** as **Mstislav Keldysh, Andrey Kolmogorov, Nikolai Bogolyubov, Mikhail Lavrentyev, Nikolai Muskheleshvili, Ivan Petrovsky, Sergey Sobolev, Lev Pontryagin, Izrail Gelfand, David Sherman, Alexandre Gelfond, Sergey Bernstein, Sergey Nikolsky,**

Andrey Tikhonov, Lazer Lusternik, Yaroslav Lopatinsky, Anatolii Maltsev, Georgiy Shilov, Yuriy Mitropolski, Yuriy Prokhorov, Valentin Maslov, Khalil Rahmatulin, Aleksey Ilyushin, Yuriy Rabotnov, Pyotr Ogibalov, Qoremir Chorniy, Gennadiy Cherepanov, Yuri Savin, Alexandre Guz, Viktor Moskvitin, Vitaut Tamuj and others had great merits in creation of scientific schools in mathematics and mechanics and development of IMM and in training highly qualified scientific personnel for our Republic.

In 1960 in our country a new generation of mathematicians and mechanical engineers school began to be formed. The academicians **Majid Rasulov, Azad Mirzajanzadeh, Jalal Allahverdiyev, Mirabbas Gasimov, Faramaz Maksudov, Akif Gadjiev, corr. Member of the Academy Goshgar Ahmedov, Arif Babayev, Yahya Mammadov, Yusif Amenzadeh, Mayis Javadov** with great services in further development of mathematics and mechanical engineering in our Republic were at the head of this school.

In 1959-1969 **3** collaborators of IMM, **Rashid Mammadov** (1964), **Mais Javadov** (1966), **Sasun Yakubov** (1969) defended **doctoral** dissertation and got academic degree of doctor of physical-mathematical sciences (doctor of mathematical sciences), **54** collaborators defended **candidate** dissertation and got the academic degree of candidate of physical-mathematical sciences (doctor of philosophy in mathematics).

THE INSTITUTE IN 1970-1980

From 1970 to February 1974, **IMM** and at the same time **“Functional Analysis”** department was headed by acad. **Zahid Khalilov**.

From 1974 **Faramaz Gazanfar oglu Maksudov** was director of IMM and the head of “Functional analysis” department. At that time there was a great revival in training doctor of sciences in the field of mathematics and mechanics. Though in **1959-1974** years only **three doctoral** dissertations were defended in Azerbaijan, in the next **fifteen years** the collaborators of IMM defended **fifteen doctor’s** dissertations. In 1976 **Faramaz Maksudov** was elected a corr.-member, in 1980 academician of the Academy, and acad.-Secretary of the department of “ Physical-mathematical and Technical Sciences”.

By the order of the Council of Ministers on June 23, 1978, a **Special Design Office** was created in IMM. The main goal of this SDO was to work out application of results of scientific and practical-constructive research works to economy.

In 1983, a group of collaborators (**Faramaz Maksudov, Vagif Mirsalimov, Valeh Guliyev, Fuad Iskenderzadeh**) were awarded the prize of the Council of Ministers for their work “Elaboration and application of scientific bases for increasing strength of energy equipments based on crack resistance criteria”.

In 1970-1980 at the Institute of Mathematics and Mechanics, **78** collaborators defended **candidate** of physical-mathematical sciences (doctor of philosophy in mathematics), **5** collaborators Allahveren Jabrayilov (1972), Faramaz Maksudov

(1974), Hashim agayev (1975), Valeh Guliyev and Vagif Mirsalimov (1980) defended **doctor's** dissertation and got academic degree of doctor of physical-mathematical sciences (doctor of sciences in mathematics and mechanics).

In 1981-1990 **107** collaborators of IMM defended **candidate** dissertation and became candidate of physical-mathematical sciences (doctor of philosophy in mathematics), **13** collaborators, Jafar Agalarov, Vagif Gadjiev, Hatam Guliyev, Ilham Mammadov (1981), Akif Gadjiev, Yuriy Domshlak (1982), Hamlet Isayev, Frunze Shamiyev (1983), Akif Ibrahimov, Musa Ilyasov, Rafig Feyzullayev (1985), Varga Kalantarov, Barat Nuriyev (1986) defended **doctor's** dissertation and got academic degree of doctor of physical-mathematical sciences (doctor of sciences in mathematics and mechanics) .

THE INSTITUTE IN ITS INDEPENDENCE YEARS

Political confrontation during the years of independence of Azerbaijan, economic recession, Armenia –Azerbaijan, Nagorno-Karabakh conflict had a negative impact on the development of science. Hundreds of well-known scientists left the country in those years and the attitude to the Academy's work was not so good, even in 1992, the government put forward a proposal to close the Academy.

Fortunately, as a result of the wise domestic and foreign policy of Heydar Aliyev who returned to the country for the second time in June, 1993 at the request of the people, political stability was restored in the country and foundation for economic development was laid. Special attention was paid to the development of science, and large-scale reforms in all areas of life including science and education get started.

It is gratifying that despite the political and economical difficulties that have emerged since independence, our scientists successfully carry out scientific research in the country and in many of the world's most developed countries and a generation of mathematicians and mechanical engineers that represent the name of Azerbaijan at a higher level has been brought up.

In **1991-2000**, **40** collaborators of IMM defended **candidate** dissertation and became candidate of physical-mathematical sciences (doctor of philosophy in mathematics).

At those years **5** collaborators, (Tulparkhan Salavatov (1991), Misreddin Sadykhov (1993), Bilander Allahverdiyev, Hamidulla Aslanov (1996), Vahid Gadimov (1998) defended **doctor's** dissertation and got academic degree of doctor of physical-mathematical sciences (doctor of mathematical and mechanical sciences).

In 2001-2003, corr.-member of ANAS, the known specialist in the field of differential equations **Ilham Tofig oglu Mammadov** was at the head of the Institute. He defended his doctor's dissertation in 1981 in his **26** years at the specialized Council of Defense of Ukranian Mathematics Institute and in 2001 was elected a corr. Member of ANAS.

In 2004-2013, the Institute was headed by the known scientist in the field of functions theory, theory of linear operators in functional spaces and harmonic analysis academician **Akif Jafar Khandan oglu Gadjiev** that at the same time was the head of "Functional Analysis" department. At the same time he was acad.-secretary of the department of Physical-Mathematical and Technical Sciences and in 2001 was elected academician of ANAS.

In 2001-2010 **124** collaborators of IMM defended **candidate's** dissertation and got academic degree of candidate of physical-mathematical sciences (doctor of philosophy in mathematics).

At the same years **24** collaborators, Fuad Latifov, Tahir Gadjiev, Asaf Zamanov, Ilham Adil oglu Aliyev, Nizameddin Iskenderov, Fakhraddin Abdullayev (2003), Orujali Rzayev (2004), Elshar Orujov, Farman Mammadov (2005), Sadig Veliyev, Mahammad Guliyev (2006), Nazila Rassoulova, Rakhshanda Jabbarzadeh, Ibrahim Nabiyev, Rauf Amirov (2007), Agil Khanmammadov, Fada Rahimov, Heybatgulu Mustafayev, Adalet Akhundov (2008), Araz Aliyev, Natik Ahmedov (2009), Daniel Israfilov, Turab Ahmedov, Ilham Pirmamedov (2010) defended **doctor's** dissertation and got academic degree of doctor of physical-mathematical sciences.

Since 2013 corr.-member of ANAS prof. Misir Jumail oglu Mardanov, the known scientist in the field of mathematical methods of optimal control and variation is the head of the Institute and from 2015 **"Optimal Control"** department created by his initiative. In 1996-1998 he was Rector of Baku State University, in 1998-2013 minister of Education of the Republic of Azerbaijan, in 2011-2013 President of Mathematical Society of Turkish World.

In **2011-2018**, **89** collaborators defended candidates dissertation (doctor of philosophy in mathematical and mechanics).

In the same years, at the Council of Defense of the Institute **46 doctors** dissertations (doctor of sciences in mathematics and mechanics) were defended Rabil Amanov, Ilham Aliyev, Magsud Najafov, Anar Dosiyev, Ziyathan Aliyev, Yuriy Turovskii, Bakhtiyar Rustamov, Rovshan Humbataliyev, Bahram Aliyev, Mubariz Garayev, Nigar Aslanova, Valeh Gadjiev, Nihan Aliyev, Mugan Guliyev, Mahir Sabzaliyev, Vugar Ismayilov, Alesker Gulgezli, Ramiz Iskenderov, Elmaga Gasimov, Khanlar Mammadov, Rafig Rasulov, Rovshan Bandaliyev, Yagub Mammadov, Yagub Sharifov, Ilgar Mammadov, Sadi Bayramov, Naila Kalantarli, Laura Fatullayeva, Elshad Eyvazov, Etibar Panahov, Rovshan Aliyev, Elman Ibrahimov, Maleyka Mammadova, Shakir Yusubov, Mubariz Hajibeyov, Rashid Aliyev, Gavanshir Hasanov, Elkhan Abbasov, Musa Almammadov, Telman Gasimov, Togrul Muradov, Alizade Seyfullayev.

Now scientific-research works in IMM in the field of **mathematics and mechanics** include seven scientific directions (**6 of them in mathematics, 1 in mechanics**):

- **Harmonic and nonharmonic analysis**. Corr.-members of ANAS, Bilal Bilalov, Vagif Guliyev, prof. Hamidulla Aslanov.
- **Approximation by Ridge functions and neural networks.** Prof. of ANAS Vugar Ismayilov.
- **Spectral and quality theory of differential equations.** Corr.-member of ANAS Rauf Huseynov, prof. Akper Aliyev, ass.prof. Abdurrahim Guliyev.
- **Optimal control and variation calculus.** Corr.-member of ANAS prof. Misir Mardanov.

- **Visualization and control in nonlinear dynamics problems and computer technologies.** Ass. Prof. Hasan Nagiyev.
- **Development history of mathematics in Azerbaijan.** Ass. Prof. Ali Babayev, prof. Ramiz Aslanov.
- **Theoretical and applied problems of deformable solid, fluid and gas mechanics.** Corr.-member of ANAS Geylani Panahov, prof. Jafar Agalarov, Vagif Gadjiyev, Gabil Aliyev, Latif Talybly.

At present the staff of the institute consists of **213** employees including **127** research associates, **47** doctor of sciences, **61** doctors of philosophy, **5** corresponding members of ANAS (Misir Mardanov, Vagif guliyev, Geylani Panahov, Bilal Bilalov, Kamil Aydazadeh).

At present there are **15 departments** and **1 laboratory** in the institute.

1. **"Functional Analysis"** department headed by prof. **Hamidulla Aslanov.**
2. **"Mathematical Analysis"** department headed by corr.-member of ANAS **Vagif Guliyev.**
3. **"Functions Theory"** department headed by prof. of ANAS doct.math.sci. **Vugar Ismayilov.**
4. **"Differential Equations"** department headed by prof. **Akper Aliyev.**
5. **"Optimal Control"** department headed by corr.-member of ANAS prof. **Misir Mardanov.**
6. **"Mathematical Physics Equations"** department headed by ass. prof. **Abdurrahim Guliyev.**

7. **“Nonharmonic Analysis”** department headed by corr.-member of ANAS **Bilal Bilalov.**
8. **“Algebra and Mathematical logic”** department headed by ass. prof. **Ali Babayev.**
9. **“Computational mathematics and Informatics”** department headed by Hasan Nagiyev.
10. **“Fluid and gas mechanics”** department headed by corr.-member of ANAS **Geylani Panahov.**
11. **“Theory of Elasticity and Plasticiy”** department headed by prof. **Vagif Gadjiev.**
12. **“Wave dynamics”** department headed by prof**. Jafar Agalarov.**
13. **“Creeping Theory”** department headed by prof. **Latif Talybly.**
14. **“Applied Mathematics”** department, headed by prof. **Gabil Aliyev.**
15. **“Mathematical problems of signal processing”** laboratory headed by ass.prof. **Sabina Sadigova.**

At present the Institute publishes **four scientific journals**: **“Proceedings of the Institute of Mathematics and Mechanics”**, **“Transactions of ANAS** (issue of mathematics), **“Azerbaijan Journal of Mathematics”**, **“Transactions of ANAS** (issue of mechanics).

The **“Proceedings of the Institute of Mathematics and Mechanics”** has been publishing since 1946, **“Transactions of ANAS”** (issues of mathematics and mechanics) since 1992, **“Azerbaijan Journal of Mathematics”** since 2011. All three journals are published twice a year in the English language.

It should be noted that even in 1970 years **“Transactions of ANAS”** (issues of mathematics and mechanics) was implicated in Thomson Reuters database (Clarivate Analytics). On December 22, 2015, **“Azerbaijan Journal of Mathematics”** of Azerbaijan Mathematical Society published in IMM was implicated in **“Emerging Sources Citation Index”** database of Clarivate Analytics Agency.

On June 1, 2016, **“Proceeding the Institute of Mathematics and Mechanics”** was implicated in the **“Emerging Sources Citation Index” (ESCI)** database of “Clarivate analytics” Agency. This is the first journal in 70 years history of ANAS implicated in this database among the ANAS Institute journals.

“Transactions of ANAS” (physico-mathematical and technical sciences series) from 2015 is published under the name **“Transactions of ANAS”** (issue of mathematics) and **“Transactions of ANAS”** (issue of mechanics).

From January 01, 2018, “Transactions of National Academy of Sciences of Azerbaijan” (series of physical technical and mathematical sciences) of IMM was implicated in **Scopus** database.

In the last four years, over **200** papers of the institute collaborators were published in scientific journals implicated in **Web of Sciences** and **Scopus** list.

From the first day of its functioning, every Wednesday at 10^{00} the institute holds a seminar. The institute collaborators, the known scientists from the Republic and foreign countries participate at these seminars.

Since 1968 till today **candidate (doctor of philosophy)** and **doctor's (doctor of sciences) dissertations council** functions in our Institute. During this period **649** dissertations, were defended. **556** of them got the academic degree of doctor of philosophy and **93** doctor of sciences.

In 2015, for the first time in the history of the National Academy of Sciences **magistracy** was created, **"Education", "International Relations"** departments get started their activity.

The Institute of Mathematics and Mechanics co-operates with scientific research institutions of the USA, Germany, India, England, Iran, Italy, Kazakhstan, Egyptian Arab Republic, Uzbekistan, Poland, Russian Federation, Turkey, Ukraine, Japan and other countries of Asia.

During 60 years the Institute of Mathematics and Mechanics has gone a glorious way, played an important role in development of mathematics and mechanics sciences in our country and had a decisive effect on the development of other sciences.

ALGEBRA AND LOGIC;OLD AND NEW CONNECTIONS

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I will discuss relations between two of this important parts of mathematics during ages.

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OPTIMAL CONTROL OF DIFFERENTIAL INCLUSIONS

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This talk is devoted to optimal control of dynamical systems governed by differential inclusions in both frameworks of Lipschitz continuous and discontinuous velocity mappings. The latter framework mostly concerns a new class of optimal control problems described by various versions of the so-called sweeping/Moreau processes that are very challenging mathematically and highly important in applications to mechanics, engineering, economics, robotics, etc. Our approach is based on developing the method of discrete approximations for optimal control problems of such differential inclusions that addresses both numerical and qualitative aspects of optimal control. In this way we derive necessary optimality conditions for optimal solutions to differential inclusions and discuss their various applications. Deriving necessary optimality conditions strongly involves advanced tools of first-order and second-order variational analysis and generalized differentiation.

ON NONLOCAL STABILIZATION PROBLEM FOR ONE HYDRODYNAMIC TYPE SYSTEM

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Let consider 3D Helmholtz system that describes evolution of the velocity vortex of viscous incompressible fluid with periodic boundary conditions for spatial variables (i.e. defined on 3D torus T^3) and with arbitrary smooth initial condition. One has to find impulse control supported in a given subdomain of $\omega \subset T^3$ that will ensure the tendency to zero of the solution's $L_2(T^3)$ - norm with increasing time. This problem is full of content because the millennium problem is not solved yet, i.e. existence in whole of smooth solutions for 3D Helmholtz system (or, what is equivalent, for 3D Navier-Stokes system) is not proved.

The quadratic operator in the Helmholtz system consists of the sum of a normal operator Φ(y)y whose image is collinear to the argument y, and the tangential operator B_τ(y), whose image is orthogonal to yin$L_2(T^3)$. At the first stage, we omit the operator B_τ(y) of the Helmholtz system with the aim to stabilize by impulse control the obtained boundary value problem. As is known, (see [1]) the solution of the boundary value problem obtained has an explicit formula that allows us to solve the stabilization problem (see [2, 3]). At the second stage, after returning the operator B_τ(y), only the first steps were made, more precisely, the stabilization problem solution was obtained, but not for the Helmholtz system. Solution of stabilization problem has been obtained only in the case of a model problem for the differentiated Burgers equation (see [4]).The main content of the report is related to the presentation of the results of the second stage.

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CONTINUOUS DEPENDENCE AND DECAY ESTIMATES FOR EULER-BERNOULLI TYPE EQUATIONS

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The talk is devoted to the Cauchy problem for the damped second order differential-operator equation in a Hilbert space of the form

$$u_{tt} + \nu A^2 u - \alpha A u + \kappa B u_t + b A^{\theta} u_t + dF(u) = f(t) \quad (1)$$

$$u(0) = u_0 \quad u_t(0) = u_1 \tag{2}$$

in a Hilbert space H with inner product $(.,.)$ and the corresponding norm $\|\ \|$.

Here $f \in L^\infty(R^+; H)$ is a given vector function and

$$\nu > 0, b > 0, d \geq 0, \alpha \in (-\infty, \frac{\nu}{\lambda_1}), \theta \in [0,2], \kappa, \mu \in R$$

are given parameters, $A: D \to H$ is a positive definite selfadjoint operator with dense domain $D \subset H$ and a compact inverse, $B: D(A^{1/2}) \to H$ is a skew-symmetric operator, that satisfies the conditions

$$(Au,u) \geq a_0 \|u\|^2, \ (Bu,u) \leq d_0 \left\|A^{1/2}u\right\|, \ \forall u \in D \ \ G(.): D \to R$$

$F(.): D \to H$ is a gradient operator with a potential $G(.): D \to R$ that satisfies some growth and monotonicity conditions.

We obtained uniform estimates for solutions of the problem (1), (2) and established continuous dependence of solutions on parameters α and b of the equation. It is shown also that when the source term tends to zero as $t \to \infty$ the corresponding solutions of the Cauchy problem for this equation tends to zero as $t \to \infty$. Applications to the initial boundary value problems for linear and nonlinear Euler-Bernoulli type equations, Boussinesq type equations, pipe equations and some related system of equations are given.

SHARP SPECTRAL STABILITY ESTIMATES FOR HIGHER ORDER ELLIPTIC operators

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We consider the eigenvalue problem for elliptic operators of order 2m, where $m \in N$, whose coeficients are real-valued Lipschitz continuous functions satisfying the symmetry condition, subject to the homogeneous Dirichlet or Neumann boundary conditions, in a bounded open set Ω. We consider open sets Ω for which the spectrum is discrete and can be represented by means of a non-decreasing sequence of non-negative eigenvalues $\lambda n[\Omega]$, n ∈N, of finite multiplicity. The aim is to obtain sharp estimates for the variation $|\lambda n[\Omega]-\lambda n[G]|$ of the eigenvalues corresponding to two open sets Ω and G via geometric charactristics of the vicinity of these sets. There is vast literature on spectral stability problems for elliptic operators. However, very little attention has been devoted to the problem of spectral stability for higher order operators and, in particular, to the problem of finding explicit qualified estimates for the variation of the eigenvalues. Moreover, most of the existing qualified estimates for second order operators were obtained under certain regularity assumptions on the boundaries.

Our analysis comprehends elliptic operators of arbitrary even order, with homogeneous Dirichlet or Neumann boundary conditions, and open sets admitting arbitrarily strong degeneration.

MULTISCALE MODELS AND MODEL LEARNING FOR FLOWS IN POROUS MEDIA

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Abstract. I will discuss a novel multi-phase upscaling technique, which uses rigorous multiscale concepts based on Constraint Energy Minimization (CEM-GMsFEM). To design an upscaled model, we modify multiscale basis functions such that the degrees of freedom have physical meanings, in particular, the averages of the solution in each continua. This allows rigorous upscaled models and account both local and non-local effects. The transmissibilities in our upscaled models are non-local and account for non-neighboring connections. To extend to nonlinear two-phase flow problems, we develop non-linear upscaling, where the pressures and saturations are interpolated in an oversampled region based on averages of these quantities. Multicontinua concepts are used to localize the problem to the oversampled regions. Our upscaled model shares some similarities with pseudo-relative permeability approach, which will be discussed. I will also discuss learning coarse-grid models. Using deep learning algorithms, the proposed model parameters are computed and the models are modified.

GRADIENT ESTIMATES FOR NONLINEAR ELLIPTIC EQUATIONS IN MORREY TYPE SPACES

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We obtain Calder_on-Zygmund type estimates in generalized Morrey spaces for nonlinear equations of p-Laplacian type. Our result is obtained under minimal regularity assumptions both on the operator and on the domain. This result allows us to study asymptotically regular operators. As a byproduct, we obtain also generalized H□older regularity of the solutions under some minimal restrictions of the weight functions.

MULTIPLICATIVE CONGRUENCES AND POINTS ON EXPONENTIAL CURVES MODULO A PRIME

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Let p be a large prime number, h be a positive integer, $h<p$ and g be a primitive root modulo p. Let also $s,\ s_1$ be integers and

$$K=\{s+1,\ s+2,\dots,s+h\}\times\{s_1+1,\ s_1+2,\dots s_1+h\}.$$

Denote by J the number of solutions to the congruence

$$y\equiv g^x \pmod p,\qquad (x,y)\in K.$$

The problem of estimating the quantity J dates back to the earlier works of Vinogradov [5], where one can find the asymptotic formula

$$J = \frac{h^2}{p} + O\left(p^{1/2}\log^2 p\right).$$

This is very strong when h is considerably larger than $p^{3/4}\log p$ since it shows that J asymptotically behaves as $h^2/p.$ On the other hand, in view of the trivial bound $J \le h$ this formula does not give information in the range $h < p^{1/2}\log^2 p.$

The quantity J has been investigated in a series of works, and in 2010 Chan and Shparlinski [3] initiated the problem of obtaining nontrivial upper bounds to J for all ranges of h. With an elegant application of sum-product estimates in prime fields, they proved that

$$J \le h^{10/11+o(1)} \quad \text{if} \quad h < p^{11/19}.$$

The result of [3] was subsequently refined by Cilleruelo and Garaev [4], and by Bourgain, Garaev, Konyagin and Shparlinski [1,2]. For instance, from [4] and [1] it is known that for any integer constant $n \ge 2$ one has the bound

$$J \le h^{1/n+o(1)} \quad \text{if} \quad h < p^{1/\left(n^2-1\right)}.$$

This result for $n \in \{2,3\}$ was established in [4], using the tools from the Dirichlet's approximation theory and properties of the generalized Pell equation. The case $n \ge 4$ corresponds to the work [1], where a connection with the geometry of numbers, the arithmetic of algebraic numbers and metric properties of polynomials has been revealed.

The ideas of [1] has been further developed in [2]. Let χ be an arbitrary subset of integers of the interval

$[s+1,\ s+h]$ with $|\chi| = \#\chi$ elements. Denote by L_n the number of solutions to the congruence

$$x_1x_2...x_n \equiv y_1y_2...y_n \equiv 0 \quad (\mathrm{mod}\ p), \quad x_i, y_j \in \chi.$$

Bourgain et al. [2] proved that

$$L_2 \le |\chi|^2 h^{o(1)} \quad \text{if} \quad \frac{h^3}{|\chi|} < p$$

and

$$L_3 \le |\chi|^3 h^{o(1)} \quad \text{if} \quad \frac{h^8}{|\chi|^4} < p.$$

As a consequence of these estimates, they showed that

$$J \le \begin{cases} h^{1/2+o(1)} & \text{if} \quad h < p^{2/5}; \\ h^{1/3+o(1)} & \text{if} \quad h < p^{3/20}. \end{cases}$$

This refines the aforementioned results of [4].

Based on the ideas of [1,2], we have the following statement.

Theorem. *Let* $\chi \subseteq [s+1, s+h]$ *be a set of integers such that*

$$\frac{h^{14}}{|\chi|^6} + \frac{h^{15}}{|\chi|^9} < p.$$

Then

$$L_4 \le |\chi|^4 h^{o(1)}.$$

Corollary. *If* $h < p^{4/51}$, *then* $J \le h^{1/4+o(1)}$.

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THE EIGENVALUE PROBLEM IN THE CASE OF COMPLEX-VALUED ROOTS OF THE CHARACTERISTIC EQUATION

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We consider the problem

$$y'' - \lambda^2 \theta^2 (x) y = 0,\ y(0) = y(1) = 0,\ x \in [0,1], \qquad (1)$$

where $\theta(x)$ - complex-valued function $\theta(x) = \theta_1(x) + i\theta_2(x)$ such that

$$\theta(x) \in C^2[0,1],\ \theta_1(x) \neq 0,\ V(\theta_1, \theta_2) =$$

$$= \theta_1'(x)\theta_2(x) - \theta_1(x)\theta_2'(x) \neq 0 \qquad (2)$$

and

$$\int_0^1 \theta_2(x) dx = 0. \qquad (3)$$

It is obviously that problems without condition (3) can be reduced to this case by replacing the complex parameter λ by μ :

$$\lambda = \mu e^{-i\alpha}, \quad \alpha = \arg \int_0^1 \theta(x)dx.$$

In addition, instead of condition $\theta_1(x) \neq 0$, one could assume that one of the functions $\theta_j(x)$ $(j = 1,2)$ does not vanish, since the case $\theta_2(x) \neq 0$ (when instead of (3) there is an assumption that the integral of $\theta_1(x)$ is equal to zero) also leads to the previous by changing the parameter $\lambda = i\mu.$

We reduce the following theorem.

Theorem. Let conditions (2) and (3) be satisfied. Then the set of eigenvalues λ_k^2 of problem (1) is countable and the asymptotic representation holds:

$$\lambda_k = \left(\int_0^1 \theta_1(x)dx \right)^{-1} k\pi i \left[1 + O(\frac{1}{k}) \right] \ (|k| \to \infty) . \qquad (4)$$

It is know that there are no results regarding the problem of the form (1) with the condition $V(\theta_1, \theta_2) \neq 0$ (which entails $\arg \theta(x) \neq const$) in the literature.

Separate special cases ($\theta(x) = x + b$ and $\theta(x) = \sqrt{x + b}$, where $b = b_1 + ib_2 -$ constants) are studied in [1] and [2]. However in [3] $\theta(x)$ is considered an analytical function, and the fulfillment of additional restrictions imposed on it can be checked only for specific functions.

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ABSTRACTS

SIMULATION OF FLUID FLOW IN CONJUGATERESERVOIR-WELL-PIPELINE SYSTEM

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There are frequent cases of connecting or withdrawing gas, liquid from existing pipelines. In this case, all changes occurring in the pipeline are transferred to the reservoir and thus the wells are transferred to another mode of operation.

The definition of transient modes of operation of wells associated with the consideration of fluid flow in the associated system of the reservoir-well-pipeline Until now, the work of the authors [1-3] has been devoted to studying the movement of fluid in the reservoir, in the pipeline separately and in some simple cases in combination.

Unsteady motion of fluid in the interfaced system of the reservoir – well pipeline remains poorly understood.

Therefore, modeling and studying the movement of fluid in the interfaced reservoir-well-pipeline system presented to both the large scientific and practical interest to which this work is devoted.

Consider the movement of fluid in the associated reservoir-well-pipeline system. At some point in time the pipeline with performance G was connected to the main pipeline at a distance

from the wellhead l_2. At the same time, we will determine what changes will occur in the well operation mode.
In the first approximation, we take the liquid homogeneous. Then the equation of the flat-radial filtration of the fluid will be:

$$\frac{\partial^2 \Delta P}{\partial r^2}+\frac{1}{r}\frac{\partial \Delta P}{\partial r}=\frac{1}{\chi}\frac{\partial \Delta P}{\partial t} \qquad r_c \le r \le R_k;\, t>0. \tag{1}$$

Initial and boundary conditions

$$\Delta P\big|_{t=0}=\frac{P_k-P_c(0)}{\ln\left(\frac{R_k}{r_c}\right)}\ln\left(\frac{R_k}{r}\right) \qquad r_c \le r \le R_k\,. \tag{2}$$

$$\Delta P\big|_{r=R_k}=0 \qquad t>0. \tag{3}$$

$$\Delta P\big|_{r=r_c}=P_k-P_c(t) \qquad t>0. \tag{4}$$

The movement of fluid in the riser pipes is [2]

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}-2a\frac{\partial u}{\partial t} \tag{5}$$

$$u=u_e+u_r \tag{6}$$

Then substituting expression (6) into equation (5), we get:

$$\frac{\partial^2 u_e}{\partial t^2}+\frac{\partial^2 u_r}{\partial t^2}=c^2\frac{\partial^2 u_r}{\partial x^2}-2a\left(\frac{\partial u_e}{\partial t}+\frac{\partial u_r}{\partial t}\right) \tag{7}$$

Since equation (7) is linear, it splits into two equations

$$\frac{\partial^2 u_e}{\partial t^2}+2a\frac{\partial u_e}{\partial t}=\frac{\dot{P}_c-\dot{P}_y}{\rho l} \tag{8}$$

$$\frac{\partial^2 u_r}{\partial t^2} = c^2 \frac{\partial^2 u_r}{\partial x^2} - 2a \frac{\partial u_r}{\partial t} + \frac{\dot{P}_y - \dot{P}_c}{\rho l} \tag{9}$$

Initial and boundary conditions

$$u_e\big|_{t=0} = \frac{G_0}{f} \tag{10}$$

$$\left.\frac{du_e}{dt}\right|_{t=0} = 0 \tag{11}$$

$$u_r\big|_{t=0} = 0, \qquad \left.\frac{\partial u_r}{\partial t}\right|_{x=0} = 0 \tag{12}$$

$$\left.\frac{\partial u_r}{\partial x}\right|_{x=l} = 0 \tag{13}$$

$$u_r\big|_{x=0} = 0 \tag{14}$$

$$\left.\frac{\partial \varphi}{\partial t}\right|_{x=0} = 0 \tag{15}$$

$$fu\big|_{x=0} = -\frac{2\pi k}{\mu} r_c h \left.\frac{\partial \Delta P}{\partial r}\right|_{r=r_c} \tag{16}$$

The movement of fluid in the pipeline is

$$\frac{\partial^2 P}{\partial t^2} = c^2 \frac{\partial^2 P}{\partial x^2} - 2a_1 \frac{\partial P}{\partial t} - \frac{2a_1 c^2 G}{f_1} \delta(x - l_2) \tag{17}$$

Initial and boundary conditions

$$\left.\frac{\partial P}{\partial t}\right|_{t=0} = -c^2 \frac{G}{f} \delta(x - l_2) \tag{18}$$

$$P(x,0)\big|_{t=0} = P_{yc}(0) - 2a_1 Q_0 x \tag{19}$$

$$P\big|_{x=0} = P_{yc}(t) \tag{20}$$

$$P\big|_{x=l_1} = P_2 = const \tag{21}$$

The definitions of a change in the mode of operation of a well can be determined by jointly solving the related equations (1), (5), (17) and this is done in representative work.

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ON THE RATE OF COMPONENTVISE EQUICONVERGENCE FOR A THIRD-ORDER DIFFERENTIAL OPERATOR WITH MATRIX COEFFICIENTS

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On the interval $G=(0,1)$ we consider the differential operator $L\psi=\psi^{(3)}+U_2(x)\psi^{(1)}+U_3(x)\psi$ with matrix coefficients $U_l(x)=\{u_{lij}(x)\}_{i,j=1}^{m}$, $l=2,3;$ $u_{lij}(x)\in L_1(G)$.

Let the system $\{\psi_k(x)\}_{k=1}^{\infty}$ $\psi_k(x)=\left(\psi_{k1}(x),\psi_{k2}(x),...,\psi_{k,m}(x)\right)^T$

satisfy the conditions A_p [1].

For an arbitrary vector-function $f(x) \in L_p^m(G)$ for the fixed $p \geq 1$, as in the co0nditions A_p, we constitute a partial sum of order ν of biorthogonal expansion in the system $\{\psi_k(x)\}_{k=1}^{\infty}$:

$$\sigma_\nu(x, f) = \sum_{\rho_k \leq \nu} (f, \varphi_k)\psi_k(x), \quad \nu > 0,$$

$$\varphi_k(x) = (\varphi_{k1}(x), \varphi_{k2}(x), \ldots, \varphi_{km}(x))^T;$$

$$\sigma_\nu(x, f) = \left(\sigma_\nu^1(x, f), \sigma_\nu^2(x, f), \ldots, \sigma_\nu^m(x, f)\right)^T$$

Denote by $S_\nu(x, f_j)$ a partial sum of the ν – th order of the trigonometric Fourier series of the function $f_j(x),\ j = \overline{1, m}$.

Let us introduce some denotation that we will need in the sequel:

$$\Delta_\nu^j(f, K) = \left\| \sigma_\nu^j(\cdot, f) - S_\nu(\cdot, f_j) \right\|_{C(K)}, \quad j = \overline{1, m}$$

$$\hat{f}_k = f_k \|\varphi_k\|_{q,m}^{-1} = (f, \varphi_k)\|\varphi_k\|_{q,m}^{-1};$$

$$\psi(f, \nu/2, \gamma) = \nu^{-1} \sum_{1 \leq \rho_k \leq \nu/2} \rho_k^{-\gamma} \left| \hat{f}_k \right|, \quad \gamma \geq 0,$$

$$\Phi_p(f, \nu) = \nu^{-1} \|f\|_{p,m} + \max_{\rho_k \geq \nu/2} \left| \hat{f}_k \right|; Q_p(f_j, \nu) = \nu^{-1} \|f_j\|_p + \max_{2\pi k \geq \nu/2} \left| \tilde{f}_{jk} \right|,$$

where $\tilde{f}_{jk}$-are the Fourier coefficient of the function $f_j(x)$ in the following trigonometric system normalized in $L_q(G)$;

$$D(\nu, U_2) = \inf_{\substack{\alpha > 1 \\ n \geq 2}} \left\{ \Omega_{1j}\left(U_2, n^{-1}\right)\psi(f, \nu/2, 0) + \right.$$

$$\left. + n^{2\left(1-\alpha^{-1}\right)} \|U_2\|_{1j}\, \psi\left(f, \nu/2, 1-\alpha^{-1}\right) \right\},$$

where $\Omega_{1j}(U_2, \delta) = \max\limits_{1 \leq l \leq m} \omega_1(u_{2jl}, \delta)$, $\|U_2\|_{rj} = \max\limits_{1 \leq l \leq m} \|u_{2jl}\|_{L_r(G)}$;

$$T(f,\nu,r)=\psi\left(f,\nu/2,1-r^{-1}\right)+\Phi_p(f,\nu),$$
$$T(f,\nu,\infty)=\psi\left(f,\nu/2,1\right)+\Phi_p(f,\nu)$$

Theorem. Let all elemenys of the j*-th row of the matrix* $U_2(x)$ *belong to* $L_r(G),\ r\geq 1;\ U_3(x)\in L_1(G)\otimes C^m$ *and the system* $\{\psi_k(x)\}_{k=1}^{\infty}$ *satisfy the conditions* A_p *for some fixed* $p\geq 1$. *Then the* j*-th component of biorthogonal expansion of any vector-function* $f(x)\in L_p^m(G)$ *uniformly eguicanvergers on any compact* $K\subset G$ *with expansion in trigonometric Fourier series corresponding to the* j*-th component* $f_j(x)$ *of the vector-function* $f(x)$ *and the following estimations are valid :*

$$\Delta_\nu^j(f,K)\leq C(K)\left\{\|U_2\|_{rj}T\left(f,\nu,r\right)+\|U_3\|_{1j}T(f,\nu,\infty)+\right.$$
$$\left.+\Phi_p(f,\nu)+Q_p(f_j,\nu)\right\}\ \textit{for}\ \ r>1;$$
$$\Delta_\nu^j(f,K)\leq C(K)\left\{D\left(\nu,U_2\right)+\|U_3\|_{1j}T(f,\nu,\infty)+\right.$$
$$\left.+\Phi_p(f,\nu)+Q_p(f_j,\nu)\right\}\ \textit{npu}\ \ r=1;$$

where $C(K)$ *-is a constant independents of* $f(x)$ *and* ν.

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INTEGRATION OF A CERTAIN CLASS OF SECOND-ORDER QUASILINEAR DIFFERENTIAL-OPERATOR EQUATION

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Consider the Cauchy problem for second-order differential operator equations on Banach space E

$$A(t)u''(t)= f(t,\ B(t)u'(t),\ C(t)u(t))\ (t_0 \le t \le T), \tag{1}$$

$$u(t_0)=u_0,\quad u'(t_0)=u, \tag{2}$$

where, $-\infty < t_0 < T < \infty$, $A(t), B(t), C(t)$ are the linear operators acting in E, $f(t,u,v)$ is a function with values from E and $u_0,\ u_1 \in E$ are the given elements.

By solution of the problem (1)-(2) we mean twice continuously differentiable function $u(t)$, which the functions $A(t)u''(t), B(t)u'(t), C(t)u(t), f(t, B(t)u'(t), C(t)u(t))$ exist, continuous and are satisfied by relations (1), (2).

The main conditions for operators are follows:

1^0. $A(t), B(t), C(t)$ - linear operators acting in a Banach space E with everywhere dense domains $D(A), D(B), D(C)$, such that $D(A)\subset D(B)$, $D(A)\subset D(C)$ and for any $t\in[t_0,T]$ the operator $A(t)$ have bounded inverse $A^{-1}(t)$, also operator-functions

$$A(t)A^{-1}(s),\ B(t)A^{-1}(s), C(t)A^{-1}(s)$$

are strongly continuous with respect to variables $t,\ s\ (t_0 \le t, s \le T)$.

2^0 . The function $f(t,u_1,u_2)$ is continuous with respect to aggregate variables on the $[t_0,T]\times E\times E$ and on the every ball $S_R=\{\|u\|\le R\}$ satisfied Lipshits condition

$\|f(t,\ u_1,u_2)-f(t,v_1,v_2)\|\le C(R)(\|u_1-v_1\|+\|u_2-v_2\|)$ for all $u_i,v_i\in S_R\ \ (i=1,2)$, where $C(R)$ is a positive constant depending of the radius R.

Теорема 1. Let the conditions $1^0, 2^0,\ u_0,\ u_1\in D(A)$ be satisfied. Then for any $R>0$ there exists $\tau>0$ such that the problem (1) –(2) have only one solution $v(t)$ which is continuous on interval $[t_0,\ t_0+\tau]$ with a norm $\|v\|_{C_{t_0+\tau}}\le R$, such that is a limit some sequences of approximations with respect to $C_{t_0+\tau}$ norm.

To establish the solvable equation problems (1)-(2) on the a whole interval $[t_0, T]$, for the operator $f(t,\ v_1,v_2)$ put on additionally conditions:

3^0. $\|f(t,\ v_1,v_2)\|\le c(1+\|v_1\|+\|v_2\|)$ for all $t\in[t_0,T,],\ v_1,\ v_2\in E$.

The last condition shows that the nonlinear part of equation (1) can have only linear growth. Note that in this case, too, the low results are new.

Теорема 2. Let the conditions $1^0-3^0, u_0,\ u_1\in D(A)$ be satisfied. Then problem (1)-(2) has a unique solution on $[t_0,\ T]$.

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ON THE BOUNDEDNESS OF AN n-FOLD SINGULAR INTEGRAL OPERATOR IN GENERALIZED HÖLDER SPACES

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We denote by C_{T^n} the space of continuous 2π -periodic in each of the variables functions on $T^n=[-\pi,\pi]^n$ with the max-norm.

Consider a n-fold singular integral

$$\tilde{f}(x)=\frac{(-1)^n}{(2\pi)^n}\int\limits_{T^n} f(s)\prod_{i=1}^{n}\cot\frac{s_i-x_i}{2}\,ds_i. \qquad (1)$$

Suppose that for the vector $i=(i_1,i_2,\ldots,i_n)$ any coordinate $i_k, k=\overline{1,n},$ is either zero or one. For $f\in C_{T^n}$, a mixed modulus of continuity $\omega_f^{i_1,i_2,\ldots,i_n}(i_1\delta_1,i_2\delta_2,\ldots,i_n\delta_n)$ of the function f is introduced by the totality of those variables for which $i_k\neq 0$.

We introduced an analogue of the generalized Hölder space by using a mixed modulus of continuity and proved the boundedness of the operator (1) in this space.

ON THE UNIFORM CONVERGENCE OF BIEBERBACH POLYNOMIALS IN REGIONS WITH FINITE NUMBER ZERO ANGLES

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Let $\mathbb{C}$ be a complex plane, $\overline{\mathbb{C}}=\mathbb{C}\cup\{\infty\}$ and let $G\subset\mathbb{C}$ be a finite Jordan region with $0\in G$; $L:=\partial G,\ \Omega:=\overline{\mathbb{C}}\setminus G$; $w=\varphi(z)$ be the conformal mapping of G onto the disk $B(0,\rho_0):=\{w:|w|<\rho_0\}$ normalized by $\varphi(0)=0,\ \varphi'(0)=1$; where $\rho_0=\rho_0(0,G)$ is a conformal radius of G with respect to 0. For $p>0$, we denote:

$$A_p^1(G):=\left\{\begin{array}{l} f:\text{analytic in } G,\ f(0)=0,\ f'(0)=1 \text{ and} \\ \|f\|_{A_p}:=\|f\|_{A_p^1}:=\left(\iint\limits_G |f'(z)|^p\, d\sigma_z\right)^{1/p}<\infty \end{array}\right\}$$

where σ denotes two dimensional Lebesque measure.

Let us denote by $\wp_n$ the class of all polynomials $P_n(z),\ degP_n(z)\le n,$ satisfying the conditions: $P_n(0)=0,\ P_n'(0)=1.$ We consider the following extremal problem:

$$\{\|P_n\|_{A_2},\ P_n\in\wp_n\}\to inf. \qquad (1)$$

It is well known that the solution of the extremal problem (1) is called the n-th Bieberbach polynomial for the pair $(G,0)$ (see for more literature, for example, [1]) and denoted by $\pi_n(z,G,0)=:\pi_n(z)$. On the other hand, the solution of the extremal problem (1) in the class $A_2^1(G)$ is achieved by Riemann function $\varphi(z)$ and $\|\varphi\|_{A_2}=\pi\rho_0{}^2$. Also we can be take $\pi_n(z)-$ Bieberbach polynomials as the solutions of the following (equivalent to (1)) extremal problem:

$$\{\|\varphi-P_n\|_{A_2},\ P_n\in\wp_n\}\to inf.$$

Therefore $inf\{\|\varphi-P_n\|_{A_2},\ P_n\in\wp_n\}=\|\varphi-\pi_n\|_{A_2}$.

Let us set: $E_n(\varphi,\overline{G}):=inf\{\|\varphi-P_n\|_{C(\overline{G})},\ P_n\in\wp_n\}$.

In the present work, we will continue of investigate the estimate $E_n(\varphi,\overline{G})=O(\varepsilon_n)$ in the regions with finite number interior and exterior zero angles of complex plane, where $\varepsilon_n=\varepsilon_n(\varphi,G)\to 0,\ n\to\infty,$ depending on the geometric properties of the given region.

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SOME ESTIMATES FOR THE GENERALIZED OPERATORS OF LAPLACE-BESSEL HARMONIC ANALYSIS

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Let $R^{+}_{m+k,k}=\{x=(x_1,...,x_m,x_{m+1},...,x_{m+k}):x_{m+1}>0,...,x_{m+k}>0\}$,

T^{s} - is generalized shift operator, generalized by the Laplace-Bessel differential operator [1,2]:

$$\Delta_B=\sum_{i=1}^{m}\frac{\partial^2}{\partial x_i^2}+\sum_{i=m+1}^{m+k}\left[\frac{\partial^2}{\partial x_i^2}+\frac{2v_i}{x_i}\frac{\partial}{\partial x_i}\right],$$

$$v_{m+1}>0,...,v_{m+k}>0,\ 1\le p<\infty,$$

$$d\mu(x)=\prod_{i=1}^{m+k}d\mu(x_i),d\mu(x_i)=\begin{cases}dx_i, & i\in\{1,...,m\};\\ x_i^{2v_i}dx_i, & i\in\{m+1,...,m+k\},\end{cases}$$

$$L_{p,v}\overset{\mathrm{df}}{=}\left(u-\text{measurable.}:\|u:L_{p,v}\|\overset{\mathrm{df}}{=}\left(\int_{R^{+}_{m+k,k}}|u(x)|^{p}d\mu(x)\right)^{\frac{1}{p}}<+\infty\right).$$

Let $1<p\le q<\infty$. By definition a sublinear operator A belongs to $\overline{K}_v(p,q)$ class [2], if $A:L_{p,v}\to L_{q,v}$ bounded and for any function $u\in L_{p,v}$ with a compact support $|Au(x)|\le c\int_{R^{+}_{m+k,k}}|s|^{-\beta}T^{s}|u(x)|d\mu(s)$ for $x\in\operatorname{supp}u$, where $\beta=(m+k+2|v|-\alpha)$.

The following characterization is introduced

$$\Omega^*_{p,i}(\upsilon,\xi)=\left\{\int_{\{R^+_{m+k,k}:|x_i|\le\xi\}}|\upsilon(x)|^p\,d\mu(x)\right\}^{\frac{1}{p}},\quad \xi>0,\quad i=\overline{1,m+k}\ .$$

Theorem. Let $1<p\le q<\infty$, $A\in\overline{K}_\nu(p,q)$, $a_i=0$ for $i\in\{1,\ldots,m\}$, $a_i=2\nu_i$ for $i\in\{m+1,\ldots,m+k\}$ and let $\int_\xi^\infty \Omega^*_{p,i}(\upsilon,t)\cdot t^{-\left(\frac{1+a_i}{q}+1\right)}dt$ is convergent. Then for almost every $x\in R^+_{m+k,k}$ there exists $\upsilon(x)=Au(x)$ and the following estimation holds:

$$\Omega^*_{q,i}(\upsilon,\xi)\le c\cdot\xi^{\frac{1+a_i}{q}}\int_\xi^\infty \Omega^*_{p,i}(\upsilon,t)\cdot t^{-\left(\frac{1+a_i}{q}+1\right)}dt,\quad \xi>0$$

where $i=\overline{1,m+k}$, the constant c is not dependent of u and ξ.

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ON APPLICATION OF METHOD OF LINES IN CONTROLLING LOADED SYSTEMS

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The following optimal control problem for objects described by loaded parabolic equations

$$u_t(x,t) - u_{xx}(x,t) - \sum_{s=1}^{k} B^s(x,t)u(x_s,t) = v(x,t),$$

$$(x,t) \in \Omega = \{0 < x < l,\ 0 < t \le T\}, \tag{1}$$

at initial conditions

$$u(x,0) = \varphi(x)\ , \quad 0 \le x \le l\ , \tag{2}$$

and non-local boundary conditions

$$\begin{aligned} &\alpha_{11}(t)u(0,t) + \alpha_{12}(t)u_x(0,t) + \\ &\quad + \gamma_{11}(t)u(l,t) + \gamma_{12}(t)u_x(l,t) = \xi_1(t), \quad 0 \le t \le T, \end{aligned} \tag{3}$$

$$\begin{aligned} &\alpha_{21}(t)u(0,t) + \alpha_{22}(t)u_x(0,t) + \\ &\quad + \gamma_{21}(t)u(l,t) + \gamma_{22}(t)u_x(l,t) = \xi_2(t), \quad 0 \le t \le T, \end{aligned} \tag{4}$$

is considered. Here t and x are correspondingly time and space coordinates; T and l are given positive quantities; $u(x,t)$ is temperature at point x of the considered environment at point t; $x_s \in (0,l), s = 1,2,...k$ are given loading points; $B^s(x,t)$, $\alpha_{ij}(t), \gamma_{ij}(t)$, $i = 1,2,\ j = 1,2$, $\xi_1(t)$, $\xi_2(t)$ are given continuous functions; $\varphi(x) \in L_2[0,l]$ is the given function, defining the initial state; control function

$$v = v(x,t) \in V = \{v = v(x,t) \in L_2(\Omega);\ \|v\|_{L_2(\Omega)} \le R\} \qquad (5)$$

The problem at conditions (1)-(5) lies in seeking control $v \in V$ at which the given functional

$$J(v) = \int_0^l [u(x,T,v) - U(x)]^2 dx + \alpha \|v\|_{L_2(\Omega)},\ \alpha \ge 0 \qquad (6)$$

takes on a minimal value, here $U(x)$ is the given function, $u(x,T;v)$ is the solution to boundary problem (1)-(4).

Necessary optimality conditions for the considered problem, which are used for building numerical solution later on, were obtained.

In order to solve problem (1)-(6) numerically with the application of standard procedures of first-order optimization, at each iteration of the iterative procedure it is necessary to determine the gradient of the functional. With this purpose at the current control it is necessary to solve the straight loaded differential equations problem and the conjugate integral and differential equations problem with non-local conditions. In order to solve the straight and conjugate problems, the schemes developed by the author of the proposed approach to numerical solving on the base of method of lines [1] and method of conditions shift, proposed in work [2]. Then standard procedures of numerical first-order optimization methods are used.

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ON CONVERGENCE OF BERNSTEIN-CHLODOWSKY-KANTOROVICH OPERATORS SEQUENCE IN VARIABLE LEBESGUE SPACES

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Let f be a continuous real function defined on $[0,\infty)$. In 1932, Bernstein's follower Chlodowsky had constructed the increasing sequences of polynomials

$$B_n(f;x)=\sum_{k=0}^{n}p_{n,k}(x)f\left(\frac{kb_n}{n}\right),\ 0\le x\le b_n,$$

that was named as Bernstein-Chlodowsky polynomials some later.

Here

$$p_{n,k}(x)=C_n^k\left(\frac{x}{b_n}\right)^k\left(1-\frac{x}{b_n}\right)^{n-k}$$

and

$$\lim_{n\to\infty}b_n=\infty,\qquad \lim_{n\to\infty}\frac{b_n}{n}=0.$$

Note that, the condition imposed on b_n in the definition of classic Bernstein-Chlodovsky operators doesn't provide uniformly convergence of $B_n(f;x)$ polynomials to the function $f(x)$ on $[0,\infty)$. This fact it is valid on each finite segment $[0,A]$ (see, [1]).

We put

$$\Delta_{n,k}=\left[\frac{kb_n}{n+1},\frac{(k+1)b_n}{n+1}\right].$$

Let $f \in L_1^{loc}(0,\infty)$ and we denote

$$A_n(f;x) = \frac{n+1}{b_n} \sum_{k=0}^{n} p_{n,k}(x) \int_{\Delta_{n,k}} f(t)dt .$$

In this abstract we reduce the folowing problem :
Under some condintions on variable exponent function the convergence to zero of the difference

$$A_n(f;x) - f(x)$$

in variable Lebesgue spaces norm is investigated. (see , [2])

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INTEGRAL REPRESENTATION OF SOLUTION OF GEOMETRIC MIDDLE BOUNDARY PROBLEM IN THE NON-CLASSICAL TREATMENT FOR ONE 3D BIANCHI INTEGRO DIFFERENTIAL EQUATION

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Problem statement. Consider 3D (three dimensional) Bianchi integro-differential equation:

$$(V_{1,1,1}u)(x,y,z) \equiv \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} A_{i,j,k}(x,y,z) D_x^i D_y^j D_z^k u(x,y,z)$$

$$+\int_{\sqrt{x_0x_1}}^{x}\int_{\sqrt{y_0y_1}}^{y}\int_{\sqrt{z_0z_1}}^{z}\Big[K_{0,0,0}(\tau,\xi,\eta;x,y,z)u(\tau,\xi,\eta)$$
$$+K_{1,0,0}(\tau,\xi,\eta;x,y,z)u_x(\tau,\xi,\eta)$$
$$+K_{0,1,0}(\tau,\xi,\eta;x,y,z)u_y(\tau,\xi,\eta)+K_{0,0,1}(\tau,\xi,\eta;x,y,z)u_z(\tau,\xi,\eta)$$
$$+K_{1,1,0}(\tau,\xi,\eta;x,y,z)u_{xy}(\tau,\xi,\eta+K_{0,1,1}(\tau,\xi,\eta;x,y,z)u_{yz}(\tau,\xi,\eta)$$
$$+K_{1,0,1}(\tau,\xi,\eta;x,y,z)u_{xz}(\tau,\xi,\eta)\Big]d\tau d\xi d\eta$$
$$=\varphi_{1,1,1}(x,y,z),\ \ (A_{1,1,1}(x,y,z)\equiv 1),(x,y,z)\in G, \tag{1}$$

with geometric middle boundary conditions in the non-classical form

$$\begin{cases} V_{0,0,0}u\equiv u(\sqrt{x_0x_1},\sqrt{y_0y_1},\sqrt{z_0z_1})=\varphi_{0,0,0}, \\ (V_{1,0,0}u)(x)\equiv u_x(x,\sqrt{y_0y_1},\sqrt{z_0z_1})=\varphi_{1,0,0}(x), \\ (V_{0,1,0}u)(y)\equiv u_y(\sqrt{x_0x_1},y,\sqrt{z_0z_1})=\varphi_{0,1,0}(y), \\ (V_{0,0,1}u)(z)\equiv u_z(\sqrt{x_0x_1},\sqrt{y_0y_1},z)=\varphi_{0,0,1}(z), \\ (V_{1,1,0}u)(x,y)\equiv u_{xy}(x,y,\sqrt{z_0z_1})=\varphi_{1,1,0}(x,y), \\ (V_{0,1,1}u)(y,z)\equiv u_{yz}(\sqrt{x_0x_1},y,z)=\varphi_{0,1,1}(y,z), \\ (V_{1,0,1}u)(x,z)\equiv u_{xz}(x,\sqrt{y_0y_1},z)=\varphi_{1,0,1}(x,z), \end{cases} \tag{2}$$

Here $u(x,y,z)$ is a desired function determined on G; $A_{i,j,k}(x,y,z)$ are the given measurable functions on $G=G_1\times G_2\times G_3$, $K_{i,j,k}(\tau,\xi,\eta;x,y,z)\in L_\infty(G\times G)$, where $G_1=(x_0,x_1)$, $G_2=(y_0,y_1)$, $G_3=(z_0,z_1)$; $\varphi_{i,j,k}$ is a given measurable functions on G; In addition, it is assumed that $x_0\geq 0$, $y_0\geq 0$, $z_0\geq 0$.

The 3D Bianchi equation in various points of view was studied in the papers [1-5] and etc.

In the present work integro-differential equation 3D Bianchi (1) is considered in the general case when the coefficients $A_{i,j,k}(x,y,z)$ are non-smooth functions satisfying only the following conditions:

$$A_{0,0,0}(x,y,z)\in L_p(G), A_{1,0,0}(x,y,z)$$
$$\in L^{x,y,z}_{\infty,p,p}(G), A_{0,1,0}(x,y,z)\in L^{x,y,z}_{p,\infty,p}(G),$$
$$A_{0,0,1}(x,y,z)\in L^{x,y,z}_{p,p,\infty}(G), A_{1,1,0}(x,y,z)$$
$$\in L^{x,y,z}_{\infty,\infty,p}(G), A_{0,1,1}(x,y,z)\in L^{x,y,z}_{p,\infty,\infty}(G),$$
$$A_{1,0,1}(x,y,z)\in L^{x,y,z}_{\infty,p,\infty}(G).$$

Under these conditions, we'll look for the solution $u(x,y,z)$ of integro-differential equation 3D Bianchi (1) in S.L.Sobolev isotropic space $W_p^{(1,1,1)}(G)\equiv\{u\in L_p(G)/D_x^iD_y^jD_z^ku\in L_p(G);$ $i,\ j,k=0,1\}$, where $1\le p\le\infty$. We'll define the norm in the space $W_p^{(1,1,1)}(G)$ by the equality

$$\|u\|_{W_p^{(1,1,1)}(G)}=\sum_{i,j,k=0}^{1}\left\|D_x^iD_y^jD_z^ku\right\|_{L_p(G)}.$$

In this work the integral representation of solution of geometric middle boundary problem (1)-(2) for integro-differential equation of type 3D Bianchi (1) with nonsmooth coefficients is constructed.It should be particularly noted that, in particular, conditions (2) coincide with Goursat conditions for the 3D Bianchi integro-differential equation (1). In this sense, geometric middle boundary problem (1)-(2) by statement generalizes the Goursat problem for 3D Bianchi integro-differential equation (1).

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ON EQUICONVERGENCE RATE FOR THE DIRAC OPERATOR

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On the interval $G=(0,2\pi)$ we consider the one-dimensional Dirac operator

$$Dy = By' + P(x)y, \qquad y(x) = (y_1(x), y_2(x))^T,$$

where $B=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$, $P(x)=\begin{pmatrix}p(x) & 0\\ 0 & q(x)\end{pmatrix}$, $p(x)$ and $q(x)$ are real-valued functions from the class $L_\alpha(G),\ \alpha \ge 1$.

Following V.A. Il'in [1,2], under the eigen vector-function of the operator D corresponding to the real eigen-value λ we will understand any identically non-zero vector-function $y(x)$ that is absolutely continuous in $\overline{G}=[0,2\pi]$ and always every where in G satisfies the equation $Dy=\lambda y.$

Let $L_p^2(G),\ p\geq 1$ be a space of two-component vector-functions $f(x)=\left(f_1(x),f_2(x)\right)^T$ with the norm

$$\|f\|_{p,2,G}=\|f\|_{p,2}=\left(\int_G |f(x)|^p\,dx\right)^{1/p},\ \left(\|f\|_{\infty,2}=\sup_{x\in\overline{G}} vrai|f(x)|\right).$$

Obviously, for $f(x)\in L_p^2(G),\ g(x)\in L_q^2(G),\ p^{-1}+q^{-1}=1,$ $p\geq 1,$ there exists a « scalar product »

$$(f,g)=\int_G \langle f(x),g(x)\rangle dx=\int_G\sum_{j=1}^{2} f_j(x)\overline{g_j(x)}dx.$$

Let $\{u_n(x)\}_{n=1}^{\infty}$ be a complete orthonormend in $L_2^2(G)$ system of eigen vector- functions of the operator D and $\{\lambda_n\}_{n=1}^{\infty},\ \lambda_n\in R$ corresponding system of eigen-values.

By $W_{p,2}^1(G),\ p\geq 1$ we denote a space of absolutely continuous on $\overline{G}$ two-component vector-functions $f(x)$ for which $f'(x)\in L_p^2(G).$

We introduce partial sum of spectral expansion of the vector-function $f(x)\in W_{1,2}^1(G)$ in the system $\{u_n(x)\}_{n=1}^{\infty}$:

$$\sigma_\nu(x,f)=\left(\sigma_\nu^1(x,f),\sigma_\nu^2(x,f)\right)^T,\quad \sigma_\nu^j(x,f)=\sum_{|\lambda_n|\leq\nu}(f,u_n)u_n^j(x),\ j=1,2;$$

$$\left(u_n^1(x),u_n^2(x)\right)^T=u_n(x),\quad f(x)=\left(f_1(x),f_2(x)\right)^T.$$

Alongside with the partial sum $\sigma_\nu^j(x, f)$ we introduce a modified partial sum of trigonometric Fourier series of the function $f_j(x)$, i.e.

$$S_\nu(x, f_j) = \frac{1}{\pi}\int_G \frac{\sin\nu(x-y)}{x-y} f_j(y)dy, \quad j = 1,2;$$

$$S_\nu(x, f) = \left(S_\nu(x, f_1),\ S_\nu(x, f_2)\right)^T.$$

Theorem. Let $f(x) \in W_{1,2}^1(G),$ the coefficients $p(x)$ and $q(x)$ belong to $L_\alpha(G), \alpha > 1$. Then the *j*-th component of spectral expansion of the vector-function $f(x)$ in the system $\{u_n(x)\}_{n=1}^\infty$ uniformly equiconverges on any compact $K \subset G$ with expansion in trigonometric Fourier series corresponding to the j-th component $f_j(x)$ of the vector-function $f(x)$, and the following estimation

$$\left\|\sigma_\nu^j(\cdot, f) - S_\nu(\cdot, f_j)\right\|_{C(K)} = \begin{cases} O\left(\nu^{1/\alpha-1}\ln\nu\right) & for\ \alpha \in (1, \infty), \\ O\left(\nu^{-1}\ln^2\nu\right) & for\ \alpha = \infty, \end{cases}$$

is true as.

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ASYMPTOTIC FORMULAS FOR EIGENVALUES AND EIGENFUNCTIONS OF A FOURTH-ORDER EIGENVALUE PROBLEM WITH A SPECTRAL PARAMETER IN THE BOUNDARY CONDITION

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We consider the following boundary value problem

$$y^{(4)}(x) - (q(x)y'(x))' = \lambda\, y(x),\ 0 < x < 1, \tag{1}$$

$$y(0) = y'(0) = y''(1) = 0, \tag{2}$$

$$(a\lambda + b)y(1) - (c\lambda + d)Ty(1) = 0, \tag{3}$$

where $\lambda \in C$ is a spectral parameter, $Ty \equiv y''' - qy'$, $q(x)$ is a positive absolutely continuous function on the interval $[0, 1]$, a, b, c and d are real constants such that $bc - ad \neq 0$ and $c \neq 0$.

In this note, are refined asymptotic formulas for eigenvalues and eigenfunctions of problem (1)-(3) that obtained in papers [1, 2].

Theorem. *For eigenvalues* λ_k *and corresponding eigenfunctions* $y_k(x)$ *of problem (1)-(3) one has the following asymptotic formulas:*

$$\lambda_k = (k - 3/2)\,\pi + q_0/4k\pi + O\left(\frac{1}{k^2}\right),$$

$$y_k(x) = \left(1 + \frac{q_0 x - q_0(x)}{4k\pi}\right) \sin\left(k - \frac{3}{2}\right)\pi x - \left(1 - \frac{q_0 x - q_0(x)}{4k\pi}\right) \times$$

$$\times\cos\left(k-\frac{3}{2}\right)\pi x+\left(1-\frac{q_0 x+q_0(x)}{4k\pi}\right)e^{-\left(k-\frac{3}{2}\right)\pi x}-$$

$$-(-1)^k\left(1+\frac{q_0(x-1)-q_1(x)}{4k\pi}\right)e^{-\left(k-\frac{3}{2}\right)\pi x}+O\left(\frac{1}{k^2}\right),$$

where $q_0=\int_0^1 q(x)dx,\ q_0(x)=\int_0^x q(t)dt,\ q_1(x)=\int_x^1 q(t)dt$ *and the second relation holds uniformly for* $x\in[0,1]$.

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DISCHARGE WAVE IN FLAT NET

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Summary. Based on net motion equations, flat mesh motion equations are generally constructed. Wave propagation options are determined in case of net base made of elastic fibres. The problem of propagation of unloading waves in a pre-stretched net has been solved. The task is solved by the method of characteristics. The task is illustrated by calculations.

Keywords: net, wave, deformation, radius, speed

Articles [2-5] examined waves in nets in a rectangular cartesian coordinate system. Waves in the flat coordinate system are examined here. Obviously, when stretched, the flat net will taper; placed on a rigid pipe it will be subjected to friction forces between it and the pipe when moving.

The purpose of the work is to study waves in flat nets. Given the many wave propagation options in flat nets, an attempt is made to solve the problem of continuous waves.

1. Basic equations.

A flat net is considered. The equation of one-dimensional motion is given in [1], where the automatic motion of the floor of the infinite net is investigated:

$$\begin{aligned}&\frac{\partial}{\partial z}(\sigma_1 \cos\gamma_1)\sin\alpha + \frac{\partial}{\partial z}(\sigma_2 \sin\gamma_2)\cos\alpha = 2\rho\frac{\partial^2 x}{\partial t^2}\\ &\frac{\partial}{\partial z}(\sigma_1 \sin\gamma_1)\sin\alpha + \frac{\partial}{\partial z}(\sigma_2 \cos\gamma_2)\cos\alpha = 2\rho\frac{\partial^2 y}{\partial t^2}\end{aligned} \qquad (1)$$

$$\begin{aligned}&1+\frac{\partial x}{\partial z}\sin\alpha = (1+e_1)\cos\gamma_1; \quad \frac{\partial y}{\partial z}\sin\alpha = (1+e_1)\sin\gamma_1;\\ &\frac{\partial x}{\partial z}\cos\alpha = (1+e_2)\sin\gamma_2; \quad 1+\frac{\partial y}{\partial z}\cos\alpha = (1+e_2)\cos\gamma_2;\end{aligned} \qquad (2)$$

where σ_1, σ_2 - conditional stresses are defined as sum of tension of individual threads of one family crossing section of threads of another family, related to initial length of the element under consideration. e_1, e_2 - relative elongations, ρ - mass of net per unit area in initial compression, x, y- coordinate particles of net, t - time, γ_1, γ_2 -angles of thread rotation of corresponding families.

$$z = s_1 \sin\alpha + s_2 \cos\alpha$$

where s_1, s_2 - Lagrangian coordinates of thread particles, counted from selected threads, α is angle of inclination of one of families to straight reference.

The following is the diagonal movement of the net: i.e. $\alpha = \pi/4$.

Then equations (1) take the form:

$$\frac{\partial}{\partial z}\left[\sigma(\cos\gamma + \sin\gamma)\right] = 2\sqrt{2}\rho\frac{\partial^2 x}{\partial t^2} ...... (3)$$

From (2) follows

$$1+\frac{\partial x}{\partial z}\frac{\sqrt{2}}{2} = (1+e)\cos\gamma; \quad \frac{\partial x}{\partial z}\frac{\sqrt{2}}{2} = (1+e)\sin\gamma; \quad (4)$$

If the edge of the net is stretched, a severe rupture will propagate in the plane of the net, since the velocity of the wave will obviously increase with stretching. If the net is pre-stretched and then unloaded through the net, continuous waves will be found.

2. Net stretching.

Let the net from the edge be considered. D -the speed of propagation of a strong fracture, υ - the speed of particles of the net.

According to the law of mass preservation

$$D\rho_0 = (D+\upsilon)\rho \tag{5}$$

and qualities of the movement

$$\vartheta\rho(D+\upsilon) = \rho_0 D\upsilon = -2\sigma\cos\gamma \tag{6}$$

Then $\vartheta_0 = \dfrac{dM}{\cos\gamma_0 dz}$ and $\rho = \dfrac{dM}{(1+e)\cos(\gamma_0-\gamma)dz}$ (7)

$$\frac{2\cos(\gamma_0-\gamma)}{\rho D} = D\left[\frac{(1+e)\cos(\gamma_0-\gamma)}{\cos\gamma_0} - 1\right]$$

Where $\sigma = Ee$ we get

$$D = a\sqrt{\frac{\cos\gamma_0\cos(\gamma_0-\gamma)(1-\cos\gamma+\sin\gamma)}{\cos(\gamma_0-\gamma)-\cos\gamma\cos\gamma_0+\sin\gamma-\cos\gamma}} \tag{8}$$

In fig. 1 the dependence of speed of the strong gap D from γ is presented at $\gamma_0 = \dfrac{\pi}{4}$.

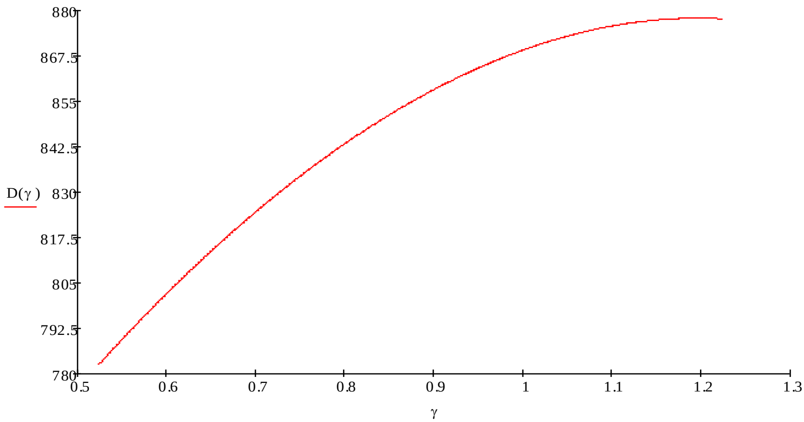

Fig. 1

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ON THE SOLVABILITY DIRICHLET PROBLEM FOR THE LAPLACE EQUATION WITH THE BOUNDARY VALUE IN GRAND-LEBESGUE SPACE

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Let $\omega = \{z \in C : |z| < 1\}$ be the unit disk on C and $\gamma = \partial\omega$ be its circumference.

Consider the following Dirichlet problem for the Laplace equation

$$\left.\begin{array}{l} \Delta u = 0, \ in \ \ \omega, \\ u/_{\gamma} = f, \end{array}\right\} \tag{1}$$

where $f : \gamma \to R$ is a real-valued function. Let $u_r(t) = u(re^{it})$ and

$$h_p = \{u : \Delta u = 0 \quad in \quad \omega, \quad and \quad \|u\|_{h_p} < +\infty\},$$

where

$$\|u\|_{h_p} = \sup_{0<r<1} \|u_r\|_p ,$$

$$\|g\|_p = \left(\int_{-\pi}^{\pi} |g(t)|^p \, dt \right)^{\frac{1}{p}} , \; 1 \le p < +\infty .$$

Let $w : [-\pi, \pi] \to R_+$ be a weight function. Consider the weighted grand space $h_w^{p),\theta}$ of harmonic functions in w equipped with the norm

$$\|u\|_{h_w^{p,\theta}} = \sup_{0<r<1} \|u_r(\cdot) w(\cdot)\|_{L_w^{p,\theta}} ,$$

where $u_r(t) = u\left(re^{it}\right) = u(r\cos t; r\sin t)$.

Assume that the weight $w(\cdot)$ satisfies the following condition

$$w^{-1}(\cdot) \in L_{p'+0} , \tag{2}$$

i.e. $\exists \varepsilon_0 > 0 : w^{-1}(\cdot) \in L_{p'+\varepsilon_0}$.

Let us define the subspace $M_w^{p),\theta}$ of the space $L_w^{p),\theta}$ which consist of such functions $f \in L_w^{p),\theta}$, that satisfies

$$\sup_{|s|\le\delta} \| f(\cdot+s) - f(\cdot) \|_{L_w^{p),\theta}} \to 0, \delta \to 0.$$

Denote by $P_z(\varphi)$ the Poisson kernel for the unit circle

$$P_z(\varphi) = \operatorname{Re} \frac{e^{i\varphi} + re^{it}}{e^{i\varphi} - re^{it}} = \frac{1 - r^2}{1 - 2r\cos(t - \varphi) + r^2} , \; z = re^{it} \in \omega .$$

The following properties of the Poisson kernel for the unit disk are true:

$$i) \; \frac{1}{2\pi} \int_{-\pi}^{\pi} P_r(t) dt = 1, \; \forall r \in [0;1) ;$$

$ii)\ \sup\limits_{|t|>\delta} P_r(t) \to 0$, as $r \to 1-0$, $\forall \delta > 0$;

$iii)\ \int\limits_{|t|>\delta} P_r(t)dt \to 0$, as $r \to 1-0$, $\forall \delta > 0$.

main theorem is proved.

Theorem **.** *Let* $f \in M_w^{p),\theta}, 1 < p < +\infty,$ *the weight* $w(\cdot)$ *belongs to the class* A_p *and the condition (2) holds. Then it holds*

$$\left\| P_r * f - f \right\|_{L_w^{p),\theta}} \to 0, r < 1-0.$$

This theorem has the following

Corollary **.** *Let* $f \in M_w^{p),\theta}, 1 < p < +\infty,$ $w(\cdot)$ *belongs to the class* A_p *and the condition (2) holds. Then the Dirichlet problem (1) is solvable in the classes* $h_w^{p),\theta}$.

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ONE FEEDBACK PROBLEM EXTINGUISHING OF OSCILLATIONS OF A MEMBRANE WITH POINTED PULSE INFLUENCES

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The problem of damping of transverse oscillation of a given thin homogenous membrane with a fixed boundary is studed. It is assumed that oscillations arise as a result of simultaneous actions of external sources at the initial moment of time of the neighborhood of some points θ^{ν}, $\nu=1,\dots,N_b$ of the membrane. Oscillations are damped by stabilizers with local effects at the neighborhood of membrane's points, η^{i}, $i=1,\dots,N_c$. For the formulation of the modes of control of the stabilizers, there are used information from the sensors measuring of the current values of displacement at the neighborhood of the points ξ^{j}, $j=1,\dots,N_o$.

This process for $t>0$ can be described by the following initial-boundary value problem:

$$u_{tt}(x,t)=a^2Lu(x,t)-\lambda u_t+\sum_{s=1}^{N_t}\delta\left(t;O_{\varepsilon_t}(\tau_s)\right)\sum_{i=1}^{N_c}\vartheta_s^i\delta\left(x;O_{\varepsilon_x}(\eta^i)\right) \quad (1)$$

$$x\in\Omega\subset R^2,$$

$$u(x,0)=0,\quad u_t(x,0)=\sum_{\nu=1}^{N_b}q^{\nu}\delta\left(x;O_{\varepsilon_x}(\theta^{\nu})\right),\quad x\in\Omega, \quad (2)$$

$$u(x,t)=0,\ x\in\Gamma. \quad (3)$$

Here $u(x,t)$ is the a displacement of the membrane at the a point of $x\in\Omega$ in the time of t; $L=\partial^2/\partial x_1^2+\partial^2/\partial x_2^2$; a^2, $\lambda\geq 0$ are the given constants; Γ is almost everywhere smooth border of area of Ω; q^{ν} is intensity of ν-th an external source, the lumped at the point $\theta^{\nu}=(\theta_1^{\nu},\theta_2^{\nu})\in\Omega$, $\nu=1,\dots,N_b$, $\vartheta=(\vartheta_1^1,\dots,\vartheta_1^{N_c},\dots,\vartheta_{N_t}^1,\dots,\vartheta_{N_t}^{N_c})\in R^{N_tN_c}$ is the vector determining the effects of stabilizers at the neighborhood of the points $\eta^i=(\eta_1^i,\eta_2^i)\in\Omega$, $i=1,\dots,N_c$, $\eta=(\eta^1,\dots,\eta^{N_c})$, $\tau=(\tau_1,\dots,\tau_{N_t})$ is the given time moment in which at the neighborhood take place effect of the stabilizers, where $\tau_s>\tau_{s-1}>0$, $s=1,\dots,N_t$, $\tau_0=0$, $\tau_{N_t}=T_f$; N_t is number of impulses; T_f is duration of time of observation of the process.

Continuously differentiable function $\delta(x;O_{\varepsilon_x}(\tilde{\eta}))$

determines the distribution intensity of the source at the neighborhood of $O_{\varepsilon_x}(\tilde{\eta})$ near the point $\tilde{\eta} \in \Omega$. Function $\delta(t; O_{\varepsilon_t}(\tilde{\tau}))$ is continuous for all $t \in [0, T_f]$.

Let the values of the powers of the sources q^ν and their locations θ^ν, $\nu = 1, \ldots, N_b$, are known exactly, but there are given sets Q^ν of possible values of q^ν, and the density functions of the distribution $\rho_{Q^\nu}(q) \geq 0$.

Locations θ^ν possible placement of external sources of influences are defined by sets $\Theta^\nu \subset \Omega$, $\nu = 1, \ldots, N_b$, $\Theta = \Theta^1 \times \ldots \Theta^{N_b}$ with the given density function of distribution of $\rho_{\Theta^\nu}(\theta) \geq 0$.

The problem of control process of oscillation of membrane damping for a given time T_f is to minimize the functional [1]:

$$J(\vartheta, \eta) = \int_Q \int_\Theta I(\vartheta, \eta; q, \theta) \rho_{\Theta^\nu}(\theta) \rho_{Q^\nu}(q) d\theta dq,$$

$$I(\vartheta, \eta; q, \theta) = \int_T^{T_1} \iint_\Omega \mu(x) [u(x, t; \vartheta, \eta, q, \theta)]^2 dx dt + \varepsilon_1 \left\| \vartheta - \hat{\vartheta} \right\|^2_{R^{N_t N_c}} + \varepsilon_2 \left\| \eta - \hat{\eta} \right\|^2_{R^{2N_c}}$$

.

Here, the function $u(x, t) = u(x, t; \vartheta, \eta, q, \theta)$ is the solution of the initial-boundary value problem (1)–(3) of given external effect with the power q^ν at the initial moment of time, $\nu = 1, \ldots, N_b$, and damping modes ϑ; The value of ΔT defines time duration of the interval $[T_f, T_1]$, where $T_1 = T_f + \Delta T$, in which it must be observe state of the membrane on this interval.

Let at the points $\xi^j = (\xi_1^j, \xi_2^j) \in \Omega$, $j = 1, \ldots, N_o$, of the membrane are installed sensors that measure the values of the displacement at the neighborhood of these points at times $\tau_s \in (0, T_f]$, $s = 1, \ldots, N_t$:

$$\hat{u}_s^j = \int_{O_{\varepsilon_t}(\tau_s)} \iint_{O_{\varepsilon_x}(\xi^j)} u(\hat{x},\hat{t})\delta(\hat{x};O_{\varepsilon_x}(\xi^j))\delta(\hat{t};O_{\varepsilon_t}(\tau_s))d\hat{x}d\hat{t} \,,$$

$$j = 1,\dots,N_o\,, \quad s = 1,\dots,N_t\,,$$

and it is possible to promptly define admissible stabilization modes ϑ_s^i, $i = 1,\dots,N_c$, $s = 1,\dots,N_t$ based on the results of these measurements.

To assign current values of stabilizer modes, we use the following function, which determines the feedback of control effect with the state of the membrane at the neighborhood of observation points:

$$\vartheta_s^i = \sum_{j=1}^{N_o} k^{ij}\left[\int_{O_{\varepsilon_t}(\tau_s)} \iint_{O_{\varepsilon_x}(\xi^j)} u(\hat{x},\hat{t})\delta(\hat{x};O_{\varepsilon_x}(\xi^j))\delta(\hat{t};O_{\varepsilon_t}(\tau_s))d\hat{x}d\hat{t} - z^{ij}\right], \qquad (4)$$

$$i = 1,\dots,N_c\,, \quad s = 1,\dots,N_t\,.$$

Here $k = ((k^{ij}))$ are reinforcing coefficient; $z = ((z^{ij}))$, are the nominal values of the displacement at the point ξ^j with respect to the stabilizer at the point η^i; k, z are optimized feedback parameters.

Substituting formula (4) into equation (1), we obtain loaded differential equations [2]-[4]:

$$u_{tt}(x,t) = a^2 Lu(x,t) - \lambda u_t + \sum_{s=1}^{N_t}\delta(t;O_{\varepsilon_t}(\tau_s))\sum_{i=1}^{N_c}\vartheta_s^i\delta(x;O_{\varepsilon_x}(\eta^i)) \cdot \qquad (5)$$

$$\cdot\sum_{j=1}^{N_o} k^{ij}\left[\int_{O_{\varepsilon_t}(\tau_s)} \iint_{O_{\varepsilon_x}(\xi^j)} u(\hat{x},\hat{t})\delta(\hat{x};O_{\varepsilon_x}(\xi^j))\delta(\hat{t};O_{\varepsilon_t}(\tau_s))d\hat{x}d\hat{t} - z^{ij}\right]$$

$$, \quad x \in \Omega\,.$$

The problem consists to finding of optimal values of parameters of a feedback control of $k \in R^{N_c N_o}$, $z \in R^{N_c N_o}$, locations of measurement points ξ and stabilization η at which (5), (2), (3) are satisfied. The obtained problem of synthesis the control of stabilization of the oscillations belongs to the class of parametric optimal control problems for distributed systems.

The considered problem is reduced to a parametric optimal control of a system with distributed parameters. Gradient formula for the functional of the considering problem in the space of synthesized parameters are obtained. The formulas made it possible to use efficient first-order optimization methods for the numerical solution of the synthesis problem. The results of numerical experiments are given.

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ON INVERSE SOURCE PROBLEMS FOR A PARABOLIC EQUATION WITH NONLOCAL CONDITIONS

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The investigation of the inverse source problems of mathematical physics is conducted in various directions, and in the recent years, the number of scientific works devoted to theoretical as well as specific applied problems has essentially increased [1-5]. In this work, we consider the solution to classes of inverse problems in which the identifiable coefficients depend only on the time or only on the space. This specific character allows reducing the initial problems to specially built Cauchy problems with respect to a system of ordinary differential equations. It is important to note that the proposed approach does not use any iterative procedures.

Problem 1. We consider the following inverse problem to determine an unknown coefficient $B_0(t)$ of a linear parabolic equation:

$$\frac{\partial v(x,t)}{\partial t} = a_0(t)\frac{\partial^2 v(x,t)}{\partial x^2} + a_1(t)\frac{\partial v(x,t)}{\partial x} + a_2(t)v(x,t) + f(x,t) + B_0(t)C_0(x,t), \quad (1)$$

$$(x,t) \in \Omega = \{(x,t): 0 < x < l,\ 0 < t \le T\},$$

under the following conditions:

$$\int_0^l e^{k\xi} v(\xi,t)d\xi = \psi(t),\ k = const,\quad t \in [0,T], \quad (2)$$

$$v(0,t) = \psi_0(t),\quad v(l,t) = \psi_1(t),\ t \in [0,T], \quad (3)$$

$$v(x,0) = \varphi_0(x),\quad x \in [0,l]. \quad (4)$$

Here functions $a_0(t) > 0, a_1(t), a_2(t), a_3(t), f(x,t), C(x,t),$ $\psi(x), \psi_0(t), \psi_1(t), \varphi_0(x)$ and constant k are given, the functions $\varphi_0(x), \psi(t), \psi_0(t), \psi_1(t)$ satisfy the following consistency conditions:

$$\varphi_0(0) = \psi_0(0),\quad \varphi_0(l) = \psi_1(0),\ \int_0^l e^{k\xi}\varphi_0(\xi)d\xi = \psi(0)$$

and all other necessary conditions of existence and uniqueness of the solution to the inverse problem (1)-(4). The problem (1)-(4) consists in determining the unknown continuous function $B_0(t)$ and the corresponding solution to the boundary value problem $v(x,t)$, which is twice continuously differentiable with respect to x and once continuously differentiable with respect to t for $(x,t)\in\Omega$, and satisfies conditions (1)-(4).

Problem 2. The following equation is given:

$$\frac{\partial v(x,t)}{\partial t} = a_0(x)\frac{\partial^2 v(x,t)}{\partial x^2} + a_1(x)\frac{\partial v(x,t)}{\partial x} + a_2(x)v(x,t) +$$
$$+ f(x,t) + B_0(x,t)C_0(x), \tag{5}$$
$$(x,t)\in\Omega = \{(x,t): 0\le x\le l,\ 0\le t\le T\},$$

under the following conditions:

$$k_1 v(x,0) + \int_0^T e^{k\tau} v(x,\tau)d\tau = \varphi_0(x), \quad x\in[0,l], \tag{6}$$

$$v(0,t)=\psi_0(t), \quad v(l,t)=\psi_1(t), \quad t\in[0,T], \tag{7}$$

$$v(x,T)=\varphi_T(x), \quad x\in[0,l]. \tag{8}$$

It is required to determine the pair of functions $(C_0(x), v(x,t))$.

In the work, we suggest a numerical approach to solving problems (1)-(4) and (5)-(8). At first, the problems with integral conditions are reduced to problems with point conditions, but with an additional unknown function as a parameter in the equation. Then, using the method of lines, the problems are reduced to a system of ordinary differential equations with unknown numerical parameters. To determine these parameters, a method based on the author’s results [6-9] is suggested.

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FORCED VIBRATION OF THE "HOLLOW CYLINDER + SURROUNDING ELASTIC MEDIUM" SYSTEM WITH INHOMOGENEOUS INITIAL STRESSES

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The paper studies forced vibration of the "hollow cylinder + surrounding elastic medium" with non-homogeneous initial stresses caused by the uniformly distributed radial compressional forces acting at infinity. It is assumed that after appearing the mentioned initial stresses on the interior of the cylinder the time-harmonic ring load acts and it is required to investigate how the initial stresses influence on the dynamic stress field appearing by the action of the additional time-harmonic forces. This investigation is made with utilizing the method developed in the paper [1, 2], according to which the corresponding problem is formulated within the scope of the three-dimensional linearized theory elastic waves in bodies with initial stresses the basic equations and relations of which are described in many monographs (for instance in the monograph [3] and others listed therein). The mathematical method applied in the present work it is called the "discrete analytical" one, according to which, the solution of the differential equations with variable coefficients is reduced to the solution of the corresponding series differential equations with constant coefficients.

Numerical results on the dynamic stresses acting on the interface surface between the cylinder and elastic medium are presented and discussed. It is also discussed the recommendation on the possible application of the obtained results on the dynamics of the underground structures used in various branches of the modern industry.

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ON THE "GYROSCOPIC EFFECT" UNDER DYNAMICS OF AN OSCILLATING MOVING LOAD ACTING ON THE ELASTIC PLATE LAYING ON THE COMPRESSIBLE VISCOUS FLUID WITH FINITE DEPTH

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The abstract of the investigations related to the forced vibration of the hydro-elastic and hydro–viscoelastic systems consisting on the elastic (or viscoelastic) plate, compressible viscous fluid and rigid wall have been made in the paper [1]. According to this review and to analyses of the results of the papers [2, 3], the main non-trivial and non-predicting results in these investigations are obtained in the case where the time-harmonic force acting on the mentioned system moves simultaneously with constant velocity. Namely, as a result of this simultaneous vibration and move with certain constant velocity of the external force the so-called "gyroscopic effect appear. The aim of the paper is to focus on this effect with respect to the hydro-elastic system described above. The corresponding investigations to achieve this aim are made with the scope of the exact field equations and relations of the elastodynamics (for describing the motion of the plate) and linearized Navier-Stokes equations of the barotropic compressible viscous fluid (for describing the fluid flow). The plane-strain state in the plate and the plane flow of the fluid is considered. The solution of the corresponding boundary-value problems is solved by employing the variable separation method and by employing the exponential Fourier integral transform with respect with the space variable indicated the coordinate of the point along the plate laying direction. The originals of the sought values are found numerically, according to which, it is discussed the aforementioned "gyroscopic effect" with respect to the interface pressure and fluid velocity on the plate-fluid interface. In particular, it is established that the influence of the motion of the oscillating load on the distribution of the stress and velocities depends on the vibration phase of the system.

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ON THE GENERALIZED SPECTRAL OPERATORS

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As is well known, one achievement of the theory of normal operators is the spectral representation and the possibility of constructing a functional calculus for such operators. For nonselfadjoint operators these and related questions have been intensively developed by the efforts of many mathematicians. The class of spectral operators introduced and studied by Dunford [1] and his colleagues occupies a noteworthy place in this area. But the requirement of boundedness of the resolution of the it identity or the spectral measure restricts the area of application of such operators. Therefore, attempts were made to get rid of this restriction. Lyantse [2], [3], Foias [4], Colojoara and Foias [5],

[6], Allakhverdiev and Akhmedov [7], and Stranss and Trunk [8] took essential steps in this direction.

Lyantse treated operators with unbounded spectral measures, and in our view his papers spurred the development of the theory of spectral operators to a considerable degree. Foias and his colleagues singled out and studied the so-called decomposable operators. A functional calculus for such operators was constructed, and some spectral properties were investigated.

In [7] is investigated a class, consisting of closed linear operators that are close in some ways to spectral operators but make up a broader class and coincide in general neither with the generalized spectral operators in the Lyantse sense nor with the decomposable operators.

In [8] is introduced the notion of spectralizable operators, which a closed operator A in a Hilbert space is called spectralizable of there exists a nonconstant polynomial p such that the operator $p(A)$, is a scalar spectral operator and was showed that such operators belongs to the class of operators in the sence Colojoara and Foias.

In this work we show that the class studing in [8] being a generalization of a class of sealar spectral operators is a subclass of class **A** [7] consisting of operators having decomposition of commutative spectral operators.

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EXPANSION OF THE APPLIED CAPABILITIES OF THE THEORY DIFFERENTIAL EQUATIONS WITH A NEW CONSEPT OF UNIVERSAL TRAJECTORY

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The goal is to justify ideas that make changes to the two basic principles of the theory of differential equations in order to increase the applied capabilities of this theory. Here are the following (accepted by default):

I. In the derivation of the equations, not taken into account or only incompletely taken into account the linear dimensions of the studied object.

II. The material point chosen to derive the equations assumed that to be hard connected with only one point of the object (or stream) under study.

Usually, by choosing a point is hard connected with an object (often by default implying that the point was selected), a transition to mathematical research is carried out in order to determine the properties of this object. It was in this way, existing since the time of Newton and Leibniz, that brilliant successes were achieved. However, for a number of reasons, the classical path is not successful in attempts to describe turbulent phenomena. To get out of this difficult situation, are changes introduced into the classical principles I and II. Note that this is not a rejection of the theory of differential equations; on the contrary, we only increase the applied capabilities of this theory. Moreover, the classical approach itself turns out to be a particular limiting case of the new approach. New analogues of the basic principles of I and II:

In. The linear dimensions of the studied real objects are taken into account (defined by a single rule).

IIn. We select (in a general way) any point *A* of the object under study. A hard link between point *A* and the object is not required. We will use point *A* for modeling the behavior of the object.

Point *A* can immediately participate in two movements: in movement with the object, as well as in movement inside the object. We can set the movement of point *A* inside the object ourselves, which opens up new possibilities. The trace of the movement the object we will be call the lane. Any trajectory of point *A* in the lane we will be call a universal trajectory. Scalar ordinary differential equation $dx(t)/dt = f(x(t),t)$ with the initial condition $x(t_0) = x_0$ in the transition from I, II to the positions In, IIn, takes the form:

$$dz_\mu(t)/dt = f\left(z_\mu(t) - \alpha\,\mu(t), t\right) + \alpha\, d\mu(t)/dt$$

with the initial condition $z_\mu(t_0) = x_0 + \alpha\,\mu(t_0)$, where α is the linear size of a one-dimensional object, μ a smooth control function, $|\mu(t)| \le 1$, $z_\mu(t) = \hat{x}(t) + \alpha\,\mu(t)$ determine all smooth universal trajectories in the lane and $\hat{x}(t)$- the solution of the original Cauchy problem. If the control functions are pointwise discontinuous, then the corresponding universal trajectories are also pointwise discontinuous [1, 2].

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THE INVERSE PROBLEM FOR PARABOLIC EQUATION IN A DOMAIN WITH MOVING BOUNDARY

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The goal of this paper is to study the well-posedness of an inverse problem of determining the unknown coefficient on the right side of parabolic equation. The inverse problem in a domain with moving boundary in case of the Dirichlet boundary condition with additional integral information.

We consider an inverse problem of determining a pair of functions

$$\{f(x),\ u(x,t)\}$$

from conditions

$$u_t - u_{xx} = f(x)g(t),\ (x,t) \in D = \{s_1(t) < x < s_2(t),\ 0 < t \le T\},$$

$$u(x,0) = \varphi(x),\ x \in [s_1(0),\ s_2(0)]$$

$$u(s_1(t),t)=\psi_1(t),\ \ u(s_2(t),t)=\psi_2(t),\ t\in[0,T]$$

$$\int_0^T u(x,t)dt=h(x),\ \ x\in[s_1(t),s_2(t)]$$

where the data $g(t),\varphi(x),\psi_1(t),\psi_2(t),h(x)$ are continuous functions whose degree of continuity we shall make precise later. Recall that we have assumed that $s_i(t)\in C^1([0,T]),\ i=1,2.$ Here, we shall also assume that

$$\inf_{0\le t\le T}|s_1(t)-s_2(t)|>0,\ \ \sup_{0\le t\le T}|s_1(t)-s_2(t)|<\infty$$

The additional condition represents the specification of a relative heat content of a portion of the conductor. For diffusion, the condition is equivalent to the specification of mass in a portion of the domain of diffusion.

The uniqueness theorem and the estimation of stability of the solution of inverse problems occupy a central place in investigation of their well-posedness. In the paper, the uniqueness of solution for considering problem is proved under more general assumptions and the estimation characterizing the "conditional" stability of the problem is established.

CONVERGENCE OF SPECTRAL EXPANSION IN EIGENFUNCTIONS OF THIRD ORDER ORDINARY DIFFERENTIAL OPERATOR

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Consider on the interval $G=(0,1)$ a formal differential operator

$$Lu=u^{(3)}+P_1(x)u^{(2)}+P_2(x)u^{(1)}+P_3(x)u$$

with summable coefficients $P_1(x)\in L_2(G),\ P_\ell(x)\in L_1(G),\ l=2,3.$

Denote by $D(G)$ the class of functions absolutely continuous together with its derivatives to second order inclusively, on the closed interval $\overline{G}=[0,1]$.

Under the eigenfunction of the operator L responding to the eigenvalue λ, we understand any function $u(x)\in D(G)$ identically not equal to zero and satisfying almost everywhere in G the equation (see [1])

$$Lu+\lambda u=0.$$

Let $\{u_n(x)\}_{n=1}^{\infty}$ be a complete system, orthonormalized in $L_2(G)$, consisting of eigenfunctions of the operator L, and $\{\lambda_n\}_{n=1}^{\infty}$ be a corresponding system of eigenvalues $(\operatorname{Re}\lambda_n=0)$.

Denote

$$\mu_k=\begin{cases}(-i\lambda_n)^{1/3}, & if\ \operatorname{Im}\lambda_n\ge 0,\\ (i\lambda_n)^{1/3}, & if\ \operatorname{Im}\lambda_n<0,\end{cases}$$

and consider the partial sum of orthogonal expansion of the function $f(x)\in W_1^1(G)$ in the system $\{u_n(x)\}_{n=1}^{\infty}$:

$$\sigma_\nu(x,f)=\sum_{\mu_n\le\nu} f_n u_n(x),\ \nu>0,$$

where $f_n=(f,u_n)=\int_0^1 f(x)\overline{u}_n(x)dx.$

Study the behavior of the difference $R_\nu(x,f)=\sigma_\nu(x,f)-f(x)$.

In the paper, the following theorem is proved.

Theorem. *Let the function $f(x)$ belong to the class $W_1^1(G)$, the system $\{u_n(x)\}_{n=1}^{\infty}$ by uniformly bounded, and the following conditions be fulfilled:*

$$\left|\,f(x)u_n^{(2)}(x)\Big|_0^1\right|\le C(f)\mu_n^{\alpha},\ \ 0\le\alpha<2,\ \mu_n\ge 4\pi;$$

$$\sum_{k=1}^{\infty}k^{-1}\omega_1\left(f',k^{-1}\right)<\infty,\quad \sum_{k=1}^{\infty}k^{-1}\omega_1\left(f\,\overline{P}_1,k^{-1}\right)<\infty.$$

Then the expansion of the function $f(x)$ in the system $\{u_n(x)\}_{n=1}^{\infty}$ converges absolutely and uniformly on $\overline{G}=[0,1]$.

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ON BASIS PROPERTY FOR A CLASS STURM-LIOUVILLE EQUATION WITH DISCONTINUOUS POTENTIAL

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Vibrating string problems when the string loaded with point masses are reduced to Sturm-Liouville problems containing spectral parameter in boundary conditions. Investigating the spectral properties of such problems is very important to find the solution of physical problem.

In this study, we consider a boundary value problem for a class Sturm-Liouville equation with discontinuous potential. Taking into account conjugate conditions at discontinuous points, we examine the basis properties of eigenfunctions of boundary value problem.

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EXISTENCE OF STRONG SOLUTIONS FOR THE COUPLED SUSPENSION BRIDGE EQUATIONS

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We consider the following mathematical model for the oscillations of the bridge which has one common point with the cable:

$$\begin{cases} u_{tt} + u_{xxxx} + (u-v)_+ + \Phi(u_t) = f(x), \\ v_{tt} - v_{xx} + (v-u)_+ + G(v_t) = g(x), \end{cases}$$

$$\begin{cases} u(\xi-0,t) = u(\xi+0,t) = v(\xi-0,t) = v(\xi+0,t), \\ u'''(\xi-0,t) - u'''(\xi+0,t) - v'(\xi-0,t) + v'(\xi+0,t) = 0, \\ u''(\xi-0,t) = u''(\xi+0,t), \end{cases}$$

$$u(0,t) = u'(0,t) = u(l,t) = u'(l,t) = v(0,t) = v(l,t) = 0,$$

$$u(x,0) = u_0(x), u'(x,0) = u_1(x), v(x,0) = v_0(x), v'(x,0) = v_1(x),$$

where $0 \le x \le l,\ t > 0$, $0 < \xi < l$. Here $u(x,t)$ is state function of the road bed and $v(x,t)$ is that of the main cable, $\Phi(u_t), G(v_t)$ are the aerodynamically dampings. The solution of the problem is reduced to the Cauchy problem for the operator differential equation in some functional spaces.

In this study, we show the existence of global solutions of the considered problem and investigate the asymptotics of these solutions.

MIXED PROBLEM FOR THE STRONGLY DAMPED NONLINEAR WAVE EQUATION WITH DYNAMIC BOUNDARY CONDITIONS

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We consider a strong damped nonlinear wave equation with dynamic boundary conditions

$$u_{tt} - u_{xxt} - u_{xx} = f(x),\ (t,x) \in Q_T = [0,T]\times[0,1], \quad (1)$$

$$u(t,0) = 0,\ t > 0, \quad (2)$$

$$u_{tt}(t,1) + u_{xt}(t,1) + u_x(t,1) = g(t),\ t > 0, \quad (3)$$

$$u(0,x) = \varphi(x),\ u_t(0,x) = \psi(x),\ 0 \le x \le 1, \quad (4)$$

$$u(1,0) = h, \quad (5)$$

where $f(x) \in C^1(R),\ f'(x) \le c,\ c \ge 0,\ \sup \lim\limits_{x\to+\infty} \frac{f(x)}{x} \le 0,\ \ f(0) = 0,$

$g(t) \in C^1[0,T],\ \varphi(\cdot) \in W_p^2(0,1),\ \psi(\cdot) \in L_p(0,1),\ h \in R$.

We prove that the problem (1) - (5) has a unique solution $u \in C([0,T]\times W_p^2[0,1] \cap {}_0W_p^1[0,1]) \cap C^1([0,T]\times L_p[0,1]) \cap C^1((0,T)\times W_p^2[0,1] \cap {}_0W_p^1[0,1]) \cap C^2((0,T)\times L_p[0,1])$, such that $u_t(t,1), u_x(t,1) \in C[0,T]$, $u_{tt}(t,1),\ u_{tx}(t,1) \in C(0,T)$.

FIRST-ORDER REGULARIZED TRACE OF THE STURM–LIOUVILLE OPERATOR WITH POINT OF δ' -INTERACTION

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Inthework, we consider a boundary-value problem for the differential equation

$$-y'' = \lambda y\,, \quad x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right), \tag{1}$$

subject to the boundary conditions

$$y'(0) = y(\pi) = 0\,, \tag{2}$$

and to the jump conditions at the point $x = \frac{\pi}{2}$

$$\begin{cases} y'\left(\frac{\pi}{2}+0\right) = y'\left(\frac{\pi}{2}-0\right) \equiv y'\left(\frac{\pi}{2}\right), \\ y\left(\frac{\pi}{2}+0\right) - y\left(\frac{\pi}{2}-0\right) = \alpha y'\left(\frac{\pi}{2}\right), \end{cases} \tag{3}$$

where $\alpha \neq 0$ is a real value, and λ is a spectral parameter.

Theproblem (1) underthe conditions (3) can be reduced to the equation

$$-y'' + q(x)y = \lambda y\,, \quad x \in (0, \pi)\,,$$

where $q(x) = \alpha\delta'\left(x - \frac{\pi}{2}\right)$ is the potential, and $\delta'(x)$ is the derivative of the Dirac's function(see [1]).

The primary objective of the work is to calculate a first-order regularized trace of the Sturm-Liouville operator with point of δ' - interaction.

Note that the eigenvalues λ_n, $n = 1, 2, ...$ of the operator L, generated by the differential expression $-y'' + \alpha\delta'\left(x - \frac{\pi}{2}\right)y$ on the finite interval $(0, \pi)$ with the boundary conditions (2), have an asymptotic

$$\lambda_n = n^2 + \frac{4}{\pi\alpha} + \frac{\xi_n}{n},\ \{\xi_n\} \in l_2,$$

which is established by direct calculations.

Further, by a first-order regularized trace of the operator L we mean the sum of the series

$$\sum_{n=1}^{\infty}\left(\lambda_n - n^2 - \frac{4}{\pi\alpha}\right). \tag{4}$$

The following theorem is true.

Theorem. *The series* (4) *is convergent, and its sum is equal to* $-\frac{2}{\pi\alpha} - \frac{2}{\alpha^2}$.

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SOLVABILITY OF A BOUNDARY VALUE PROBLEM FOR SECOND ORDER ELLIPTIC DIFFERENTIAL OPERATOR EQUATIONS WITH A QUADRATIC COMPLEX PARAMETER

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In the paper [1] in a separable Hilbert space H the solvability of the following boundary value problem is studied for a second order elliptic differential-operator equation :

$$L(\lambda, D)u := \lambda u(x) - u''(x) + Au(x) = f(x), \; x \in (0,1), \quad (1)$$

$$\begin{aligned} L_1(\lambda)u &:= u'(1) + \lambda Bu(0) = f_1, \\ L_2 u &:= u'(0) = f_2, \end{aligned} \quad (2)$$

where λ is a complex parameter ; A is a φ - positive operator in H ; B is a linear bounded operator in H ; $D := \dfrac{d}{dx}$. It is proved that coercive solvability in space $L_p((0,1);H), p \in (1,+\infty)$ does not hold for boundary value problem (1)-(2).

In this note in H we study solvability of the following boundary value problem

$$L(\lambda, D)u := \lambda^2 u(x) - u''(x) + Au(x) = f(x), \; x \in (0,1), \quad (3)$$

$$\begin{aligned} L_1(\lambda)u &:= u'(1) + \lambda Bu(0) = f_1, \\ L_2 u &:= u'(0) = f_2. \end{aligned} \quad (4)$$

For boundary value problem (3),(4) we proved the following

Theorem 1. Let the following conditions be fulfillied :

1) A is a φ - positive operator in H ;

2) The linear operator B boundedly acts from H to H and from $H(A)$ to $H(A)$.

Then, the operator $\mathsf{L}(\lambda): u \to \mathsf{L}(\lambda)u := (L(\lambda, D)u, L_1(\lambda)u, L_2 u)$ for sufficiently large $|\lambda|$ from the angle $|\arg\lambda| \le \varphi < \frac{\pi}{2}$ is an isomoprhism from $W_p^2((0,1); H(A), H)$ to $L_p((0,1);H) \dot{+} (H(A),H)_{\frac{1}{2}+\frac{1}{2p},p} \dot{+} (H(A),H)_{\frac{1}{2}+\frac{1}{2p},p}$ and for such λ the following coercive estimation is valid for solving problem (3),(4)

$$|\lambda|^2 \|u\|_{L_p((0,1);H)} + \|u''\|_{L_p((0,1);H)} + \|Au\|_{L_p((0,1);H)} \le$$

$$\le c\left[|\lambda| \|f\|_{L_p((0,1);H)} + \sum_{k=1}^{2}\left(\|f_k\|_{(H(A),H)_{\frac{1}{2}+\frac{1}{2},p}} + |\lambda|^{1-\frac{1}{p}} \|f_k\|_H\right)\right],$$

where $c > 0$ is a constant independent of λ.

As is seen, problem (3),(4) differes from boundary value problem(1),(2) by the fact that the complex parameter λ unikle equation (1) participates in equation (3) quadratically. But this change strongly affects on the character of solvability of boundary value problem (3),(4), i.e. problem (3),(4) becomes coercively solvable in the space $L_p((0,1);H),\ p \in (1,+\infty)$.

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LMI METHOD FOR SOLVING BHH EQUATIONS IN THE NORMAL CASE

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In this paper, the continuous and discrete BHH (Bevis–Hall–Hartwig equation**)** matrix equation is solved by linear matrix inequality (LMI), which gives a more accurate result than [1]. A computational algorithm is suggested. In [1] it is shown that the discrete BHH equation have been reduced to to the Stein equation and for its solution have been used the standard procedure dlyap.m of the MATLAB application package, but in the considered work we show that the accuracy of the solution is lost when used the method [1].In the numerical example of the third order it is shown that the proposed LMI algorithm produces a result which is the order of accuracy higher accuracy [1].

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ON APPROXIMATION OF THE HILBERT TRANSFORM

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Let $L_p(R)$, $1 \le p < \infty$, – the space of the functions, with finite norm $\|u\|_{L_p(R)} = \left(\int_R |u(\tau)|^p \, d\tau \right)^{1/p}$. The Hilbert transform of the function $u \in L_p(R)$, $1 \le p < \infty$, is called the singular integral

$$(Hu)(t) = \frac{1}{\pi} \int_R \frac{u(\tau)}{t-\tau} d\tau, \quad t \in R.$$

It is known (see [1, Ch. III, §2]) that the Hilbert transform of the function $u \in L_p(R)$, $1 \le p < \infty$, exists for almost all values $t \in R$. In the case $1 < p < \infty$, the Hilbert transform is a bounded operator in the space $L_p(R)$ and satisfies the equality $H^2 = -I$.

In [2], for the analytic functions in a strip $\{z \in C : |\mathrm{Im} z| < d\}$ (under certain additional restrictions) it was proved that the series $\frac{1}{\pi} \sum_{k=-\infty}^{\infty} \frac{u(t+(k+1/2)\delta)}{-k-1/2}$ converges uniformly to $(Hu)(t)$ on $\delta \to 0$.

The report is devoted to the approximation of the Hilbert transform of arbitrary functions from $L_2(R)$ by operators of the form $(H_\delta u)(t) = \frac{1}{\pi}\sum_{k=-\infty}^{\infty}\frac{u(t+(k+1/2)\delta)}{-k-1/2}$, $\delta > 0$, which were introduced in [2]. It is proved that the operators H_δ is bounded in $L_p(R)$, $1 < p < \infty$, satisfy equality $H_\delta^2 = -I$ in $L_p(R)$, and for every $\delta > 0$ the sequences of the operators $\{H_{\delta/n}\}_{n\in N}$ converges strongly to the operator H in $L_2(R)$.

Theorem 1. For any $\delta > 0$ the operators H_δ are bounded in the space $L_p(R)$, $1 < p < \infty$, and the inequality

$$\|H_\delta\|_{L_p(R)\to L_p(R)} \le \|\tilde{h}\|_{l_p \to l_p},$$

holds, where $\tilde{h}$ is modified discrete Hilbert transform defined by $\tilde{h}(b) = \{(\tilde{h}(b))_n\}_{n\in Z}$, $\quad (\tilde{h}(b))_n = \sum_{m\in Z}\frac{b_m}{n-m-1/2}$, $\quad n \in Z$, for $b = \{b_n\}_{n\in Z} \in l_1$.

Theorem 2. For any $\delta > 0$ and $u \in L_p(R)$, $1 < p < \infty$ following inequality hold:

$$H_\delta(H_\delta u)(t) = -u(t).$$

Theorem 3. For any $\delta > 0$ the sequence of the operators $\{H_{\delta/n}\}_{n\in N}$ strongly converges to the operator H in $L_2(R)$, that is for any $u \in L_2(R)$ following inequality hold:

$$\lim_{n\to\infty}\|H_{\delta/n}u - Hu\|_{L_2(R)} = 0.$$

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CONSTRUCTING THE SYSTEM OF CHEBYSHEV-HERMITE POLYNOMIALS

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It is known that the weight function, the system of orthogonal and orthonormal polynomials with respect to this function was constructed, their algebraic and asymptotic properties were proved, the appropriate Fourier series were studied [1]. In the paper, we will consider the existence and uniqueness condition of the system of orthonormal polynomials with respect to the weight function. In the special case, we will construct the first Rodrig formula of Chebyshev-Hermite polynomials.

Theorem. There exists a unique sequence of polynomials $\{p_n(x)\}$ for any weight function $h(x)$ with positive higher term coefficient and satisfying the orthonormality condition [1].

Using this theorem, we prove the Chebyshev-Hermite orthogonal and orthonormal polynomials and study the construction of the system of polynomials. The polynomial

$H_n(x) = (-1)^n e^{x^2}\left(e^{-x^2}\right)^{(n)}$ is said to be a system Chebyshev-Hermite polynomials with respect to the weight function. We can prove that for $m < n$, if we take into account, $H_m^{(n)}(x) = 0$, then

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x) dx = 0, \quad m < n.$$

So, the polynomial $\{H_n(x)\}$ is orthogonal.

Then the orthonormal Chebyshev-Hermite polynomial is as follows:

$$\hat{H}_n(x) = \frac{H_n(x)}{\sqrt{n! 2^n \sqrt{\pi}}}, \ \mu_n = \frac{2^n}{\sqrt{n! 2^n \sqrt{\pi}}} = \sqrt{\frac{2^n}{n! \sqrt{\pi}}}$$

We determine some terms of the system of Chebyshev-Hermite polynomials. By the theorem $H_0(x) = C_{00} \succ 0$ and

$$\int_{-\infty}^{+\infty} h(x) H_0^2(x) dx = C_{00}^2 \int_{-\infty}^{+\infty} e^{-x^2} dx = 1 \Rightarrow C_{00} = \frac{1}{\sqrt[4]{\pi}}.$$

Using the known lemma, we express the term $H_1(x)$ of the Chebyshev-Hermite polynomial as follows:

$$H_1(x) = C_{01} H_0(x) + C_{11} x.$$

Use orthogonality and orthonormality

$$\begin{cases} \int_{-\infty}^{+\infty} h(x) H_0(x) H_1(x) dx = 0 \\ \int_{-\infty}^{+\infty} h(x) H_1^2(x) dx = 1 \end{cases} \quad \begin{cases} \int_{-\infty}^{+\infty} e^{-x^2} \frac{1}{\sqrt[4]{\pi}} \left(C_{01} \frac{1}{\sqrt[4]{\pi}} + C_{11} x \right) dx = 0 \\ \int_{-\infty}^{+\infty} e^{-x^2} \left(C_{01} \frac{1}{\sqrt[4]{\pi}} + C_{11} x \right)^2 dx = 1 \end{cases}$$

As a result, $\begin{cases} \frac{1}{\sqrt[4]{\pi}} C_{01} \cdot \frac{1}{\sqrt[4]{\pi}} \cdot \sqrt{\pi} = 0 \\ C_{01}^2 \frac{1}{\sqrt{\pi}} \sqrt{\pi} + C_{11}^2 \cdot \frac{\sqrt{\pi}}{2} = 1 \end{cases}$ $\begin{cases} C_{01} = 0 \\ C_{11} = \sqrt{\frac{2}{\sqrt{\pi}}} \end{cases}$ then

$$H_1(x) = \frac{2x}{\sqrt{2\sqrt{\pi}}} \quad \begin{cases} \int_{-\infty}^{+\infty} e^{-x^2} H_0(x) H_2(x) dx = 0 \\ \int_{-\infty}^{+\infty} e^{-x^2} H_1(x) H_2(x) dx = 0 \\ \int_{-\infty}^{+\infty} e^{-x^2} H_2^2(x) H_2(x) dx = 0 \end{cases}$$

We can show that $H_2(x) = \left(4x^2 - 2\right) / \sqrt{8\sqrt{\pi}}$. In the same way, we can construct the polynomials $H_3(x), H_4(x), \ldots$ and others.

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BRANCHING PROCESS WITH SPECIAL GENERATING FUNCTION

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In general, various aspects of branching random processes in many works [1,2] were investigated. It is known that for investigation of branching processes there is a suitable apparatus – generating functions. All characteristics of branching processes are given by generating functions. In this work we consider the case when generating function of branching process has a special form.

Let ξ_n, $n = 0,1,2,...$ be a branching process with the non-negative integers as state space, i.e. ξ_n is the size of the n-th generation, $f_n(s)$ is the corresponding generating function.

We denote the generating function of density probabilities p_k by

$$f(s) = \sum_{k=0}^{\infty} p_k s^k = M(s^{\xi_1} \big| \xi_0 = 1),\ |s| < 1.$$

Assume that $p_1 \neq 1$ and the mean

$$f'(1) = \sum_{k=1}^{\infty} k p_k = 1.$$

Then $f_n(s)$ increases to 1 as $n \to \infty$ for every $s \in [0,1)$.
We consider the partial case, when generating function of the branching process has the following form

$$f(s) = s + (1-s)^{1+\alpha} L(1-s),\ |s| < 1, \quad (1)$$

where $L(x)$ is a slowly varying function as $x \to 0$ and $0 < \alpha \leq 1$.

This includes the case $f''(1) < \infty$, when $\alpha = 1$ and $L(x) \to \frac{1}{2} f''(1)$ as $x \to 0+$.

Denote $F_n(u) = M\left[e^{-u(1-f_n(u))}\xi_n \big| \xi_n > 0\right]$, $u > 0$.

The following theorem is proved.

Theorem: If(1) holds, then

$$F_n(u) \to F(u) = 1 - u(1+u^{\alpha})^{-\frac{1}{\alpha}} \text{ as } n \to \infty.$$

Corollary: $\lim_{n\to\infty} P\left[(1-f_n(0))\xi_n < x \big| \xi_n > 0\right] = G(x)$, $x > 0$, where $G(x)$ has the Laplace transform $F(u)$.

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THE SCATTERING PROBLEM FOR THE SYSTEM OF SIX ORDINARY DIFFERENTIAL EQUATIONS ON SEMI-AXIS

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In this paper we study the scattering problem for the system of equations on the semi-axis $x>0$

$$-i\frac{dy_k(x)}{dx}+\sum_{j=1}^{5}u_{kj}(x)y_j(x)=\lambda\xi_k y_k(x),\ \ k=\overline{1,6} \tag{1}$$

where $\left\|u_{kj}(x)\right\|_{k,j=1}^{6}$ is a matrix with zero diagonal elements; its elements are measurable complex-valued functions satisfying conditions:

$$\left|c_{kj}(x)\right|\le ce^{-\varepsilon x},\ c>0,\ \varepsilon>0 \tag{2}$$

λ is a spectral parameter; $\xi_1>\xi_2>\xi_3>0>\xi_4>\xi_5>\xi_6$.

The solution of the system (1) is a absolutely continuous vector-function $\{y_1(x),...,y_6(x)\}$ almost everywhere satisfying (1).

We can prove that any substantially bounded solution of the system (1) under the condition (2) and $\operatorname{Im}\lambda=0$ assumes the following asymptotic representations:

$$y_j(x,\lambda) = A_j e^{i\lambda\xi jx} + 0(1),\ j = 1,2,3 \tag{3}$$

$$y_j(x,\lambda) = B_j e^{i\lambda\xi jx} + 0(1),\ j = 4,5,6 \tag{4}$$

We consider two problems for the system (1) on a semi-axis. The k-th problem is following. We need to find the solution of the system (1) satisfying the boundary conditions:

$$z_2^k(0,\lambda) = h_k z_1^k(0,\lambda),\ k = 1,2. \tag{5}$$

where $z_2^k(x,\lambda) = (y_4^k(x,\lambda), y_5^k(x,\lambda), y_6^k(x,\lambda))$,

$z_1^k(x,\lambda) = (y_1^k(x,\lambda), y_2^k(x,\lambda), y_3^k(x,\lambda))$,

$$h_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad h_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

by given asymptotics (3).

Matrix-function $S(\lambda) = (S_1(\lambda), S_2(\lambda), S_3(\lambda))$, where $S_k(\lambda) = \left\| S_{11}^k(\lambda) \right\|_{k=1}^3$ takes $(A_1, A_2, A_3)^t$ to $(B_1, B_2, B_3)^t$ (t is transposition), is called scattering matrix for the system (1) on a semi-axis.

Theorem. Let the coefficients of the equations system (1) satisfy the conditions (2) and the condition $c_{61}(x) = \ldots = c_{65}(x) = 0$ and the discrete spectrum is absent. Then the rest coefficients of this system are uniquely determined by the scattering matrix.

The inverse problem for the system (1) on the whole axis was studied in [1].

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SOME PROPERTIES OF THE RIESZ POTENTIAL

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Let R^n be an n dimensional Euclidean space of points $x=(x_1,x_2,\ldots,x_n)$, and $B(a,r):=\{x\in R^n : |x-a|\le r\}$ be a closed ball in R^n centered at $a\in R^n$ and radius $r>0$. The class of all locally integrable of ρ-th degree $(1\le p<\infty)$ functions defined in R^n is denoted by $L^p_{loc}(R^n)$, and the class of all locally bounded functions is denoted by $L^\infty_{loc}(R^n)$.

For the function $f\in L^1_{loc}(R^n)$ assume that ([1],[3])

$$P_{k,B(a,r)}f(x):=\sum_{|\nu|\le k}\left(\frac{1}{|B(a,r)|}\int_{B(a,r)} f(t)\varphi_\nu\left(\frac{t-a}{r}\right)dt\right)\varphi_\nu\left(\frac{x-a}{r}\right).$$

It is easy to see that $P_{k,B(a,r)}f\in P_k$, where P_k is the set of all polynomials in R^n of degree at most k.

Let $x\in R^n$, $k\in N$ and suppose that for any ν with the condition $|\nu|\le k-1$ there is a finite limit

$$\lim_{x\to\infty} D^\nu P_{k-1,B(x,r)}f(x)=:D_\nu f(x) \tag{1}$$

where $D^\nu g:=\dfrac{\partial^{|\nu|}g}{\partial x_1^{\nu_1}\partial x_2^{\nu_2}\ldots\partial x_n^{\nu_n}}$.

Let us denote

$$P_{k-1,x} f(t) := \sum_{|\nu| \le k-1} D_\nu f(x) \cdot \frac{(t-x)^\nu}{\nu!}\ , \qquad (2)$$

$$n_f^k(x;\delta)_p := \sup\left\{ |B(x,r)|^{-\frac{1}{p}} \cdot \left\| f - P_{k-1,x} f \right\|_{L^p(B(x,r))} : r \le \delta \right\},$$

$$\left(x \in R^n, \delta > 0\right),$$

where $\nu! = \nu_1! \nu_2! \ldots \nu_n!$.

Consider the following integral operator of potential type (see. [2], [4])

$$R_{\alpha,k} f(x) = \int_{R^n} \left\{ K_\alpha(x-y) - \left(\sum_{|\nu| \le k-1} \frac{x^\nu}{\nu!} D^\nu K_\alpha(-y) \right) X_{\{|t|>1\}}(y) \right\} f(y) dy$$

,

where $K_\alpha(x) = |x|^{\alpha-n}$, $0 < \alpha < n$, $k \in N$, $X_{\{|t|>1\}}$ is the characteristic function of $\{t \in R^n : |t| > 1\}$.

The operator $R_{\alpha,k} f$ is a modification of the Riesz potential

$$I_\alpha f(x) = \int_{R^n} |x-y|^{\alpha-n} f(y) dy .$$

Theorem 2.4. Let $f \in L^p_{loc}(R^n)$, $1 \le p \le \infty$, $k, l \in N$, $k \ge [\alpha] + l$, $x \in R^n$, $\overline{f} := R_{\alpha,k} f$ and

$$\int_0^1 \frac{n_f^l(x;t)}{t^{k-\alpha}} dt < +\infty\ , \ \int_1^\infty \frac{n_f^l(x;t)}{t^{k-\alpha+1}} dt < +\infty\ .$$

Then the following inequality holds true

$$n_{\overline{f}}^k(x;\delta)_p \le c \left(\delta^{k-1} \int_0^\delta \frac{n_f^l(x;t)_p}{t^{k-\alpha}} dt + \delta^k \int_\delta^\infty \frac{n_f^l(x;t)_p}{t^{k-\alpha+1}} dt \right), \qquad (3)$$

where constant $c>0$ does not depend on f, δ and x.

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BASE OF NANOHYDRODYNAMICS OF VISCOUS FLUID WITH REGARD TO QUANTUM-MECHANICAL EFFECTS

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In the paper we offer determining equations of the motion of viscous fluid in nano-channels ($10^{-9}m \le d \le 10^{-4}m$), and also give generalizated Navier condition of fluid slippage with regard to

influence of physical field stress penetrating deep and holding on the boundary between the vessel wall and fluid, in the form:
- equation of motion of viscous compressible fluid in

$$\begin{cases} \dfrac{\partial \upsilon_x}{\partial t} + \upsilon_x \dfrac{\partial \upsilon_x}{\partial x} + \upsilon_y \dfrac{\partial \upsilon_x}{\partial y} + \upsilon_z \dfrac{\partial \upsilon_x}{\partial z} = \\ = X - \dfrac{1}{\rho_0 (1-\tilde{E})} \cdot \dfrac{\partial p}{\partial x} + \nu_0 \cdot [\Delta \upsilon_x + \dfrac{1}{3} \cdot \dfrac{\partial div\vec{\upsilon}}{\partial x}] + + \dfrac{2}{3} \cdot \dfrac{\nu_0}{(1-\tilde{E})} \cdot \dfrac{\partial \tilde{E}}{\partial x} \cdot (div\vec{\upsilon} - 3\dfrac{\partial \upsilon_x}{\partial x}) \\ \\ \dfrac{\partial \upsilon_y}{\partial t} + \upsilon_x \dfrac{\partial \upsilon_y}{\partial x} + \upsilon_y \dfrac{\partial \upsilon_y}{\partial y} + \upsilon_z \dfrac{\partial \upsilon_y}{\partial z} = \\ = Y - \dfrac{1}{\rho_0 (1-\tilde{E})} \cdot \dfrac{\partial p}{\partial y} + \nu_0 \cdot [\Delta \upsilon_y + \dfrac{1}{3} \cdot \dfrac{\partial div\vec{\upsilon}}{\partial y}] - \dfrac{\nu_0}{(1-\tilde{E})} \cdot \dfrac{\partial \tilde{E}}{\partial x} \cdot (\dfrac{\partial \upsilon_x}{\partial y} + \dfrac{\partial \upsilon_y}{\partial x}) \\ \\ \dfrac{\partial \upsilon_z}{\partial t} + \upsilon_x \dfrac{\partial \upsilon_z}{\partial x} + \upsilon_y \dfrac{\partial \upsilon_z}{\partial y} + \upsilon_z \dfrac{\partial \upsilon_z}{\partial z} = \\ = Z - \dfrac{1}{\rho_0 (1-\tilde{E})} \dfrac{\partial p}{\partial z} + \nu_0 \cdot [\Delta \upsilon_z + \dfrac{1}{3} \dfrac{\partial div\vec{\upsilon}}{\partial z}] - \dfrac{\nu_0}{(1-\tilde{E})} \cdot \dfrac{\partial \tilde{E}}{\partial x} \cdot (\dfrac{\partial \upsilon_x}{\partial z} + \dfrac{\partial \upsilon_z}{\partial x}) \end{cases}$$

-substance conservation equation

$$\frac{\partial \rho_0}{\partial t} + \rho_0 \cdot [div \cdot \vec{\upsilon} - \frac{1}{1-\tilde{E}(x)} \cdot \frac{\partial \tilde{E}(x)}{\partial x} \cdot \upsilon_x] = 0$$

- generalized Navier boundary condition in nanohydrodynamics with regard to influence of quantum mechanical effects of transformation of homogeneous fluid into inhomogeneous one:

$$\upsilon(x) = \upsilon_0 + L \cdot \frac{\partial \upsilon}{\partial x}$$

Here $\rho = \rho_0 \cdot [1 - \tilde{E}(x)]$ and $\mu = \mu_0 \cdot [1 - \tilde{E}(x)]$ is the function of density and viscosity of the fluid in depth of fluid, $\nu = \frac{\mu_0}{\rho_0}$ is kinematic viscosity. All of them depend on experimentally given quantum-mechanical effects given from the density of the action of the physical field tension $\vec{E} = \frac{E(x)}{E_0}$ of hydrodynamics of viscous.

The basic quality and quantity results of hydrodynamics of viscous fluids in small-dimensional systems ($10^{-9} m \le d \le 10^{-4} m$) are the followings:

- in formation of empty space between the vessel wall and fluid of size $\Delta = 0{,}12 \cdot R_0$,
- in the depth of fluid close to the wall the homogeneous fluid will be transformed into inhomogeneous one,
- mechanical characteristics of inhomogeneous part of fluid (of density $\rho(r)$ and viscosity $\mu(r)$) in depth, depending on the action of physical field strain will change in the form:

$$\rho = \rho_0 \cdot [1 - \tilde{E}(r)], \qquad \mu = \mu_0 \cdot [1 - \tilde{E}(r)]$$

- The fluid slippage holds at the expense of influence of quantum-mechanical effects.

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TWO – FOLD COMPLETENESS OF A SYSTEM OF ROOT FUNCTIONS OF THE PROBLEM OF THE IRREGULAR BOUNDARY VALUE PREBLEMS OF ORDINARY DIFFERENTIAL EQUATIONS

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Consider a nonlocal boundary value problem:

$$L(\lambda, D)u \coloneqq 2\lambda^2 u(x) - \lambda u'(x) - u''(x) + b(x)u(x) = 0 \quad x \in (0,1) \quad (1)$$

$$L_1 u: u'(x) - u'(1) = 0 \quad L_2 u \coloneqq u(0) - 2u(1) = 0 \quad (2)$$

where$b$ (x) is given function. The root of the equations $-\omega^2- \omega+2 = 0$ are $\omega_1 = -2$, $\omega_2 = 1$. Problem (1)-(2) is Birkhoff-irregular [1]. Let us show that problem (1)-(2) is 1-regular with respect to a system of numbers $\omega_1 = -2$, $\omega_2 = 1$ (see [2]) and is not 1-regular with respect to a system of numbers $\omega_2 = 1$, $\omega_1 = -2$.

Inside the angle

$$-\frac{\pi}{2} + \varepsilon < arg\lambda < \frac{\pi}{2} - \varepsilon, \quad (3)$$

the resolvent of problem (1)-(2) decreases like to the regular case and, inside the angle

$$\frac{\pi}{2} + \varepsilon < arg\lambda < \frac{3\pi}{2} - \varepsilon, \quad (4)$$

the resolvent of problem (1)-(2) increases with respect to the spectral parameter λ.

Theorem. Let $b \in W_q^{k+1}(0,1), where\ 1 < q \leq \infty$, $b^{(j)}(0) = b^{(j)}(1) = 0, j = 0, \ldots, k-1$, $b^{(k)}(1) + 2(-1)^{k+1} b^{(k)}(0) \neq 0$for some$k \in N$.

Then the spectrum of problem (1)-(2) is discrete and the system of root functions of problem (1)-(2) is two fold complete in the space $W_2^1\big((0,1); u(0)-2u(1)=0\big) \oplus L_2(0,1)$.

Proof. Let us denote

$$H_k := W_2^k(0,1),\ \ k=0,1,2,\ \ H^\nu := H_0^\nu := C,\ \ \nu = 1,2$$

Consider an operatorsA_1 and A_2 defined by the equality.

$$A_1 u := -\frac{1}{2}u'(x), \qquad A_2 := -\frac{1}{2}u''(x) + \frac{b(x)}{2}u(x),$$

And functionalA_{10}, A_{11}, A_{20}defined by the equalities

$$A_{10}u := u'(0) + u'(1), \qquad A_{20}u := u(0) - 2u(1).$$

Then problem **(1)-(2)**is rewritten in the operator form as the following system of pencil equations

$$L(\lambda)u := \lambda^2 u + \lambda A_1 u + A_2 u = 0, L_1 u := A_{10}u = 0, L_1 u := A_{10}u = 0,$$

where$u \in H_2 = W_2^2(0,1)$.

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A BOUNDARY VALUE PROBLEM FOR A FIRST ORDER ORDINARY DIFFERENTIAL EQUATION WITH CONTINUOUSLY CHANGING ORDER OF DERIVATIVE

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We consider the following boundary value problem:

$$a_1\int_0^{\frac{1}{3}} D_x{}^{\alpha} y(x,\lambda)d\alpha + a_2\int_{\frac{1}{3}}^{\frac{2}{3}} D_x{}^{\alpha} y(x,\lambda)d\alpha + a_3\int_{\frac{2}{3}}^{1} D_x{}^{\alpha} y(x,\lambda)d\alpha = 0,$$

$$0 < x_0 < x < x_1, \qquad (1)$$

$$\alpha_1 y(x_0,\lambda) + \alpha_2 y(x_1,\lambda) = \varphi_1,$$

$$\beta_1 D_x^{1/3} y(x,\lambda)\big|_{x=x_0} + \beta_2 D_x^{1/3} y_i(x,\lambda)\big|_{x=x_1} = \varphi_2, \qquad (2)$$

$$\gamma_1 D_x^{2/3} y(x,\lambda)\big|_{x=x_0} + \gamma_2 D_x^{2/3} y(x,\lambda)\big|_{x=x_1} = \varphi_3.$$

It is known [1], that the solution of a first order differential equation:

$$\int_0^1 a(\alpha) D^{\alpha} y(x) d\alpha = 0, \qquad 0 < x_0 < x, \quad (3)$$

where the coefficients $a(\alpha)$ is a step function, is sought in the form of the Volterra function, i.e. in the form:

$$y(x) \equiv y(x,\lambda) = \int_{-1}^{\infty} \frac{x^{\beta}}{\beta!}\lambda^{\beta} d\beta = \int_{-1}^{\infty} \frac{(\lambda x)^{\beta}}{\beta!} d\beta.$$

Then,

$$D^{\alpha} y(x) = D^{\alpha} y(x,\lambda) = \int_{-1}^{\infty} \frac{\lambda^{\beta} x^{\beta-\alpha}}{(\beta-\alpha)!} d\beta\,,$$

and the characteristic equation will have the form:

$$a_3\rho^3 + (a_2 - a_3)\rho^2 + (a_1 - a_2)\rho - a_1 = 0$$

By the found λ_1, λ_2 and λ_3, the general solution of the equation (1), is determined in the following form :

$$y(x,\lambda) = c_1 y_1(x,\lambda_1) + c_2 y_2(x,\lambda_2) + c_3 y_3(x,\lambda_3)\,. \quad (4)$$

To solve problem (1) - (2), it is necessary to determine the coefficients c_1, c_2, c_3 and they are determined from the following system:

$$\sum_{i=1}^{3} c_i \left[\alpha_1 y_i(x_0,\lambda_i) + \alpha_2 y_i(x_1,\lambda_i)\right] = \varphi_1, \quad (5)$$

$$\sum_{i=1}^{3} c_i \left[\beta_1 D_x^{1/3} y_i(x,\lambda_i)\Big|_{x=x_0} + \beta_2 D_x^{1/3} y_i(x,\lambda_i)\Big|_{x=x_1}\right] = \varphi_2, \quad (6)$$

$$\sum_{i=1}^{3} c_i \left[\gamma_1 D_x^{2/3} y_i(x,\lambda_i)\Big|_{x=x_0} + \gamma_2 D_x^{2/3} y_i(x,\lambda_i)\Big|_{x=x_1}\right] = \varphi_3, \quad (7)$$

where $0 < x_0 \le x.$

Subject to the solvability of the system (5)-(6)-(7), substituting the obtained values for c_1, c_2 and c_3, from this system to general solution (4) we determine the solution of problem (1)-(2).

Under certain conditions on the problem data, i.e. a_1, a_2, a_3, φ_1, φ_2, φ_3, and also α_1, α_2, β_1, β_2, γ_1, γ_2, we proved a theorem on the existence of the unique solution of problem (1)-(2).

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NECESSARY CONDITIONS CONNECTED WITH HELMHOLTZ EQUATION ON SEMI-LINEAR

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1.Statement of the problem

The following problem for the Helmholtz equation with nonlocal boundary conditions is considered in this paper:

$$\sum_{i,j=1}^{2} a_{ij}\frac{\partial^2 u(x)}{\partial x_i \partial x_j} + k^2 u(x) = 0, \qquad x = (x_1, x_2) \in D \subset R^2, \tag{1}$$

$$\sum_{j=1}^{2}\left[\alpha_{ij}^{(0)}(x_2)\frac{\partial u(x)}{\partial x_j}\bigg|_{x_1=0} + \alpha_{ij}^{(1)}(x_2)\frac{\partial u(x)}{\partial x_j}\bigg|_{x_1=1}\right] + \alpha_{i0}^{(0)}(x_2)u(0,x_2) +$$

$$+\alpha_{i0}^{(1)}(x_2)u(1,x_2) = \alpha_i(x_2), \quad i=1,2; \quad x_2 \geq 0 \tag{2}$$

$$\sum_{j=1}^{2}\left[\beta_j(x_1)\frac{\partial u(x)}{\partial x_j}\bigg|_{x_2=0}\right] + \beta_0(x_1)u(x_1,0) = \beta(x_1), \quad x_1 \in [0,1]. \tag{3}$$

2.Necessary conditions

Under some restrictions on domain D,coefficients and right side of equation and boundary conditions, taking into account fundamental solution [1]

$$U(x-\xi,k) = -\frac{i}{4}\mathrm{H}_0^{(1)}\left(k\sigma(x-\xi)\right)$$

of equation (1), where $\mathrm{H}_0^{(1)}(z)$-Hankel function and

$$\sigma^2(x-\xi) = \frac{1}{a^2}\left[a_{22}(x_1-\xi_1)^2 - 2a_{12}(x_1-\xi_1)(x_2-\xi_2) + a_{11}(x_2-\xi_2)^2\right]$$

is a given metrics., for solvability of problem (1)-(3) using second Green's formula and boundary conditions was obtained necessary conditions in form:

$$u(\xi_1,0,k) = -\frac{a_{12}}{\pi}\int_0^1 \frac{u(x_1,0,k)}{x_1-\xi_1}dx_1 + ...$$

$$u(0,\xi_2,k) == \frac{a_{12}}{\pi}\int_0^\infty \frac{u(0,x_2,k)}{x_2-\xi_2}dx_2 + ...$$

$$u(1,\xi_2,k) = -\frac{a_{12}}{\pi}\int_0^\infty \frac{u(1,x_2,k)}{x_2-\xi_2}dx_2 + ...$$

$$\left.\frac{\partial u(\xi,k)}{\partial \xi_2}\right|_{\xi_2=0} = -\frac{a_{12}}{\pi}\int_0^1 \left.\frac{\partial u(x,k)}{\partial x_1}\right|_{x_2=0}\frac{dx_1}{x_1-\xi_1} - \frac{a_{22}}{\pi}\int_0^1 \left.\frac{\partial u(x,k)}{\partial x_2}\right|_{x_2=0}\frac{dx_1}{x_1-\xi_1} + ...$$

$$\left.\frac{\partial u(\xi,k)}{\partial \xi_1}\right|_{\xi_1=0} = \frac{a_{22}}{\pi}\int_0^\infty \left.\frac{\partial u(x,k)}{\partial x_2}\right|_{x_1=0}\frac{dx_2}{x_2-\xi_2} + \frac{a_{12}}{\pi}\int_0^1 \left.\frac{\partial u(x,k)}{\partial x_1}\right|_{x_1=0}\frac{dx_2}{x_2-\xi_2} + ...$$

$$\left.\frac{\partial u(\xi,k)}{\partial \xi_1}\right|_{\xi_1=1} = \frac{a_{22}}{\pi}\int_0^\infty \left.\frac{\partial u(x,k)}{\partial x_2}\right|_{x_1=1}\frac{dx_2}{x_2-\xi_2} - \frac{a_{12}}{\pi}\int_0^\infty \left.\frac{\partial u(x,k)}{\partial x_1}\right|_{x_1=1}\frac{dx_2}{x_2-\xi_2} + ...$$

$$\left.\frac{\partial u(\xi,k)}{\partial \xi_2}\right|_{\xi_2=0} = \frac{a_{12}}{\pi}\int_0^1 \left.\frac{\partial u(x,k)}{\partial x_2}\right|_{x_2=0}\frac{dx_1}{x_1-\xi_1} + \frac{a_{11}}{\pi}\int_0^1 \left.\frac{\partial u(x,k)}{\partial x_1}\right|_{x_2=0}\frac{dx_1}{x_1-\xi_1} + ...$$

$$\left.\frac{\partial u(\xi,k)}{\partial \xi_2}\right|_{\xi_1=0} = -\frac{a_{12}}{\pi}\int_0^\infty \left.\frac{\partial u(x,k)}{\partial x_2}\right|_{x_1=0}\frac{dx_2}{x_2-\xi_2} - \frac{a_{11}}{\pi}\int_0^\infty \left.\frac{\partial u(x,k)}{\partial x_1}\right|_{x_1=0}\frac{dx_2}{x_2-\xi_2} + ...$$

$$\left.\frac{\partial u(\xi,k)}{\partial \xi_2}\right|_{\xi_1=1} == \frac{a_{12}}{\pi}\int_0^\infty \left.\frac{\partial u(x,k)}{\partial x_2}\right|_{x_1=1} \frac{dx_2}{x_2-\xi_2} + \frac{a_{11}}{\pi}\int_0^\infty \left.\frac{\partial u(x,k)}{\partial x_1}\right|_{x_1=1} \frac{dx_2}{x_2-\xi_2} + \ldots$$

Note, that kernels of integrals in obtained necessary conditions has singularity and dots notes regular terms.

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NEW FUNDAMENTAL SOLUTION OF TWO-DIMENSIONAL LAPLACE EQUATION

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Let $U(x)$ be a fundamental solution of Laplace equation:

$$\Delta U(x) \equiv \frac{\partial^2 U(x)}{\partial x_1^2} + \frac{\partial^2 U(x)}{\partial x_2^2} = \delta(x) = \delta(x_1)\delta(x_2)\,. \qquad (1)$$

As

$$\Delta \equiv \left(\frac{\partial(x)}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)\left(\frac{\partial(x)}{\partial x_1} - i\frac{\partial}{\partial x_2}\right), \qquad (2)$$

then a fundamental solution of (1) will be sought in the form

$$U(x)=C\theta(x_2+ix_1)\theta(x_1+ix_2) \tag{3}$$

where $\theta(z)$ is a Heavicide function but with a complex argument.

Taking into account that

$$\frac{\partial^2 U(x)}{\partial x_1^2}=-C\delta'(x_2+ix_1)\theta(x_1+ix_2)+Ci\delta(x_2+ix_1)\delta(x_1+ix_2)+$$

$$+Ci\delta(x_2+ix_1)\delta(x_1+ix_2)+C\theta(x_2+ix_1)\delta'(x_1+ix_2), \tag{4}$$

$$\frac{\partial^2 U(x)}{\partial x_2^2}=C\delta'(x_2+ix_1)\theta(x_1+ix_2)+Ci\delta(x_2+ix_1)\delta(x_1+ix_2)+$$

$$+Ci\delta(x_2+ix_1)\delta(x_1+ix_2)-C\theta(x_2+ix_1)\delta'(x_1+ix_2). \tag{5}$$

Substituting (4) and (5) into (1) we'll have:

$$\Delta U(x)\equiv\frac{\partial^2 U(x)}{\partial x_1^2}+\frac{\partial^2 U(x)}{\partial x_2^2}=4Ci\delta(x_2+ix_1)\delta(x_1+ix_2)=$$

$$=\delta(x_1)\delta(x_2)$$

or $4Ci\delta(x_2+ix_1)\delta(2x_1)=2Ci\delta(x_2)\delta(x_1)=\delta(x_1)\delta(x_2)$

Thus, for the arbitrary constant C we have

$$C=\frac{1}{2i}\ . \tag{6}$$

Taking into account (6) in (3), for a two-dimensional Laplace equation we have the fundamental solution

$$U(x)=\frac{1}{2i}\theta(x_2+ix_1)\theta(x_1+ix_2)\ . \tag{7}$$

We should note that fundamental solution (7) is a fundamental solution neither in direction x_1 nor in direction x_2.

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ON MAPPINGS OF P-DIMENSIONAL SURFACES IN EUCLIDEAN SPACES EN

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In the paper we study one-to-one mappings of p dimensional surfaces of n dimensional euclidean spaces in 2n dimensional euclidean space. In the paper we use the method of moving frames and exterior forms. The following results are obtained:

Theorem 1. If the net $\Sigma_p \subset V_p$ is conjugate then the corresponding nets $\overline{\Sigma}_p = T(\Sigma_p)$ and $\Sigma_p^* \subset V_p^*$ are also conjugate.

Theorem 2. The conjugate net Σ_p (or $\overline{\Sigma}_p = T(\Sigma_p)$) is a basis for the mapping T if and only if the nets Σ_p and $\overline{\Sigma}_p = T(\Sigma_p)$ are orthogonal.

Theorem 3. $\overline{\Sigma'}_p = T(\Sigma'_p)$ if and only if the mapping T is conformal.

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BASIS PROPERTIES OF A BOUNDARY VALUE PROBLEM WITH BOUNDARY CONDITIONS DEPENDING ON THE EIGENPARAMETER

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We consider the following spectral problem

$$y^{(4)}(x)-(q(x)y'(x))'=\lambda y(x),\ 0<x<1, \quad (1)$$

$$y''(0)=y''(1)=Ty(0)-a\lambda y(0)=Ty(1)-c\lambda y(1)=0, \quad (2)$$

where $\lambda\in C$ is a spectral parameter, $Ty\equiv y''-qy'$, $q\in AC[0,1]$, and $q(x)>0, x\in[0,1]$, a, c are real constants such that $a>0$ and $c\neq 0$.

Problem (1)-(2) was considered in [1, 2], where it was proved that the eigenvalues of this problem are real, simple, with the exception of at least one (in the case $c>0$), and form an infinitely increasing sequence $\{\lambda_k\}_{k=1}^{\infty}$ such that $\lambda_k>0$ for $k\geq 4$. Moreover, if r and l ($r\neq l,\ r,l\geq 4$) are arbitrary fixed positive integers having the different parity, then the system of root functions $\{y_k(x)\}_{k=1,\,k\neq r,l}^{\infty}$ of problem (1)-(2) forms a basis in the space $L_p(0,1), 1<p<\infty$, which is an unconditional basis for $p=2$.

The main result of this note is the following theorem.

Theorem. *Let r and l ($r \neq l,\ r,l \geq 4$) are arbitrary fixed positive integers having the same parity, and $a \neq -c$. Thenthere exists a positive integer k_0 such that for sufficiently large integers r and l satisfying the inequality $|r-l| \geq k_0$ the root functions system $\{y_k(x)\}_{k=1,k\neq r,l}^{\infty}$ of problem (1)-(2) to form a basis in $L_p(0,1)$, $1<p<\infty$, which is an unconditional basis for $p=2$.*

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SOLUTION OF A BOUNDARY VALUE PROBLEM FOR THE BESSEL-HELMHOLTZ EQUATION

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Denote by $R^m(y), m$--dimensional Euclidean space with the point $y=(y_1,...,y_m)$, by $R^n(x)$ the same space with the point $x = (x_1, ..., x_n)$, $R_+^n(x) = \{x \in R^n(x); x = (x_1, x_2, x'), x_1 > 0, x_2 > 0\}$, $\gamma_1 > 0, \gamma_2 > 0$.

Let $\Lambda = R_+^n(x) \times \Omega$ be a cylindrical domain in $R_+^n(x) \times R_+^m(y)$ where Ω is a bounded domain in $R^m(y)$ with a smooth boundary $\partial\Omega$, where $\partial\Omega \in C^{\left[\frac{3m}{2}\right]}$.

The generalized shift operator T^y is defined by

$$T^y f(x) = C_\gamma \int_0^\pi \int_0^\pi f(\xi_1, \xi_2, x' - y') \sin^{\gamma_1 - 1} \beta_1 \sin^{\gamma_2 - 1} \beta_2 d\beta_1 d\beta_2,$$

where $\xi_i = \left(x_i^2 - 2x_i y_i \cos\beta_i + y_i^2\right)^{\frac{1}{2}}, x' = (x_3, \ldots, x_n) \in \mathbb{R}^{n-2}$ and $C_\gamma = \pi^{-1} \quad \Gamma^{-1}\left(\frac{|\gamma|}{2}\right) \Gamma\left(\frac{\gamma_1 + 1}{2}\right) \Gamma\left(\frac{\gamma_2 + 1}{2}\right)$.

The Laplace-Bessel differential operator

$$\Delta_B = \sum_{i=1}^{2} \frac{\gamma_i}{x_i} \frac{\partial}{\partial x_i} + \sum_{i=1}^{2} \frac{\partial^2}{\partial x_i^2}.$$

Consider in Λ the following boundary value problem

$$\left(\Delta_B + k^2\right) u(k, x, y) = f(x, y), \tag{1}$$

$$u(k, x, y)\big|_\Gamma = 0, \tag{2}$$

where $f(x,y) \in C_0^\infty(\Lambda), k^2$ is a constant number not necessarily real, Γ is the boundary of the cylinder Λ.

From this theorem it follows

Theorem 1. *The solution of problem (1)-(2) with a complex parameter* k_ε^2 *is represented in the form*

$$u(k_\varepsilon, x, y) = \int \cdots \int_\Lambda G(k_\varepsilon, x-\xi, y, z) f(\xi, z) d\Lambda,$$

this solution is unique, where $G(k_\varepsilon, x-\xi, y, z)$ *is Green's function of problem (1)-(2).*

HARDY-ORLICZ CLASSES AND BASES IN THEM

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In this work, basicity of classical trigonometric systems in Orlicz spaces are considered. Hardy-Orlicz classes corresponding to a unit ball on the complex plane are defined. As analog of the classical Riesz theorem of the theory of analytic functions for Hardy-Orlicz classes is defined. The validity of Cauchy formula for functions from these classes also is established.

Let us take some standard notations. N is the set of natural numbers; $Z_+ = \{0\} \cup N; Z = \{-N\} \cup Z_+$; R is the set of real numbers; C is a complex plane; $\omega = \{z \in C : |z| < 1\}$ – is a circle in C.

Definition 1. *Continuous convex function* $M(\cdot): R \to R$ *is called* N *-function, if it is even and satisfies the conditions*

$$\lim_{u\to 0}\frac{M(u)}{u}=0;\ \lim_{u\to\infty}\frac{M(u)}{u}=\infty.$$

Definition 2. N *-function* $M(\cdot)$ *satisfy the* Δ_2 *-condition for large numbers* u *,if* $\exists k>0 \wedge \exists u_0 \geq 0:$

$$M(2u)\leq kM(u), \forall u \geq u_0.$$

Let $M(\cdot)$ be some N -function, $G \subset R -$ be a (by Lebesgue) measurable set with finite measure . By $L_0(G)$ denote the set of all measurable functions G . Accept

$$\rho_M(u)=\int_G M[u(x)]dx,$$

and let

$$L_M(G)=\{u \in L_0(M): \rho_M(u)<+\infty\}.$$

$L_M(G) -$ called the Orlicz class.

By H_M^+ denote Hardy-Orlicz class of analytic functions $F(\cdot)$ inside ω with the norm

$$\|F\|_{H_M^+}=\sup_{0<r<1}\sup_{\rho_{M^*}(\nu)\leq 1}|(F_r(\cdot);\nu(\cdot))|=\sup_{0<r<1}\|F_r(\cdot)\|_M, \text{ where}$$

$$F_r(t)=F(re^{it}).$$

The following analogue of the classical Riesz theorem holds in Hardy classes.

Theorem 1.Let $M \in \Delta_2(\infty)$ be some N -function. Then for $\forall F \in H_M^+$:

$\alpha)$ $\rho_M(F_r) \to \rho(F^+),\ r \to 1-0,$

$\beta)$ $\rho_M(F_r - F^+) \to 0,\ r \to 1-0;$

are true, where $F_r\left(e^{it}\right)=F\left(re^{it}\right)$, F^+ *is a non-tangential boundary values of* F *on* γ .

From Theorem 1 immediately follows

Corollary 1.*Let* $M\in\Delta_2(\infty)$*. Then for* $\forall F\in H_M^+$ *:*

$\alpha)\ \|F_r\|_M\to\|F^+\|_M,\ r\to 1-0;$

$\beta)\ \|F_r-F^+\|_M\to 0,\ r\to 1-0,$

are true.

A HARNACK'S INEQUALITY FOR 2-ND ORDER NON-UNIFORMLY DEGENERATING ELLIPTIC EQUATIONS OF NON-DIVERGENT STRUCTURE

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In this report, we assert a Harnack's type inequality for the class of positive solutions of second order non-uniformly degenerating elliptic equations of non-divergent structure.

Let E_n be n-dimensional Euclidean space of points $x=(x_1,...,x_n), n\geq 3,$ D be a bounded domain in E_n with its boundary $\partial D\in C^2$ and $0\in\overline{D}$. Consider in D an elliptic equation

$$Lu=\sum_{i,j=1}^{n}a_{ij}(x)\frac{\partial^2 u(x)}{\partial x_i\partial x_j}+\sum_{i=1}^{n}b_i(x)\frac{\partial u(x)}{\partial x_i}c(x)u(x)=0, \qquad (1)$$

with matrix $A=\|a_{ij}(x)\|$ to be symmetric and such that for any $x\in D$ and $\xi\in E_n$ it is satisfied the conditions below.

$$\gamma\sum_{i=1}^{n}\lambda_i(x)\xi_i \le \sum_{i,j=1}^{n}a_{ij}(x)\xi_i\xi_j \le \gamma^{-1}\sum_{i=1}^{n}\lambda_i(x)\xi_i^2, \qquad (2)$$

where $\gamma\in(0,1]$ is a constant, $\lambda_i(x)=\left(\frac{\omega_i^{-1}(\rho(x))}{\rho(x)}\right)^2$, $i=1,\ldots,n,$

$\rho(x)=\sum_{i=1}^{n}\omega_i\left(|x_i|\right)$, $\omega_i(t)$ are monotony increasing functions on $[0, diam\, D]$ and $\omega_i(0)=0$. The $\omega_i^{-1}(t)$ are inverse functions to $\omega_i(t)$. Furthermore, there exist $\alpha,\beta\in(1,\infty)$, $A>0$, and $n<q<\frac{n^2}{n-2}$ such that for all $t\in\left[0,\frac{1}{2}diam\ D\right]$

$$\alpha\omega_i(t)\le\omega_i(2t)\le\beta\omega_i(t), \qquad (3)$$

and

$$\left(\frac{\omega_i^{-1}(t)}{t}\right)^{q-1}\int_0^{\omega_i^{-1}(t)}\left(\frac{\omega_i(\tau)}{\tau}\right)^q d\tau\le At, \quad i=1,\ldots,n. \qquad (4)$$

Also assume that

$$|b_i(x)|\le B_0,\ i=1,\ldots,n;\ -C_0\le c(x)\le 0,\ x\in D \qquad (5)$$

with B_0 and C_0 are positive constants.

For $x^0\in E_n$, $R\in(0,1]$ and $K>0$ by $E_R^{x^0}(k)$ denote the ellipsoid

$$\left\{x\in E_n:\sum_{i=1}^{n}\frac{(x_i-x_0^i|)^2}{(\omega_i^{-1}(R))^2}<K^2\right\}.$$

Then following assertion takes place.

Theorem. Let $u(x)$ be a positive solution of (1) in the domain D for which $\overline{E}_R^{x^0}(k)\subset D$. The coefficients of operator L are assumed to satisfy (2)-(5). Then there exists a constant $C(\alpha,\beta,n,B_0,C_0)>0$ such that for any positive solution of (1) it holds

$$\sup_{E_R^0\left(\frac{1}{4}\right)} u(x) \le c \inf_{E_R^0\left(\frac{1}{4}\right)} u(x) .$$

We cite [1-3] for the subject and additional information for this report.

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INVERSE PROBLEMS FOR THE QUADRATIC PENCIL OF THE DISCONTINUOUS STURM-LIOUVILLE EQUATION

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We consider the following boundary value problem

$$-y'' + \{q(x) + 2\lambda p(x)\}y = \lambda^2 \rho(x)y, \quad 0 \le x \le \pi \qquad (1)$$

$$y(0) = y(\pi) = 0 \qquad (2)$$

where λ is the spectral parameter,$y = y(x,\lambda)$ is an unknown function,$q(x) \in L_2(0,\pi)$, $p(x) \in W_2^1(0,\pi)$ are real valued functions, and $\rho(x)$ is the following picewise constant function with discontinuity at the point $a \in (0,\pi)$ such that $a > \frac{\alpha\pi}{\alpha+1}$:

$$\rho(x) = \begin{cases} 1 & ,0 \le x \le a \\ \alpha^2 & ,a \le x \le \pi \end{cases}, 0 < \alpha \ne 1$$

Sturm-Liouville equations with potentials depending on the spectral parameter arise in various problems of mathematics and physics. It is well known that in the case $\rho(x) = 1$ the equation (1) appears for modelling of some problems connected with the scattering of waves and particles in physics. Direct and inverse spectral problems in a finite interval for the case $\rho(x) = 1$ was first investigated in [1,2]. For fulher discussing of the inverse spectral theory for equation (1) in a finite interval with $\rho(x) = 1$ we refer to works [3,4]. In the study [5]new integral representations was obtained for the fundamental solutions of Eq. (1). In the present study we obtain the sufficient conditions for the inverse problems by the spectral data and by two spectra.

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FRACTIONAL INTEGRAL ON CARLESON CURVES IN VANISHING GENERALIZED MORREY SPACES

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The classical Morrey spaces were introduced by Morrey [1] to study the local behavior of solutions to second-order elliptic partial differential equations. Guliyev, Mizuhara and Nakai (1994) introduced generalized Morrey spaces $M_{p,\varphi}\left(R^n\right)$.

Let $\Gamma=\{t\in C: t=t(s), 0\le s\le l\le\infty\}$ be a rectifiable Jordan curve in the complex plane C with arc-length measure $\nu(t)=s$, here $l=\nu\Gamma=$ lengths of Γ:

We denote

$$\Gamma(t,r)=\Gamma\cap B(t,r),\ t\in\Gamma,\ r>0,$$

where $B(t,r)=\{z\in C: |z-t|<r\}$.

A rectifiable Jordan curve Γ is called a Carleson curve if the condition

$$\nu\Gamma(t,r)\le c_0 r$$

holds for all $t\in\Gamma$ and , where the constant $c_0>0$ does not depend on t and r.

We consider vanishing generalized Morrey spaces defined on Carleson curves Γ. The main result of this talk is to prove the boundedness of the fractional integral operator I^{α} on Carleson curves in vanishing generalized Morrey spaces $VM_{p,\varphi}(\Gamma)$. We find the sufficient conditions on the pair (φ_1,φ_2) which ensures the boundedness of the operator I^{α} from $VM_{p,\varphi_1}(\Gamma)$to $VM_{p,\varphi_2}(\Gamma)$, $\frac{1}{p}-\frac{1}{q}=\alpha$.

GLOBAL BIFURCATION OF SOME NONLINEAR EIGENVALUE PROBLEMS WITH A SPECTRAL PARAMETER IN THE BOUNDARY CONDITION

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We consider the following nonlinear eigenvalue problem

$$(py'')'-(qy')+ry=\lambda\tau(x)y+H(x,y,y',\lambda),\ x\in(0,1),\ (1)$$

$$y'(0)\cos\alpha-p(0)y''(0)\sin\alpha=0,\ (2)$$

$$y(0)\cos\beta-Ty(0)\sin\beta=0,\ (2)$$

$$y'(1)\cos\gamma-p(1)y''(1)\sin\gamma=0,\ (3)$$

$$(a\lambda+b)y(1)-(c\lambda+d)Ty(1)=0,\ (4)$$

where $\lambda \in R$ is an eigenvalue parameter, $p(x) \in C^2[0,1]$, $q(x) \in C^1[0,1]$, $r(x), \tau(x) \in C[0,1]$, $p(x), q(x), \tau(x) > 0, r(x) \in R$, $x \in [0, 1]$, α, β, γ, a, b, c, d are real constants such that $\alpha, \beta, \gamma \in [0, \pi/2]$ and $bc - ad > 0$. The nonlinear term has a representation $H = f + g$, where $f, g \in C([0,1] \times R^5)$ are real-valued functions satisfying the following conditions:there exist $M > 0$ and sufficiently small $\rho_0 > 0$ such that

$$\left| f(x,u,s,\vartheta,w,\lambda)/u \right| \leq M,\ (x,u,s,\lambda) \in [0,\ \pi] \times R^5,$$
$$|u| + |s| + |\vartheta| + |w| \leq \rho_0\,;$$
$$g(x,u,s,\vartheta,w,\lambda) = o(|u| + |s| + |\vartheta| + |w|)$$

in a neighbourhood of $(u, s, \vartheta, w) = (0, 0, 0, 0)$ uniformly in $x \in [0, 1]$ and in $\lambda \in \Lambda$ for every bounded interval $\Lambda \subset R$.

This note is devoted to the refinement of global bifurcation results obtained in [1].

Let $E = C^3[0, 1] \cap B.C.$ be a Banach space with the norm $\| y \|_3 = \sum_{i=0}^{3} \| y^{(s)} \|_\infty$, , $\| y \|_\infty = \max_{x \in [0,\ \pi]} | y(x) |$. For each $k \in \mathrm{N}$, $\lambda \in R$ and $\nu \in \{+, -\}$ by $S_{k,\lambda}^{\nu}$, we denote the set of functions of E which satisfy all conditions for determining the set S_k^{ν} from [2, p.1635] with replacing the condition 1) by the condition $\theta(y,1) = \omega(\lambda) + k\pi$, where $\cot \omega(\lambda) = (a\lambda + b)/(c\lambda + d)$ and $\omega(-d/c) = 0$. Moreover, for $k \in \mathrm{N}$, $\nu \in \{+, -\}$, we define the sets S_k^{ν} by $S_k^{\nu} = \bigcup_{\lambda \in R} S_{k,\lambda}^{\nu}$.

Theorem 1. *For each* $k \in \mathrm{N}$ *and each* $\nu \in \{+, -\}$ *there exits the connected component* D_k^{ν} *of the solutions set of the nonlinear eigenvalue problem (1)-(5) which contain* $J_k \times \{0\}$, *lies in* $(R \times S_k^{\sigma}) \cup (J_k \times \{0\})$ *and is unbounded in* $R \times E$, *where*

λ_k *is a* k*-th eigenvalue of the linear eigenvalue problem (1)-(5) with* $H \equiv 0$ *and* $J_k = [\lambda_k - M/\tau_0, \lambda_k + M/\tau_0]$, $\tau_0 = \min_{x\in[0,1]} \tau(x)$.

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HOLDER CONTINUITY IN RIDGE FUNCTION REPRESENTATION

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A ridge function is a multivariate function of the form

$$g(\mathbf{a}\cdot\mathbf{x}) = g(a_1x_1 + ... + a_mx_m),$$

where $g : R \to R$, $\mathbf{a} = (a_1,...,a_m)$ is a fixed vector (direction) on $R^m \setminus \{0\}$, $\mathbf{x} = (x_1,...,x_m)$ is the variable and $\mathbf{a}\cdot\mathbf{x}$ is the usual inner product in R^m. The term "ridge function" was devised by Logan and Shepp in their paper dedicated to the mathematics of computerized tomography. After a 1981 paper by Friedman and Stuetzle [1] ridge functions started to appear also in statistics, especially, in the theory of projection pursuit and projection regression. Ridge functions are used in many models in neural network theory. For example, in one of the popular models called MLP (multilayer feedforward perceptron) model, the simplest case considers functions of the form $\sum_{i=1}^{r} c_i \sigma\left(\mathbf{w}^i \cdot \mathbf{x} - \theta_i\right)$. Here the weights $\mathbf{w}^i$ are vectors in R^m, the thresholds θ_i and the coefficients c_i are real numbers and the activation function σ is a univariate function. Note that for each $\theta \in R$ and $\mathbf{w} \in R^m \setminus \{0\}$ the function $\sigma(\mathbf{w} \cdot \mathbf{x} - \theta)$ is a ridge function. Ridge functions are interesting also to approximation theorists. In approximation theory, these functions are implemented as an effective and convenient tool for approximating complicated multivariate functions.

We consider the problem of representation by sums of ridge functions with $r, r \geq 1$, fixed directions. Let the directions $\mathbf{a}^i \in R^m \setminus \{0\}$, $i = 1,...,r$ be given and pairwise linearly independent. Assume we know that a function $f(\mathbf{x})$ can be represented in the form

$$f(\mathbf{x}) = \sum_{i=1}^{r} g_i\left(\mathbf{a}^i \cdot \mathbf{x}\right) \tag{1}$$

Assume in addition that $f(\mathbf{x})$ is of the class $C^k\left(R^m\right)$. What can we say about g_i? Can we say that $g_i \in C^k(R)$? The case $r = 1$ and $r = 2$ is obvious. The above question becomes quite difficult if the number of directions $r \geq 3$. It follows from the results of Pinkus [2] that if in (1) $f \in C^k(R)$ and each g_i is measurable, then necessarily $g_i \in C^k(R)$ for $i = 1,...,r$.

The results of Pinkus [2] gave rise to the following natural and important problem. Assume in the representation (1) $f \in C^k(R)$, but the functions g_i are arbitrarily behaved. Can we write f as a sum $\sum_{i=1}^{r} f_i(\mathbf{a}^i \cdot \mathbf{x})$ but with the $f_i \in C^k(R)$, $i = 1,...,r$? In [3], Aliev and Ismailov obtained a partial solution to this problem. Their solution comprises the cases in which $k > 1$ and $r-1$ directions of given r directions are linearly independent.

In this paper, we continue investigations on ridge function representations. The question considered here is as follows. Assume f is Holder continuous on compact subsets of $\boldsymbol{R}^m$ and possesses the representation (1). Can we replace g_i with Holder continuous functions of the same degree as f? We answer this question positively in the case when $r-1$ directions of given r directions are linearly independent.

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ON THE APPROXIMATION BY RADIAL FUNCTIONS WITH FIXED CENTERS

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In modern approximation theory, radial functions play an essential role. A *radial function* is a multivariate function of the form

$$F(\mathbf{x}) = r(\|\mathbf{x}-\mathbf{c}\|),$$

where $\mathbb{R}:\mathbb{R}\to\mathbb{R}$, $\mathbf{x}=(x_1,...,x_d)$ is the variable, $\mathbf{c}\in\mathbb{R}^d$ and $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^d$. The point $\mathbf{c}$ is called the center of F. In other words, a radial function is a multivariate function constant on the spheres $\|\mathbf{x}-\mathbf{c}\|=\alpha, \alpha\in\mathbb{R}.$ These functions and their linear combinations arise naturally in many fields, especially in RBF (radial basis function) neural networks.

Consider the following set of functions

$$\mathcal{D}=\{r_1(\|\mathbf{x}-\mathbf{c}_1\|)+r_2(\|\mathbf{x}-\mathbf{c}_2\|): r_i\in C(\mathbb{R}), i=1,2\}.$$

That is, we fix centers $\mathbf{c}_1$ and $\mathbf{c}_2$ and consider linear combinations of radial functions with these centers.

Suppose Q is a compact set in $\mathbb{R}^d$ and $\mathbf{c}_1,\mathbf{c}_2\in\mathbb{R}^d$ are fixed points.

Definition 2.1. *A finite or infinite ordered set* $p=(\mathbf{p}_1,\mathbf{p}_2,...)\subset Q$ *with* $\mathbf{p}_i\neq\mathbf{p}_{i+1}$, *and either*

$$\|\mathbf{p}_1-\mathbf{c}_1\|=\|\mathbf{p}_2-\mathbf{c}_1\|, \|\mathbf{p}_2-\mathbf{c}_2\|=\|\mathbf{p}_3-\mathbf{c}_2\|, \|\mathbf{p}_3-\mathbf{c}_1\|=\|\mathbf{p}_4-\mathbf{c}_1\|,...$$

or $\|\mathbf{p}_1-\mathbf{c}_2\|=\|\mathbf{p}_2-\mathbf{c}_2\|, \|\mathbf{p}_2-\mathbf{c}_1\|=\|\mathbf{p}_3-\mathbf{c}_1\|, \|\mathbf{p}_3-\mathbf{c}_2\|=\|\mathbf{p}_4-\mathbf{c}_2\|,...$

is called a path with respect to the centers $\mathbf{c}_1$ *and* $\mathbf{c}_2$.

In the above definition, we alternate distances from two fixed points. Paths have many different variations. For example, instead of points, one can take two hyperplanes $\mathbf{a}^i \cdot \mathbf{x} = \alpha_i$, $i = 1,2,$ where "$\cdot$" denotes the standard scalar product in $\mathbb{R}^d$, and alternate distances from these two hyperplanes. Certainly, in $\mathbb{R}^2$, hyperplanes turn into straight lines, thus one can talk about distances from straight lines. Paths with respect to two straight lines in $\mathbb{R}^2$ were first considered by Braess and Pinkus [2]. They showed that paths give geometric means of deciding if a set of points $\{\mathbf{x}^i\}_{i=1}^m \subset \mathbb{R}^2$ has the "non-interpolation property" for so called ridge functions (for this terminology see [2]). Ismailov and Pinkus [3] used paths with respect to two hyperplanes in $\mathbb{R}^d$ to solve the problem of interpolation on straight lines by ridge functions. If straight lines are fixed as the coordinate lines in $\mathbb{R}^2$, then the corresponding set of points $(\mathbf{p}_1, \mathbf{p}_2, \ldots)$ turn into " bolts of lightning" (see, e.g., [1]). Ismailov [4] generalized paths to those with respect to a finite set of functions. Paths with respect to n arbitrarily fixed functions turned out to be very useful in problems of representation by linear superpositions.

In the sequel, we use the term "path" instead of the long expression "path with respect to the centers $\mathbf{c}_1$ and $\mathbf{c}_2$". A finite path $(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_{2n})$ is said to be closed if $(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_{2n}, \mathbf{p}_1)$ is also a path.

We associate a closed path $p = (\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_{2n})$ with the functional

$$G_p(f) = \frac{1}{2n} \sum_{k=1}^{2n} (-1)^{k+1} f(\mathbf{p}_k).$$

This functional has the following obvious properties:

(a) If $r \in \mathcal{D}$, then $G_p(r) = 0$.

(b) $\|G_p\| \le 1$ and if $p_i \ne p_j$ for all $i \ne j, 1 \le i, j \le 2n$, then $\|G_p\| = 1$.

The images of the distance functions $\|\mathbf{x} - \mathbf{c}_1\|$ and $\|\mathbf{x} - \mathbf{c}_2\|$ on Q denote by X_1 and X_2, respectively. For any function $h \in C(Q)$, consider the real functions

$$s_1(a) = \max_{\substack{\mathbf{x} \in Q \\ \|\mathbf{x}-\mathbf{c}_1\| = a}} h(x), s_2(a) = \min_{\substack{\mathbf{x} \in Q \\ \|\mathbf{x}-\mathbf{c}_1\| = a}} h(x), a \in X_1,$$

$$g_1(b) = \max_{\substack{\mathbf{x} \in Q \\ \|\mathbf{x}-\mathbf{c}_2\| = b}} h(x), g_2(b) = \min_{\substack{\mathbf{x} \in Q \\ \|\mathbf{x}-\mathbf{c}_2\| = b}} h(x), b \in X_2.$$

The following theorem is valid.

Theorem. *Let $Q \subset \mathbb{R}^d$ be a compact set and $f \in C(Q)$. Assume the following conditions hold.*

1) for any two points $\mathbf{x}$ and $\mathbf{y}$ in Q with $\|\mathbf{x} - \mathbf{c}_1\| = \|\mathbf{y} - \mathbf{c}_1\|$ ($\|\mathbf{x} - \mathbf{c}_2\| = \|\mathbf{y} - \mathbf{c}_2\|$) and any sequence $\{\mathbf{x}_n\}_{n=1}^{\infty}$ tending to $\mathbf{x}$, there exists a sequence $\{\mathbf{y}_n\}_{n=1}^{\infty}$ tending to $\mathbf{y}$ such that $\|\mathbf{x}_n - \mathbf{c}_1\| = \|\mathbf{y}_n - \mathbf{c}_1\|$ ($\|\mathbf{x}_n - \mathbf{c}_2\| = \|\mathbf{y}_n - \mathbf{c}_2\|$) for all $n = 1, 2, \ldots$

2) there exists an extremal element $r_0 \in \mathcal{D}$ for the function f;

3) for any path $q = (\mathbf{q}_1, \ldots, \mathbf{q}_n) \subset Q$ there exist points $\mathbf{q}_{n+1}, \mathbf{q}_{n+2}, \ldots, \mathbf{q}_{n+s} \in Q$ such that $(\mathbf{q}_1, \ldots, \mathbf{q}_n, \mathbf{q}_{n+1}, \ldots, \mathbf{q}_{n+s})$ is a closed path and s is not more than some positive integer n_0

independent of q.

Then the approximation error can be computed by the formula

$$E(f) = \sup_{p \subset Q} \left| G_p(f) \right|,$$

where the sup is taken over all closed paths.

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SOLUTION TO THE LARGE SYSTEMS OF DIFFERENTIAL EQUATIONS OF BLOCK STRUCTURE AND ITS APPLICATION

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We consider the system consisting of L independent subsystems of linear differential equations

$$\dot{y}^i(x) = A^i(x) y^i(x) + B^i(x), \quad x \in [0, l_i], \tag{1}$$

$$y^i(\cdot) \in R^{n_i}, \; i = 1, \ldots, L.$$

Here $A^i(x), B^i(x) -$ are known continuous square matrices' and vector functions dimension n_i accordingly, where $A^i(x) \neq const, x \in (0, l_i)$; unknown vector functions $y^i(x)$ dimension n_i are continuous differentiable at $x \in [0, l_i]$; $l_i > 0$ - are given; $i = 1, ..., L$. The solutions $y^i(x)$, $i = 1, ..., L$, to the subsystems in (1), are connected with initial and boundary conditions, which we will write in the following general form:

$$Gy(0) + Qy(l) = R, \tag{2}$$

Where G, Q are given square matrices dimension $n \times n$, $n = \sum_{i=1}^{L} n_i$, and the rang of extended matrices' (G, Q) is: $rang(G, Q) = n$; $R = (r^1, ..., r^n)^T -$ is given vector dimension n. The most of elements of the matrices G and Q are zero in practice, and nonzero elements match to the connection between initial and final states of corresponding distinct nodes of the complex object. The problem (1), (2) is the two-point boundary value problem and is characterized by the following specific features: 1) the subsystems of differential equations of the system (1) are mutually independent, 2) the solutions $y^i(x), i = 1, ..., L$, of the subsystems are connected by unseparated boundary conditions, 3) the great number of subsystems, and consequently in general by the large order of the system (1). The problem of calculation of unsteady fluid flows in the pipeline networks of complex structure are brought to the considered problem. The mathematical models of such processes are described by the systems of partial differential equations. These systems consist of subsystems of hyperbolic type equations, which describe the process of fluid flow in each distinct segment [1]. The condition of thread continuous and material balance is satisfied at junctions, determined by the conditions (2). The problem of calculation of the regimes of fluid flow in the pipeline network is reduced to the problem (1), (2) by the application of straight method ([2]) (analogues to the application of decomposition).

The results of numerical experiments are carried out, which are obtained by the solution to the model problem, which base is the problem of calculation of unsteady fluid flow for the segment of the pipeline network of complex structure [3,4]. The numerical experiments are carried out by applying the approach given in the work and the analysis of obtained results will be presented at the report.

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ENERGYSPECTRUM OF JOSEPHSONQUBİTS AS EİGENVALUE PROBLEM

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Possible application of Josephson junctions in the field of quantum computation seems very interesting [1,2]. It is well known that, the quantum processor then performs a quantum mechanical operation on this input state in order to derive an output which is also a quantum coherent superposition. The state of the qubit |ψ> can be presented is a linear superposition of the two quantum basis states |0> and |1> [1,2]. Josephson junctions based qubits seems as good candidates for the realization and readout operations [3,4]. Investigations of last years shows that [5-8], unconventional character of current-phase relation becomes important at operating temperatures of qubits. As a result unconventional character of current-phase relation must be taken into account in consideration of superconducting qubits. For the determination of spectrum of qubits, one has to solve the corresponding stationary Schrödinger equation

$$H\Psi = E\Psi \,, \qquad (1)$$

where H is the Hamiltonian operator of qubit. It is well known thar, the quantum dynamics of an isolated Josephson junction is described with the Mathieu-Bloch picture for a particle moving in a periodic potential [9,10]. In this study, we will describe the quantum dynamics of phase, charge and flux qubits for different types of current-phase relation. Such qubits have distinguished limiting regimes: the phase regime, $E_J = \frac{\hbar I_c}{2e} \gg E_C = \frac{e^2}{2C}$ ($I_c$critical current, C capacity of junction) is analogous to the tight-binding approximation, and the charge regime, $E_J << E_C$, is analogous to the near-free particle approximation.

Phase qubit: Hamiltonian of the phase qubits on single Josephson junction [10] can be written as ($i_B$bias burrent,ϕ Josephsonphase)

$$H = -E_C \frac{\partial^2}{\partial \phi^2} + E_J \{i_b \phi + \cos\phi\} \qquad (2)$$

Potential energy profile of such qubit correspond to tilted wash board potential. At low biascurrent i_B, the energy spectrum of phase qubit coincides with the spectrum of harmonic oscillator with the frequency $\Omega_p = \frac{I_c}{2e}(1 - i_B)^{1/4}$.

Charge qubit: The Hamiltonian of the charge qubit system (Fig. 1a) has a form [3,8]:

$$H = E_C (n - n_g)^2 - E_J \{i_b \phi + \cos\phi\} \qquad (3)$$

Splitting of energystates in chargequbitpresented in Fig 1b.

Flux qubit on single Josephson junction interfeometer: Hamiltonian of the flux qubits on

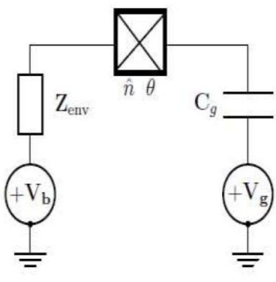

Fig.1a)

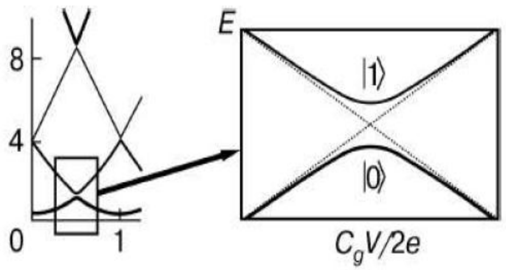

Fig.1b)

single Josepshon junction intereferometer with inductance L can be written as

$$H = -E_C \frac{\partial^2}{\partial \phi^2} - E_J \left\{ \cos\phi + \frac{(\phi - \phi_e)^2}{2l} \right\}; l = \frac{2eLI_c}{\hbar}; \phi_e = \frac{2e\Phi_e}{\hbar}$$

(4)

Flux qubit with three Josephson junction: The main draw back of the flux qubit with a single Josephson junction described above concerns the large inductance of the qubit loop, the energy of which must be comparable to the Josephson energy to form there quired double-well potential profile. This implies large size of the qubit loop, which makes the qubit-vulnerable to dephasing by magnetic fluctuations of the environment [10]. One way to over come this difficulty, replacing the large loop inductance by the Josephson inductance of an additional tunnel junction. Energy spectrum of such qubit also is analyzed.

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ON EXISTENCE OF THE SOLITION OF PARTIAL OPERATOR DIFFERENTIAL EQUATIONS IN HILBERT SPACES

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Consider the differential equation

$$\sum_{|\alpha|\le m} A_\alpha D^\alpha u = f(x), \quad x \in R^n \tag{1}$$

where A_α are bounded operators $H_{m-|\alpha|} \to H_0$. Hilbert spaces $H_i,\, i = 0,1,\dots,m$ are such that $H_{i+1} \subset H_i$. In equation (1) $f(x)$ is given function $R^n \to H_0$.

The solution of the equation (1) is such function $u(x) \in H_m$ that $D^\alpha u \in H_{m-|\alpha|}$ at $|\alpha| \le m$ and the equality (1) is satisfy at almost all $x \in R^n$.

For studying the equation (1) consider the operator function

$$R(\lambda) = \left[\sum_{|\alpha| \le m} (i\lambda)^{\alpha} A_{\alpha} \right]^{-1},$$

acting from H_0 to H_m.

Designate by $H_{k,a}$ the space with norm

$$\|u\|_{k,a}^2 = \int_{R^n} \|u(x)\|_k^2 e^{a|x|} dx,$$

where $\|\cdot\|_k$ is the norm in the space H_k.

By the Fourier transform the following theorem is proved .

Theorem. If $R(\lambda)$ exists and is an analytical function, $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ in the field $|Jm\lambda_i| < h$ and in this field

$$\|R(\lambda)\|_{H_0 \to H_j} (1 + |\lambda|)^{m-j} \le c, \ j = 0,1,\ldots,m,$$

then for any $f(x) \in H_{o,a}$, $|a| < \dfrac{h}{2}$ there exists the unique solution of the equation (1) satisfying the inequality

$$\sum_{j=1}^{m} \|u(x)\|_{H_{j,a}} \le c \|f(x)\|_{H_{o,a}}.$$

The case when $R(\lambda)$ is regular everywhere in R^n was considered in [1].

At $n > 1$ elliptical boundary problems in unbounded layer

$$\Pi = \{x : 0 < x_n < 1, \ x' = (x_1, x_2, \ldots, x_{n-1}) \in R^n\}$$

and some other problems come down to the equation (1).

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ABOUT IMPLEMENTATION OF COMPLEX ANALYSIS INFORMATIVE AND METHODICAL LINES IN THE PROBLEM BOOK OF THE COMPLEX VARIABLE THEORY

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The course of the complex variable theory represents one of the most difficult and at the same time one of the most important mathematical courses. At the same time, the complex variable theory course has unique potential in representation of logically complete and integrated informative and methodical lines. The universal problem book on the complex variable theory should contain a set of rather independent modules (chapters)the study of which can be based on a chosen methodical line. At the same time each chapter should contain different-level tasks making it possible to give the student understanding of the material in necessary extent. "Problem Book of the Complex Variable Theory" is created exactly in this way by Aslanov R.M., Gorin E.A.and Sushkov V.V. The edition is prepared by the authors on the basis of long-term complex variable theory teaching experiencein the Moscow Pedagogical State University (MPSU), the Ganja State Pedagogical Institute (nowadays Ganja state university) and the Syktyvkar State University namedafter Pitirim Sorokin.

Problem as a training aid is an object which after being studied can make the mathematical training material a student’s object of activity, and, consequently, it will be mastered well. At the same time the efficiency of learning how to solveproblems is defined not only by a set of methods, but also thematic pithiness of material. The authors divide the main material of the problem book into 7 chapters: "Plane of complex numbers. Numerical sequences and series", "Functions of complex variable.Functional sequences and series", "Analytic functions", "Elementary functions and conformal mappings", "Complex integral", "Representation of functions by ranks.Special points.Residue theory", and "Applications of the complex analysis in the theory of differential equations and operational calculation".

Each chapter is accompanied by a sufficient theoretical reference containing definitions, examples, and theorems explaining the material in detail. The problems presented in the chapters differ fundamentally in terms of levels and types –they are computational problems, problems for checking conditions, problems for evidence and construction, problems of a research nature.

The authors dedicate this book to Professor Gorin E.A., an outstanding mathematician and teacher, a bright and talented person who was excellent not only in mathematics and the history of mathematics, but also in music, literature, art. The influence of this outstanding personality on the life and pedagogical creativity not only of the mathematical faculty of the MPSU, but also of the whole Moscow Pedagogical State University as a whole will remain tangible for many years to come.

ON SELFADJOINT EXTENSIONS OF SYMMETRIC OPERATOR WITH EXIT FROM SPACE

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Considered selfadjoint extensions of closed symmetric operators generated with differential expressions with unbounded discrete operator coefficients. Given description of selfadjoint extensions with exit from space as well as description of selfadjoint extensions with discrete spectrum . It is known that selfadjoint extensions with exit from space when applying to boundary value problems for differential equations give rise to problems with eigen parameter dependent boundary conditions. For boundary value problem corresponding to one of such extensions, when one of boundary conditions depend rationally on eigen parameter, we find asymptotics of eigenvalues. We also derive formula for the first regularized trace of corresponding to that problem operator.

SOME PROPERTIES OF SCATTERING OPERATOR ON THE SEMI-AXIS FOR THE SYSTEM OF FIVE HYPERBOLIC EQUATIONS

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On a semi-axis $x \geq 0$ consider a system of equations of the form:

$$\xi_i \frac{\partial U_i(x,t)}{\partial t} - \frac{\partial U_i(x,t)}{\partial x} = \sum_{j=1}^{5} c_{ij}(x,t) U_j(x,t), i = \overline{1,5} \qquad (1)$$

where $c_{ij}(x,t)$ are complex-valued measurable functions with respect to x and t satisfying the conditions:

$$\left| c_{ij}(x,t) \right| \leq C \left[(1+|x|)(1+|t|) \right]^{-1-\varepsilon}, \qquad (2)$$

Moreover $c_{ii}(x,t) = 0$, $i = \overline{1,5}$, $\xi_1 > \xi_2 > 0 > \xi_3 > \xi_4 > \xi_5$, $-\infty < t < +\infty$.

Let us consider system (1) on a semi-axis under three different boundary conditions.

The scattering problem for system (1) is in finding the solution to the system (1) bythe given incidence waves $a(t)$ and boundary conditions for $x = 0$.

In the space of essentially bounded functions, we determined the operator $S = \left(S^1, S^2, S^3 \right)$ that takes incidence waves $a(t)$ to scattering waves $b(t)$:

$$S^k \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix} = \begin{pmatrix} b_3^k(t) \\ b_4^k(t) \\ b_5^k(t) \end{pmatrix}, k = \overline{1,3} \qquad (3)$$

Here, $S=\left(S^{1},S^{2},S^{3}\right)$ and

$$S^{k}=\begin{pmatrix} S_{11}^{k} & S_{12}^{k} \\ S_{21}^{k} & S_{22}^{k} \\ S_{31}^{k} & S_{32}^{k} \end{pmatrix}, k=\overline{1,3} \tag{4}$$

Here is studied some properties of scattering operator. Or rather the following theorem is correct:

Theorem. *Let conditions (2) are satisfied for the system (1). Then scattering operator (8), minors*

$$\begin{pmatrix} S_{21}^{k} & S_{22}^{k} \\ S_{31}^{k} & S_{32}^{k} \end{pmatrix}, \; k=\overline{1,3}$$

and operator

$$\begin{pmatrix} S_{21}^{k}-S_{21}^{1} & S_{22}^{k}-S_{22}^{1} \\ S_{31}^{k}-S_{31}^{1} & S_{32}^{k}-S_{32}^{1} \end{pmatrix}, \; k=1,2$$

have inverse. And the operators $S_{21}^{2}-{}_{21}^{1},S_{21}^{3}-{}_{21}^{1}$ *are in the form* $I+G_{k+},k=2,3.$ *The operators* G_{k+} *-Volter operators.*

FRACTIONAL MAXIMAL OPERATOR IN GENERALIZED MORREY SPACES ON HEISENBERG GROUP

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In this abstract we study the boundedness of the fractional maximal commutator $M_{b,\alpha}$ on Heisenberg group H_n in the generalized Morrey spaces.

The fractional maximal commutator is

$$M_{b,\alpha}(f)(u)=\sup_{r>0}|B(u,t)|^{-1+\frac{\alpha}{Q}}\int_{B(u,t)}|b(u)-b(v)|\,f(v)dV(v),$$

where $Q=2n+2$ is the homogeneous dimension of H_n.

We denote by Ω_p the sets of all positive measurable functions φ on $\mathrm{H}_n\times(0,\infty)$ such that for all $t>0$,

$$\sup_{u\in \mathrm{H}_n}\left\|\frac{r^{-\frac{Q}{p}}}{\varphi(u,r)}\right\|_{L_\infty(t,\infty)}<\infty,\text{ and }\sup_{u\in \mathrm{H}_n}\left\|\varphi(u,r)^{-1}\right\|_{L_\infty(0,t)}<\infty.$$

Denote by G_p the set of all almost decreasing functions $\phi:(0,\infty)\to(0,\infty)$ such that $t\in(0,\infty)\to t^{\frac{Q}{q}}\in(0,\infty)$ is almost increasing

We proved the following theorem.

Theorem. [1] Let $p,q\in[1,\infty)$, $0\le\alpha<Q$, $\phi_1\in\Omega_p$, $\phi_2\in\Omega_q$ and $b\in BMO(\mathrm{H}_n)$, $1<p<\frac{Q}{\alpha}$ and $\frac{1}{q}=\frac{1}{p}+\frac{\alpha}{Q}$. If $\phi_1\in G_p$ satisfies the condition

$$\sup_{r<t<\infty}\left(1+\ln\frac{t}{r}\right)t^{\alpha}\phi_1(t)\le Cr^{\alpha}\phi_1(r)$$

for all $r>0$, where $C>0$ does not depend on r, then the condition $t^{\alpha}\phi_1(t)\le C\phi_2(t)$ is necessary and sufficient for the boundednessof $M_{b,\alpha}$ from $M_{p,\varphi_1}(\mathrm{H}_n)$ to $M_{p,\varphi_2}(\mathrm{H}_n)$.

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ON SMOOTHNESS OF THE DERIVATIVE OF A SIMPLE LAYER POTENTIAL WITH A UNIQUE DENSITY FOR A CONTINUOUS DIFFERENTIAL FORM IN DOMAIN OF RHOMBS

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This paper is the continuation of the paper [3]. We consider a was approach for styling a derivative in the direction of a simple layer potential for continuous differential forms in domain of rhombs in R^2 and find the direction for which we prove continuity of the derivative of the potential in this direction on the boundary, including at angler points.

There exist a great number of paper devoted to the study of classic potentials in the domains with piecewise –smooth boundaries with finitely many angler points. The review of such studies is in [1]. In many problems for such domains as a rule, weight functional spaces with a weight dependent on the distance to the if points set where the smoothness condition of the boundary violates, are used. In [2] a new approach for styling simple and double layer potential in Hölder classes was offered. A new simple layer potential in $R^d, d \geq 2$, was introduced. This potential for $d = 2$ wich density ω, may be represented in the form

$$S(\omega)(x) = \frac{1}{2\pi} \int_{\partial Q} \omega(\xi) \ln \frac{1}{|\xi - x|} (d\xi_2 - d\xi_1),$$

And for the class of domains with irregular boundary including unrectifiable Indian cures, the smoothness properties of this potential are studied in Holder classes.
The direction for which the continuity of the derivative of this potential in this direction on the boundary including at angler points was found in [3].
Let Q be a rhomb on the plane, and

$$DS(x) = \frac{\partial S(x)}{\partial \overline{\upsilon}(x)} = \upsilon_1(x)\frac{\partial S(x)}{\partial x_1} + \upsilon_2(x)\frac{\partial S(x)}{\partial x_2}$$

be a derivative in the direction $\overline{\upsilon}(x) = \{\upsilon_1(x);\upsilon_2(x)\}$ of the potential $S(\omega)(x)$ at the point $x \in Q$.

Theorem. Let Q be a rhomb centered at the origin of coordinates with acute slope angle α, with sides equal to 2α, whose bases are parallel to the coordinate axis $\overline{O\xi_1}$. Then continuous confirmation of $DS(x)$ in the direction

$$\bar{v}(x) = \begin{cases} \overline{V} = (V_1, V_2),\ if\ \ x\ close later\, sides \\ \overline{W} = (W_1, W_2)\ if\ \ x\, close\ to\ bases \end{cases}$$

on ∂Q, where

$$V_{1,2} = -\frac{2}{\pi\sqrt{2}}(\cos\alpha \mp \sin\alpha)(2\pi\cos\alpha + \alpha\sin\alpha),$$

$$W_{1,2} = -\frac{2}{\pi\sqrt{2}}(\cos\alpha \mp \sin\alpha)(2\pi\cos\alpha - \alpha\sin\alpha),$$

With the exception of angler points (verties, of the rhomb) and boundary values $DS(x)$, continued in coordinaty along the boundary to the angler points of the rhomb, i.e. function from the class $H_1(\partial Q)$.

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HARDY OPERATOR IN THE LOCAL "COMPLEMENTARY" GENERALIZED VARIABLE EXPONENT WEIGHTED MORREY SPACES

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We consider the following Hardy operator

$$A_u f(x) = |x|^{-n} u(|x|) \int_{|y|\le|x|} \frac{f(y)}{u(|y|)} dy.$$

Let $p(\cdot)$ be a measurable function on Ω with values in $[1,\infty)$. An open set Ω is assumed to be bounded throughout the whole paper. We mainly suppose that $1 < p_- \le p(x) \le p_+ < \infty$, where $p_- := \operatorname{ess\,inf}_{x\in\Omega} p(x) > 1$, $p_+ := \operatorname{ess\,sup}_{x\in\Omega} p(x) < \infty$. By $L^{p(\cdot)}(\Omega)$ we denote the space of all measurable functions $f(x)$ on Ω such that the norm

$$\|f\|_{p(\cdot)} = \inf\left\{\eta > 0 : \int_\Omega \left(\frac{|f(x)|}{\eta}\right)^{p(x)} dx \le 1\right\},$$

this is a Banach function space. By $p'(\cdot) = \dfrac{p(x)}{p(x)-1}$, $x \in \Omega$, we denote the conjugate exponent.

$\mathsf{P}(\Omega)$ is the set of bounded measurable functions $p : \Omega \to [1,\infty)$;

$\mathsf{P}^{log}(\Omega)$ is the set of exponents $p \in \mathsf{P}(\Omega)$ satisfying the local log-condition

$$| p(x) - p(y) | \le \frac{A}{-\ln|x-y|}, \; |x-y| \le \frac{1}{2} \;\; x, y \in \Omega,$$

where $A = A(p) > 0$ does not depend on x, y.

We will use also the following decay condition:

$$| p(x) - p(\infty) | \le \frac{A_\infty}{|\ln|x||}, \; |x| \ge 2, \tag{1}$$

where $p_\infty = \lim_{x\to\infty} p(x) > 1$.

$\mathsf{P}^{log}(\Omega)$ is the set of exponents $p \in \mathsf{P}^{log}(\Omega)$ with $1 < p_- \le p(x) \le p_+ < \infty$;

for Ω which may be unbounded, by $\mathsf{P}_\infty(\Omega)$, $\mathsf{P}_\infty^{log}(\Omega)$, $\mathsf{P}_\infty^{log}(\Omega)$ we denote the subsets of the above sets of exponents satisfying the decay condition (1) (when Ω is unbounded).

By φ we always denote a weight, i.e. a positive, locally integrable function with R^n. The weighted Lebesgue space $L_\varphi^{p(\cdot)}(\mathsf{R}^n)$ is defined as the set of all measurable functions for which

$$\| f \|_{L_\varphi^{p(\cdot)}(\mathsf{R}^n)} = \| f\varphi \|_{L^{p(\cdot)}(\mathsf{R}^n)}.$$

Let us define the class $A_{p(\cdot)}(\mathsf{R}^n)$ to consist of those weights φ for which

$$[\varphi]_{A_{p(\cdot)}} \equiv \sup_{B(x,r} |B(x,r|^{-1} \|\varphi\|_{L^{p(\cdot)}(B(x,r))} \|\varphi^{-1}\|_{L^{p'(\cdot)}(B(x,r))} < \infty.$$

Everywhere in the sequel the functions $\omega(r)$, $\omega_1(r)$ and $\omega_2(r)$ used in the body of the paper, are non-negative measurable function on $(0,\infty)$. The local generalized weighted Morrey space $\mathrm{M}_{\{x_0\}}^{p(\cdot),\omega}(\mathrm{R}^n)$ with variable exponent is defined by the norms

$$\|f\|_{\mathrm{M}_{\{x_0\}}^{p(\cdot),\omega,\varphi}} = \sup_{r>0} \frac{1}{\omega(r)\|\varphi\|_{L^{p(\cdot)}(B(x_0,r))}} \|f\|_{L_{\varphi}^{p(\cdot)}(B(x_0,r))},$$

where $x_0 \in \Omega$ and $1 \le p_- \le p(x) \le p_+ < \infty$ for all $x \in \Omega$.

Theorem 1. *Let* $p \in \mathrm{P}_{\infty}^{log}(\mathrm{R}^n)$, $\varphi \in A_{p(\cdot)}(\mathrm{R}^n)$. *Suppose also that* $\frac{r^{\gamma}}{u(r)} \le C \frac{t^{\gamma}}{u(t)}$, $0 < t < r$, $\gamma \in \mathrm{R}$ *and the function* (ω_1, ω_2) *satisfy the conditions*

$$\int_0^t \omega_1(s)\|\varphi\|_{L^{p(\cdot)}(B(0,s))} \frac{ds}{s} \le C\omega_2(t)\|\varphi\|_{L^{p(\cdot)}(B(0,t))},$$

$$\int_0^t \frac{\omega_1(s)}{u(s)} \frac{ds}{s^{n+1}} \le C \frac{\omega_1(t)}{t^n u(t)}$$

where $t > 0$. *Then the weighted Hardy operator* A_u *is bounded from the space* $\mathrm{M}_{\{0\}}^{p(\cdot),\omega_1,\varphi}(\mathrm{R}^n)$ *to the space* $\mathrm{M}_{\{0\}}^{p(\cdot),\omega_2,\varphi}(\mathrm{R}^n)$.

ON THE IDEAS OF MATHEMATICAL LOGIC IN THE WORKS OF NASIREDDIN TUSI[1]

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Two main ideas were in the foundation of formation of mathematical logic as discipline: applying logic to the theoretical issues of mathematics and application of mathematical methods in logic.

Discussion of the problem of first direction occupied minds of philosophers and mathematicians since ancient times. These issues found great attention in the works of mathematicians of the eastern Middle Age (numerous commentaries wordings of "Element" of Euclid, especially problem of so-called 5-th postulate, measurement problem, etc.)

We are engaged in researches mathematical and logical works of Nasireddin Tusi (1201-1274). Our researches carried out in the direction to translations, comments, research and deep study of the theoretical views of this outstanding scientist, who systemized the logical and mathematical knowledges of his time.

We can conclude from our results that Tusi is trying to introduce logical rigor into the mathematics of that time, and approached to the problem of introducing mathematical methods into logic closer than his predecessors and contemporaries.

Note some facts that allow us to draw certain conclusions. In his treatise "Tahriri Uklidis" Tusi applies logical principles to the correction of "Elements". In the preface of treatise he is trying to reconcile initial concepts with logical requirements. He applies principle of logical division to the concept of "point". He postulates the existence of geometric objects and actually

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introduces an axiom of existence. He implements the principle of existence of subject of reasoning, repeatedly emphasized by him in his logical treatise. “We can say anything about nonexistent”. This principle he uses in the Clauses XXIV-XXV of 10-th books of “Tahriri Uklidis”. He precedes the proof of Euclid by the proof of existence.

In the stereometric part of his treatise Tusi formulates 3 stereometric axioms (the first stereometric axioms in the history of mathematics).

He changes the style of proofs certain Euclid’s Proposals using varieties of partitive-categorical modi of syllogism.

Tusi comes to the concept of real number as relationship using the logical categories of genus and species.

In scope of the second direction, the logical treatise of Tusi “Asas ul iktibas” is of great interest. In this work he repeatedly appeals to mathematical thought.

Tusi refines the theory of Aristotl syllogisms. He considers 4 figures of syllogism, establishes the position of large and minor premise of syllogism: minor before large, unlike Aristotl. The Aristotl’s order can be explained by his generic-species approach to the tree-like division of concepts. Tusi takes a set-theoretic approach in terms volume (denotation) of concept. As follows from examples, given by him he replaces the term “covers” with “it contains”.

In this treatise Tusi gives 43 tables, most of which are truth tables, although not in symbolic form.

Tusi gives a formal method for verifying the effectiveness of the modus of syllogism figures, which, according to him, belongs to Baghdadi. This method is based on the construction a formal tree of premises, designated by the letters of the Arabic alphabet.

Even if we will confine ourselves to the mentioned facts we can say that in Tusi’s works rudimentary ideas of mathematical logic were expressed.

THE ABSENCE OF GLOBAL SOLUTIONS TO A SEMILINEAR PARABOLIC EQUATION WITH A BIHARMONIC OPERATOR IN THE MAIN PART

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We introduce the following notation: $n > 4$, $r = |x| = \sqrt{x_1^2 + \ldots + x_n^2}$, $B_R = \{x; |x| < R\}$, $B_R' = \{x; |x| > R\}$, $Q_R' = B_R' \times (0; +\infty)$, $\partial B_R = \{x; |x| = R\}$.
In the domain Q_R' we consider the following problem:

$$\begin{cases} |x|^\lambda \dfrac{\partial u}{\partial t} = -\Delta^2 u^p + \dfrac{C_0}{|x|^4} u^p + |x|^\sigma |u|^q & (1) \\ u|_{t=0} = u_0(x) \geq 0 & (2) \\ \displaystyle\int_0^\infty \int_{\partial B_R} u dx dt \geq 0, \quad \int_0^\infty \int_{\partial B_R} \Delta u^p dx dt \leq 0, & (3) \end{cases}$$

where $q > 1$, $0 \leq C_0 \leq \left(\frac{n(n-4)}{4}\right)^2$, $\sigma > -4$.

The question of the existence of non-negative global solutions to the problem (1)-(3) is investigated. We will understand the solution of the problem in the classical sense. The function $u(x,t)$ will be called the solution to problem (1) - (3) if u (x,t) satisfies equation (1) at each point Q_R', condition (2) for t = 0 and condition (3) for $|x| = R$.
Denote:

$$(n-2)^2 + C_0 = D, \quad \sqrt{\left(\frac{n-2}{2}\right)^2 + 1 \pm \sqrt{D}} = \alpha_\pm.$$

Consider the functions: $\xi(|x|) = \frac{1}{2}\left(1 + \frac{\sqrt{D} - \alpha_+}{\alpha_-}\right) |x|^{-\frac{n-4}{2} + \alpha_-} +$

$$\frac{1}{2}\left(1-\frac{\sqrt{D}-\alpha_+}{\alpha_-}\right)|x|^{-\frac{n-4}{2}-\alpha_-}-|x|^{-\frac{n-4}{2}-\alpha_+},$$

$$\varphi(x)=\begin{cases}1, & npu\, 1\le |x|\le \rho\\ \frac{1}{2}\cos(\pi(\frac{|x|}{\rho}-1))+\frac{1}{2})^{\tau}, npu\ \ \rho\le |x|\le 2\rho\\ 0, & npu\, |x|\ge 2\rho\end{cases}$$

$$T_\rho(t)=\begin{cases}1, & npu\, 0\le t\le \rho^{k}\\ \frac{1}{2}\cos(\pi(\rho^{-k}t-1))+\frac{1}{2})^{\mu}, npu\ \ \rho^{k}\le t\le 2\rho^{k}\\ 0, npu\, t\ge 2\rho^{k}\end{cases}$$

where$\kappa=\sigma\frac{p-1}{q-p}+4\frac{q-1}{q-p}+\lambda$. Using$\psi(x,t)=T_\rho(t)\xi(x)\varphi(x)$as a test function we find a critical exponent of the absence of a global solution to the problem (1)-(3).

Теорема.*Let* $n>4,\ \ \sigma>-4,\ \ 1\le p<q,\ \ 0\le C_0\le (\frac{n(n-4)}{4})^2$*and* $q\le p+\frac{\sigma+4}{\frac{n+4}{2}+\lambda+\alpha_-}$. *If* $u(x,t)$ *is a solution to problem (1.1)-(1.3), then* $u(x,t)\equiv 0$.

ON PRECOMPACTNESS OF SETS IN MORREY SPACES

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We study totally bounded sets in special type Morrey spaces. Some characterization of this kind of sets is given for the case of Morrey spaces. Furthermore, the sufficient conditions for compactness are shown in this spaces.

This is jointly work with Vagif Guliyev and Przemyslaw Gorka.

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ON ASYMPTOTIC BEHAVIOUR OF THE NEGATIVE PART OF THE DIFFERENTIAL OPERATOR WITH OPERATORCOEFFICIENTS

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Let H be a separable Hilbert space. Let us consider the operator L in the space $H_1 = L_2(0,\infty;H)$ defined by the following differential expression:

$$-\left(p(x)y'(x)\right)' + Ay(x) - Q(x)y(x)$$

with the boundary condition $y(0)+ay'(0)=0$. Here a real number, A is a nonnegative self-adjoint operator from $D(A)$ to H with $\overline{D(A)} = H$ such that $(A+I)^{-1}$ is compact operator (I is the identically operator) and $\inf_{0 \neq f \in D(A)} (Af, f) = 0$. Assume that function $p(x)$ and operator function $Q(x)$ satisfies the following conditions:

1. For every $x \in [0, \infty)$ $Q(x): H \to H$ is a positive self-adjoint operator and $\|Q(x)\| \leq const$.
2. $Q(x)$ decreasing monotonically and the function $\left\| \left(A + CI - Q(x) \right)^{-1} \right\|$ is continuous for large x where c is some positive number.
3. $\lim_{x \to \infty} \|Q(x)\| = 0$
4. $c_1 \leq p(x) \leq c_2$ where c_1, c_2 are a positive numbers.
5. Function $p(x)$ has continious derivative on $[0, \infty)$
6. $p(x)$ is non-decreasing on $[0, \infty)$

In this paper, asymptotic formula for the $N(\varepsilon)$ which is the number of negative eigenvalues smaller than $-\varepsilon$ $(\varepsilon > 0)$ is obtained. Papers on this theme we note [1-2]. Let $\alpha_1(x) \geq \alpha_2(x) \geq \ldots$ be the eigenvalues of the operator $Q(x) - A$ (each eigenvalue is written according to its multiplicity) we have the following

Theorem. *If the conditions 1.-6. Are satisfied and for a constant* $k \in (0,2)$ *and* $h > 0$ $\lim_{x \to \infty} \alpha_1(x) x^{k-n} = \lim_{x \to \infty} \left(\alpha_1(x) x^{k+h} \right)^{-1} = 0$, *then*

$$N(\varepsilon) = \pi^{-1} \left[1 + 0\left(\varepsilon^t \right) \right] \times \sum_i \int_{\alpha_i(x) \geq \varepsilon} \sqrt{\frac{\alpha_i(x) - \varepsilon}{p(x)}} dx \quad as \quad \varepsilon \to +0.$$

Here $t > 0$ *is a constant.*

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ON A PARTICULAR SOLUTION OF HOMOGENEOUS RIEMANN BOUNDARY VALUE PROBLEMS IN HARDY-ORLICZ CLASSES

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This work considers the Orlicz space and the Hardy-Orlicz classes of analytic functions inside and outside the unit circle, generated by this space. The homogeneous and non-homogeneous Riemann boundary value problems with piecewise continuous coefficients in these classes are considered.

Definition 1. *Continuous convex function* $M(\cdot): R \to R$ *is called* N *-function, if itis even and satisfies the conditions* $\lim_{u\to 0}\frac{M(u)}{u}=0;\ \lim_{u\to\infty}\frac{M(u)}{u}=\infty.$

Definition 2. N *-function* $M(\cdot)$ *satisfy the* Δ_2*-condition for large numbers* u, *if* $\exists k>0 \wedge \exists u_0 \geq 0:$

$$M(2u)\leq kM(u), \forall u \geq u_0.$$

By H_M^+ denote Hardy-Orlicz class of analytic functions $F(\cdot)$ inside ω with the norm

$$\|F\|_{H_M^+} = \sup_{0<r<1} \sup_{\rho_{M^*}(\nu)\leq 1} |(F_r(\cdot);\nu(\cdot))| = \sup_{0<r<1} \|F_r(\cdot)\|_M, \text{ where } F_r(t)=F(re^{it}).$$

Similarly to the classical case, the Hardy-Orlicz class ${}_mH_M^-$, of analytic functions outside the unit circle that have finite order at infinity is defined.

Consider the non-homogeneous Riemann problem

$$\begin{aligned} &F^+(\tau)-G(\tau)F^-(\tau)=f(\arg\tau), \tau\in\gamma, \\ &(F^+;F^-)\in H_M^+\times{}_mH_M^-, \end{aligned} \quad (1)$$

where $f(\cdot)\in L_M$ – is some function. Suppose that the coefficient $G(\cdot)$ satisfies conditions i), ii) and $Z_\theta(\cdot)$ is the canonical solution of the homogeneous problem corresponding to the argument $\theta(\cdot)$.

We will assume that the coefficient $G(\cdot)$ satisfies the following conditions:

i) $G^{\pm 1}(\cdot)\in L_\infty(-\pi,\pi)$; *ii)* $\theta(t)=\arg G(e^{it})-$ piecewise Holder function on $[-\pi,\pi]$ with jumps $h_k=\theta(s_k+0)-\theta(s_k-0)$, $k=\overline{1,r}$, at discontinuity points $\{s_k\}_1^r:-\pi<s_1<\ldots<s_r<\pi$.

Let $\rho:[-\pi,\pi]\to(0,+\infty)-$ be some weight function and $L_{M,\rho}$ weighted Orlicz space with a norm $\|\cdot\|_{M,\rho}$:

$$\|f\|_{M,\rho}=\|f\rho\|_M,\ \forall f\in L_{M,\rho}.$$

Denote by A_M the class of weights for which the singular operator is bounded in $L_{M,\rho}$, i.e. $A_M=\{\rho:S\in[L_{M,\rho}]\}$.

For N -function $M(\cdot)$ assume $\gamma_M=\inf\{\alpha:|t|^\alpha\in L_M\}$.

The following theorem is true.

Theorem 1. *Let $M\in\Delta_2(\infty)$ be some N -function with a complement M^*. Let the coefficients $G(\cdot)$ of the problem (1)* *satisfy the conditions* $i), ii)$ *and the jumps* $\{h_k\}_0^r$ *of the argument* $\theta(\cdot)$ *satisfy the relations* $\gamma_{M^*} < \frac{h_k}{2\pi}$, $k=\overline{0,r}$; $\wedge$ $\rho_0 \in A_M$, *where weight* $\rho_0(\cdot)$ *is defined by the expression* $\rho_0(t)=\left|t^2-\pi^2\right|^{\frac{h_0}{2\pi}} \prod_{k=1}^{r}\left|t-s_k\right|^{-\frac{h_k}{2\pi}}$, $t\in[-\pi,\pi]$. *Then the function* $F_1(\cdot)$, *defined by the expression* $F_1(z)=\frac{Z_\theta(z)}{2\pi}\int_{-\pi}^{\pi}\frac{f(t)}{Z_\theta^+(e^{it})}K(t;z)dt$, $z\notin\gamma$, ($K(t;z)=\frac{e^{it}}{e^{it}-z}$ − *is a Cauchy kernel*) *is a solution of the non-homogeneous problem (1) in Hardy-Orlicz classes* $H_M^+ \times {}_{-1}H_M^-$.

ON THE BASISNESS OF THE EXPONENTIAL SYSTEM IN WEIGHTED HARDY SPACES

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We first define the weighted counterparts of the Hardy classes. Let

$$\tilde{H}^+ \equiv \left\{f\in H_1^+ : f^+ \in L_{q,\nu^+}\right\},$$

where $H_1^\pm$ − are Hardy classes of functions defined inside and outside of the unit disc, respectively, $\nu^+(\cdot)$ is a weight function defined on $[-\pi,\pi]$, L_{q,ν^+} − is a weighted Lebesgue space on

$(-\pi,\pi)$, $f^{+}\left(e^{it}\right)-$ is the non-tangential boundary value of $f\in H_1^{+}$. Equip $\tilde{H}^{+}$ with the following norm:

$$\|f\|_{\tilde{H}^{+}}\equiv\left\|f^{+}\left(e^{it}\right)\right\|_{q,\nu^{+}}, \qquad (1)$$

where $\|\cdot\|_{p,\nu^{+}}$ – is the norm of $L_{p,\nu^{+}}$:

$$\|f\|_{p,\nu^{+}}=\left(\int_{-\pi}^{\pi}|f(t)|^{p}\nu^{+}(t)dt\right)^{\frac{1}{p}}.$$

It is proved the following

Proposition 1.*Let* $\left|\nu^{-}\right|^{-\frac{p}{q}}\in L_1$, $\frac{1}{p}+\frac{1}{q}=1$, $1\le q<+\infty$. *Then* $\tilde{H}^{+}$, *endowed with the norm (1) is a Banach space which be denoted as* $H_{q,\nu^{+}}^{+}$.

Let ${}_mH_1^{-}$ be the Hardy class of the functions, which are analytic outside the unit disc and has a zero of order not greater than m at infinity.

Let ν^{-} be a weight function on $[-\pi,\pi]$. Denote

$$\tilde{H}^{-}\equiv\left\{f\in_m H_1^{-}: f^{-}\left(e^{it}\right)\in L_{q,\nu^{-}}\right\}.$$

Proposition 2. *Let* $\left|\nu^{-}\right|^{-\frac{p}{q}}\in L_1$, $\frac{1}{p}+\frac{1}{q}=1$, $1\le q<+\infty$. *Then* $\tilde{H}^{-}$, *endowed with the norm*

$$\|f\|_{\tilde{H}^{-}}\equiv\left\|f^{+}\left(e^{it}\right)\right\|_{q,\nu^{-}},$$

is a Banach space.

We denote this Banach space as ${}_mH_{q,\nu^{-}}^{-}$.

Denote the restrictions of the classes $H^+_{p,\nu}$ and ${}_mH^-_{p,\nu}$ to $\partial\omega$ by $L^+_{p,\nu}$ and ${}_mL^-_{p,\nu}$, respectively: $H^+_{p,\nu}/_{\partial\omega} = L^+_{p,\nu}$; ${}_mH^-_{p,\nu}/_{\partial\omega} = {}_m L^-_{p,\nu}$.

We say that the weight $\nu(\cdot)$ defined on $[-\pi,\pi]$ belongs to the Muckenhoupt class $A_p, 1<p<+\infty$, if

$$\sup_{I\subset[-\pi;\pi]}\left(\frac{1}{|I|}\int_I \nu(t)dt\right)\left(\frac{1}{|I|}\int_I |\nu(t)|^{-\frac{1}{\rho-1}}dt\right)^{\rho-1} < +\infty,$$

where $\sup$ takes over all subintervals $I\subset[-\pi,\pi]$, $|I|-$ is the Lebesgue measure of the interval I.

Following theorem is proved.

Theorem. *Let $\nu\in A_p, 1<p<+\infty$. Then: i) the system $\{z^n\}_{n\in Z_+}$ (i.e. $\{e^{\mathrm{int}}\}_{n\in Z_+}$) forms a basis in $H^+_{\rho,\nu}$ (i.e.in $L^+_{\rho,\nu}$); ii) the system $\{z^{-n}\}_{n\geq m}$ (i.e. $\{e^{-\mathrm{int}}\}_{n\geq m}$) forms a basis in ${}_mH^-_{\rho,\nu}$ (i.e. in ${}_mL^-_{\rho,\nu}$).*

RIESZ BASISNESS OF A DISCONTINUOUS STURM LIOUVILLE PROBLEM WITH CONJUGATE CONDITIONS

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In this work we deal with Rieszbasisness of root functions of a discontinuous Sturm Liouville operator with periodic boundary condition and with conjugate conditions. One of conjugate conditions contains differentfinite one-sided limits at point zero. We firstly acquire asymptotic formulas of eigenvalues and eigenfunctionsby using asymptotic expression of fundamental solution. By the aid of these asymptotic formulas of eigenfunctions and Bessel properties of eigenfunctions we prove the basisness of the root functions of the boundary value problem.

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ON THE QUATERNION PADOVAN NUMBERS

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It is well known that the Padovan sequence $\{P_n\}_{n\geq 0}$ is defined by the recurrence relation

$$P_0 = P_1 = P_2 = 1,\ P_n = P_{n-2} + P_{n-3}\ (n \geq 3)$$

First few members of this sequence are $1,1,1,2,2,3,4,5,7,9$. The recurrence of the Padovan sequence involves the characteristic equation $x^3 - x - 1 = 0$. The Binet formula for the Padovan sequence is

$$P_n = a\alpha^n + b\beta^n + c\gamma^n$$

where $\alpha,\ \beta,\ \gamma$ are the roots of the characteristic equation, and $a,$ b and c are constants connected to $\alpha,\ \beta$ and γ.
A quaternion is defined by the following equation

$$q = q_0 + iq_1 + jq_2 + kq_3$$

where q_0, q_1, q_2 and q_3 are real numbers and $1, i, j, k$ are the standartbasis in $\mathbb{R}^4$ (For more information see [1-4]).

The quaternion Padovan sequence $Q_P(a,b,c,d)$ is defined by

$$Q_P(a,b,c,d) = P_{d+1}P_{c+1}P_{b+1}P_a + iP_{d+1}P_{c+1}P_bP_{a+1} + jP_{d+1}P_cP_{b+1}P_{a+1} + kP_dP_{c+1}P_{b+1}P_{a+1}$$

In the present work, we study on the quaternion Padovan sequence Q_P. We obtain the plastic-like ratio and some identities for Q_P. We give the generating and exponential generating functions for this sequence. Also, we establish its series and a binomial sum formula.

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AN APPROXIMATION METHOD FOR FINDING TOTAL DOMINATING SET

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Finding a solution for the combinatorial optimization problems has always been important due to their applications. But most of them are NP-Complete and unsolvable in polynomial time. Therefore, the approximation algorithms have been designed for them. One of these problems is total dominating set problem. In this paper, we present a new quadratic integer programming model for total dominating set problem and design an approximation method to find a lower bound for total dominating number. Consider an undirected and connected graph $G=(V,E)$, where $V=\{v_1,v_2,\dots,v_n\}$ and E are respectively vertices and edges of G. A set S_t of vertices of a graph G is called a total dominating set if every vertex $v_i\in V$ is adjacent to an element of S_t. The size of total dominating set with minimum cardinality is denoted by $\gamma_t(G)$. For more details for total dominating set, we refer the reader to [1]. The open neighborhood of a vertex v consists of the set of adjacent vertices to v, that is, $N(v)=\{w\in V: vw\in E\}$ and The closed neighborhood of v is defined as $N[v]=N(v)\cup\{v\}$. The following labelling can be defined on V with respect to a subset

$$y(v_i)=\begin{cases}1 & v\in S\\ -1 & v\notin S.\end{cases}\ S\subseteq V$$

Theorem 1: $S\subseteq V$ is total dominating set if and only if it must satisfy in the following inequalities:

$$\sum_{j\in N(i)} (1 - y_i y_j) + \sum_{j\in N[i]} \frac{1+y_j}{2} \geq 2 \qquad i = 1,2,\dots,n \tag{1}$$

Now, based on the (1), the quadratic integer programming model can be written as follows:

$$\begin{array}{lll} min & \frac{1}{2}\sum_{i=1}^{n}(1+y_i) & \\ s.t. & \sum_{j\in N(i)}(1-y_i y_j) + \sum_{j\in N(i)}\frac{(1+y_j)}{2} \geq 2 & i = 1,\dots,n \\ & y_i \in \{-1,+1\} & i = 1,\dots,n \end{array} \tag{2}$$

With making many changes in (2), the following semidefinite programming model is written:

$$\begin{array}{lll} min & \frac{n}{2} + < C, X > & \\ s.t. & < A_i, X > \geq 2 & i = 1,\dots,n \\ & X_{ii} = 1 & i = 0,1,\dots,n \\ & rank(X) = 1 & \\ & X \succcurlyeq 0 & \end{array}$$

Because the constraint $rank(X) = 1$ is nonconvex, the problem is relaxed by dropping it and the following model is obtained:

$$\begin{array}{lll} min & \frac{n}{2} + < C, X > & \\ s.t. & < A_i, X > \geq 2 & i = 1,\dots,n \\ & X_{ii} = 1 & i = 0,1,\dots,n \\ & X \succcurlyeq 0 & \end{array}$$

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SOME EMBEDDIG THEOREMS ON BANACH VALUES BESOV SPACES

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We define the space $B^{s}_{p,q}\left(R^n : E_0, E\right)$ as follows.

Let E_0 and E Banach spaces, $E_0 \subset E$ continuosly embedding and E_0 in E is everywhere dense.

Definition 1. Let for $s=\left(s_1,\dots,s_n\right)$, $-\infty < s_i < \infty$, $1 < p < \infty, 1 \le q < \infty$

$$B^{s_i}_{p,q}\left(R^n : E_0, E\right) = \left\{ f \mid f \in S'\left(R^n : E\right), f = \right.$$

$$\left. = \sum_{j=0}^{\infty} a_j(x), \ \sup pFa_j \subset M_j, \ j=1,\dots,n \right\}$$

$$\|f\|_{B^{s_i}_{p,q}\left(R^n : E_0, E\right)} = \left\| \left\{ a_j \right\} \right\|_{l_q^{s_i}\left(L_p\left(R^n : E_0, E\right)\right)} =$$

$$= \|a_0\|_{L_p\left(R^n : E_0\right)} + \left(\sum_{j=1}^{\infty} \left(2^{js_i} \|a_j(x)\|_{L_p\left(R^n : E\right)} \right)^q \right)^{1/q} < \infty$$

$-\infty < s_i < \infty, 1 < p < \infty$ and for $q < \infty$

$$B^{s_i}_{p,\infty}\left(R^n : E_0, E\right) = \left\{ f \mid f \in S'\left(R^n : E\right), f = \right.$$
$$\left. = \sum_{j=0}^{\infty} a_j(x), \sup pFa_0 \subset M_0 \right\}$$
$$\| f \|^*_{B^s_{p,q}\left(R^n : E_0, E\right)} = \| a_j \|_{l^{s_i}_\infty\left(L_p\left(R^n : E_0, E\right)\right)} =$$
$$= \| a_0(x) \|_{L_p(E_0)} + \sup 2^{js_i} \| a_j(x) \|_{L_p\left(R^n : E\right)}$$

with $q = \infty$ corresponding change. So for $s = (s_1, \ldots, s_n)$

$$B^s_{p,q}\left(R^n : E_0, E\right) = \bigcap_{i=1}^{n} B^{s_i}_{p,q}\left(R^n : E_0, E\right)$$

with the appropriate norm.

We define the space $B^s_{p,q}(R^n : E_0, E)$ for $s = (s_1, \ldots, s_n), -\infty < s_j < \infty$ for functions with values from E_0 and values of derivatives from E as follows.

Definition 2. The system of functions $\{\varphi_k(x)\}_{k=0}^{\infty}$ satisfying certain conditions for any N is denoted by Φ_N, we take $\Phi = \bigcup_{N=1}^{\infty} \Phi_N$.

Theorem 1. Let $s = (s_1, \ldots, s_n), 1 < p < \infty$, $1 \le q \le \infty$. Then the Banach space $B^s_{p,q}\left(R^n : E_0, E\right)$is defined as follows. Let $\{\varphi_k\}_{k=0}^{\infty} \in \Phi$

$$B^{s_i}_{p,q}\left(R^n : E_0, E\right) = \left\{ f \mid f \in S'\left(R^n : E_0\right), \| f \|_{B^{s_i}_{p,q}\left(R^n : E_0, E\right)} = \right.$$
$$\left. = \| f * \varphi_k \|_{l^s_{q,p}\left(R^n : E\right)} < \infty \right\}$$

and this norm is equivalent to the norm $\| f \|^{*}_{B^{s}_{p,q}(R^n:E_0,E)}$.

Theorem 2. Let $s=(s_1,...,s_n)$, $-\infty<s_i<\infty$, $\varepsilon=(\varepsilon_1,...,\varepsilon_n)$, $\varepsilon_i>0$, $i=1,...,n$ $1<p<\infty$, $1\le q_1\le q_2\le\infty$. Then the following embedding are continuous

$$B^{s+\varepsilon}_{p,\infty}(R^n:E_0,E)\subset B^{s}_{p,1}(R^n:E_0,E)\subset B^{s}_{p,q_1}(R^n:E_0,E)\subset$$
$$\subset B^{s}_{p_1,q_2}(R^n:E_0,E)\subset B^{s}_{p,\infty}(R^n:E_0,E)\subset B^{s-\varepsilon}_{p,1}(R^n:E_0,E)$$

where $s\pm\varepsilon=(s_1\pm\varepsilon_1,...,s_n\pm\varepsilon_n)$.

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FINITE DEFORMATION OF INTERNALLY PRESSURIZED COMPRESSIBLE RUBBER-LIKE MATERIAL

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In this work a finite deformation of internally pressurized compressible hollow sphere and cylinder with strain energy function as proposed by Levinson and Burgess is analyzed. Stresses and displacements of the result from the second order nonlinear ordinary differential equation are determined. This is applied in determining the strain, stresses and displacements in an inflated car tyre using the finite element method with mathematica software (AceFEM).

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ONE UNIQUE SOLVABILITY OF A BOUNDARY VALUE PROBLEM FOR E FOURTH ORDER OPERATOR-DIFFERENTIAL EQUATION ON FINITE SEGMENT IN HILBERT SPACE

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A uniquely solvability problem of a fouth order operator-differential equation in the finite seqment in Hilbert space was studied in the presented paper. Usinq the Fourier transformation method a uniquely solvability of nonhomogeneous equation is proved for considered boundaryvalue problem in the Hilbert space $\overset{\circ}{W}{}_2^4([0,1];H)$.

Let H be separable Hilbert space, A be a self-adjoint positive-definite operator in H with domain of definition $D(A)$. Denote by Hilbert space

$$W_2^4([0,1];H) = \{u(t) : u^{(4)}(t) \in L_2([0,1];H),\ A^4 u(t) \in L_2([0,1];H)\}$$

with a scalar product

$$(u,v)_{W_2^4([0,1];H)} = \int_a^b \left(u^{(4)}(t), v^{(4)}(t)dt + \int_a^b \left(A^4 u(t), A^4 v(t)\right)\right) dt$$

that has the norm

$$\|u\|_{W_2^4([0,1];H)} = \left\|u^{(4)}(t)\right\|^2_{L_2([0,1];H)} + \left\|A^{(4)} u(t)\right\|^2_{L_2([a,b];H)}$$

From the theorem of trace [1] consequence, that if $u(t) \in W_2^4 L_2([0,1];H)$, then $u^{(k)}(0), u^{(k)}(1) \in H_{4-k-\frac{1}{2}}, k = 1,2,3.$

We consider the boundary value problem

$$Lu = \frac{d^4 u}{dt^4} + A^4 u = f(t),\ t \in [0,1], \tag{1}$$

$$u(0) = u(1) = 0,\ u'(0) = u'(1) = 0. \tag{2}$$

Definition. If at any $f(t) \in L_2([0,1];H)$, there exists the vector-function $u(t) \in W_2^4 L_2([0,1];H)$ which satisfies equation (1) almost everywhere in [0,1], the boundary conditions (2) in the sense

$$\lim_{t\to+0} \|u(t)\|_{H_{7/2}} = 0,\ \lim_{t\to+0} \|u'(t)\|_{H_{5/2}} = 0,$$

$$\lim_{t\to1-0} \|u(t)\|_{H_{7/2}} = 0,\ \lim_{t\to1-0} \|u'(t)\|_{H_{5/2}} = 0$$

and the inequality

$$\|u\|_{W_2^4([0,1];H)} \le const \|f\|_{L_2([0,1];H)}$$

then the problem (1)-(2) we′ll call regularly solvable or uniquely solvable.

It holds the following theorem.

Theorem. Let A be self-adjoint positive-definite operator in H space. Then the boundary value problem (1)-(2) is regularly solvable.

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FINDING THE FIRST EIGENVALUE OF THE LANDAU OPERATOR

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When studying surface superconductivity in II kind superconducting material with different cross sections in the condition of increasing of the external magnetic field strength it becomes necessary to study the first eigenvalue and its corresponding eigenfunction of the system of Ginzbury-Landau system of equations (see. [1, p.143])

$$\begin{cases} \left.\begin{array}{c} (i\nabla + hA)^2\psi(x) = \dfrac{h^2}{\sigma^2}\left(1-|\psi(x)|^2\right)\psi(x), \\ curl(curlA-\beta) = -\dfrac{1}{h}\mathrm{Re}\left[\overline{\psi(x)}(i\nabla + hA)\psi(x)\right] \end{array}\right\} \quad in \quad \Omega, \\ \left.\begin{array}{c} (i\nabla + hA)\psi(x)\cdot\nu = 0, \\ curlA = \beta \end{array}\right\} \quad on \quad \partial\Omega, \end{cases}$$

where $x = (x_1, x_2) \in R^2$, $\nabla = \left(\dfrac{\partial}{\partial x_1}, \dfrac{\partial}{\partial x_2}\right)$, β is an the external magnetic field, h is magnetic field strength, $A = (a_1(x_1,x_2), a_2(x_1,x_2))$ is induced real magnetic potential, $i = \sqrt{-1}$, $\sigma > 0$ is a Ginhzburg-Landau parameter, Ω is a cross section of the material, $\partial\Omega$ is the boundary of the domain Ω, $\psi(x)$ is a wave function, ν is an external normal vector,

$$curlA = \frac{\partial a_2(x_1, x_2)}{\partial x_1} - \frac{\partial a_1(x_1, x_2)}{\partial x_2},$$

$$curl^2 A = \left(\frac{\partial(curlA)}{\partial x_2}, -\frac{\partial(curlA)}{\partial x_1} \right).$$

From the second variation of the Ginzburg-Landau functional

$$G(\psi, A) = \int_\Omega \left\{ \left| (i\nabla + hA)\psi(x) \right|^2 + \frac{h^2}{\sigma^2} \left(|\psi(x)|^2 - 1 \right)^2 \right\} dx + h^2 \int_{R^2} |curlA - \beta|^2 dx$$

in the neighborhood of the normal state $\psi(x) = 0$ it is seen that this problem is closely related to finding the exact lower boundary of the Rayleigh magnetic quantity

$$\frac{\int_{R^2} \left| (i\nabla + hA)\psi(x) \right|^2 dx}{\int_{R^2} |\psi(x)|^2 dx}$$

in the first order Sobolev space $W_2^1(R^2) = \overset{0}{W}{}_2^1(R^2) = H^1(R^2)$.

We have the following

Theorem. *Let* $A = \omega(x) = \frac{h}{2}(-x_2, x_1)$. *Then the first eigenvalue* $\lambda_1(h)$ *of the Landau operator is equal to the number* $|h|$.

This result is well known from physical literature and in some way goes back to Landau. Some unsuccessful attempts for proving this theorem were made in [2] and [3].

This paper, in the opinion of the author, provides a simple proof of this theorem.

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ON SOLVABILITY OF AN NONLOCAL BOUNDARY VALUE PROBLEM FOR THE PARABOLIC - HYPERBOLIC EQUATION TYPE

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Consider an equation of mixed parabolic-hyperbolic type

$$Lu = \begin{cases} u_t - u_{xx} + b^2 u = 0, t > 0, \\ u_{tt} - u_{xx} + b^2 u = 0, t < 0, \end{cases} \tag{1}$$

where $b = const \geq 0$.

Let $D = \{(x,t) : 0 < x < 1, -\alpha < t < \beta\}$, where α and β - given positive real numbers.

Boundaryvalueproblem: Findfunction $u(x,t)$ in D, that satisfying the following conditions:

$$u(x,t) \in C^1(\overline{D}) \cap C^2(D_+ \cup D_-) \tag{2}$$

$$Lu = F(x,t), (x,t) \in D_+ \cup D_-, \tag{3}$$

$$u(x,\beta) = \varphi(x), u(x,-\alpha) = \psi(x), \quad 0 \le x \le 1, \tag{4}$$

$$u(0,t) = u(0,t), \; -\alpha \le t \le \beta \tag{5}$$

$$u_x(0,t) = \gamma u_x(1,t), \;\; -\alpha \le t \le \beta \tag{6}$$

where, $\gamma \ne \pm 1$ - fixed number, $\varphi(x)$, $\psi(x)$, $F(x,t)$ -given functions.

Note that the boundary-value problem for equations of mixed parabolic-hyperbolic type was studied [1] - [4] and other authors. In this paper, the uniqueness and existence of a solution to the problem is proved.

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ON FIXED POINT UNIQUENESS CONDITIONS FOR GENERALIZED WEAK CONTRACTIONS

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In this paper, by providing an example, we show that the con-dition which produced by Radenovi_c and Kadelburg in [Generalized weak contractions in partially ordered metric spaces, Comput. Math. Appl. 60(2010) pp. 1776-1783] is not sufficient for uniqueness of the fi_xed point. Fur-thermore, a new sufficient condition is introduced for the uniqueness of _fixed point. Some suitable examples are furnished to demonstrate the validity of the hypotheses of our results.

EXISTENCE OF STRONG SOLUTIONS FOR THE COUPLED SUSPENSION BRIDGE EQUATIONS

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We consider the following mathematical model for the oscillations of the bridge which has one common point with the cable:

$$\begin{cases} u_{tt} + u_{xxxx} + (u-v)_+ + \Phi(u_t) = f(x), \\ v_{tt} - v_{xx} + (v-u)_+ + G(v_t) = g(x), \end{cases}$$

$$\begin{cases} u(\xi-0,t) = u(\xi+0,t) = v(\xi-0,t) = v(\xi+0,t), \\ u'''(\xi-0,t) - u'''(\xi+0,t) - v'(\xi-0,t) + v'(\xi+0,t) = 0, \\ u''(\xi-0,t) = u''(\xi+0,t), \end{cases}$$

$$u(0,t) = u'(0,t) = u(l,t) = u'(l,t) = v(0,t) = v(l,t) = 0,$$

$$u(x,0)=u_0(x), u'(x,0)=u_1(x),$$
$$v(x,0)=v_0(x), v'(x,0)=v_1(x),$$

where $0 \le x \le l,\ t>0$, $0<\xi<l$. Here $u(x,t)$ is state function of the road bed and $v(x,t)$ is that of the main cable, $\Phi(u_t)$, $G(v_t)$ are the aerodynamically dampings. The solution of the problem is reduced to the Cauchy problem for the operator differential equation in some functional spaces.

In this study, we show the existence of global solutions of the considered problem and investigate the asymptotics of these solutions.

AN ALMOST PARA-COMPLEX STRUCTURES ON THE LINEAR COFRAME BUNDLE

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Let (M_{2k}, g) be a pseudo-Riemannian manifold admitting an almost para-complex structure φ. φ is called almost para-Nordenian if g has a signature of (k,k) and is pure with respect to φ, i.e. $g(\varphi X, Y)=g(X,\varphi Y)$ for any vector fields X, Y on M_{2k}. Almost para-Nordenian structures are used not only in mathematics, but also in theoretical physics. For this reason, a number of papers have been devoted to the study of such structures in fiber bundles (see, for example, [1], [2]). In the present report, on the linear coframe bundle equipped with Sasakian metric, almost para-Nordenian structures are determined, their properties, and also integrability conditions are studied. Let (M, g) be a n-dimensional pseudo-Riemannian manifold of class C^{∞}, $F^{*}(M)$ be a linear coframe bundle over M with Sasakian metric ${}^{S}g$ (see, [3]) . On the $F^{*}(M)$, tensor fields $F_{\alpha}, \alpha = 1,2,...,n,$ of type $(1,1)$ are determined by the following:

$$F_{\alpha}({}^{H}X) = {}^{V_{\alpha}}\tilde{X},\ F_{\alpha}({}^{V_{\beta}}\omega) = \delta_{\alpha}^{\beta}\, {}^{H}\tilde{\omega},$$

for all $X \in \Im_{0}^{1}(M)$ and $\omega \in \Im_{1}^{0}(M)$, where $\tilde{X} = g \circ X \in \Im_{1}^{0}(M)$, $\tilde{\omega} = g^{-1} \circ \omega \in \Im_{0}^{1}(M)$. It is proved that for each $\alpha = 1,2,...,n,$, F_{α} is an almost para-complex structure, i.e. $F_{\alpha}^{2} = I$. The following theorems are also proved.

Theorem 1. *For each $\alpha = 1,2,...,n,$, the triple $(F^{*}(M), {}^{S}g, F_{\alpha})$ is an almost para-Nordenian manifold.*

Theorem 2. *Let (M, g) be a pseudo-Riemannian manifold and $F^{*}(M)$ be its linear coframe bundle equipped with Sasakian metric ${}^{S}g$. Then the almost para-Nordenian manifold $(F^{*}(M), {}^{S}g, F_{\alpha})$, for each $\alpha = 1,2,...,n$, is para-Nordenian if and only if $R = 0$, where R is the curvature tensor field of the Levi-Civita connection ∇_{g}.*

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THE CONDITIONS OF ORTHOGONALITY OF THE INPUT SEQUENCES OF BINARY THREE - PARAMETERSMULTY-DIMENSIONAL NONLINEAR MODULAR DYNAMIC SYSTEMS

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The to find condition's of orthogonality of the input sequences of binary three - parameters multy-dimensional non-linear modular dynamic systems (3D–MNMDS) with memory n_0, limited connection $P = P_1 \times P_2$, degree S, r input and k output is considered [1]:

$$y_\nu[n,c_1,c_2] = \sum_{i=1}^{S} \sum_{\overline{\eta}\in\Lambda(i)} \sum_{\overline{w}\in\Psi(\overline{\eta})} \sum_{(\overline{\overline{j}},\overline{\overline{\mu}})\in L_1\times L_2} \sum_{\overline{\overline{\tau}}\in\Gamma(\gamma_1,\gamma_2,\overline{\overline{m}})[\overline{w}]} h_{\nu,i,\overline{\eta},\overline{w}}[\overline{\overline{j}},\overline{\overline{\mu}},\overline{\overline{\tau}}]$$

$$\prod_{\ell\in Q_0(\overline{\eta})} \prod_{(\alpha_\ell,\beta_\ell,\sigma)\in Q''_\ell(\eta_\ell,\gamma_1(\ell),\gamma_2(\ell),\overline{m}_\ell)} \upsilon_{\ell,i,\overline{\eta},\overline{w}}[n-$$

$$-\tau_\ell(\alpha_\ell,\beta_\ell,\sigma),c_1+p_1(j_{\alpha_\ell}(\ell)),c_2+p_2(\mu_{\beta_\ell}(\ell))],\ \ GF(2),\ \ \nu=\overline{1,k}. \tag{1}$$

In (1) $n\in[0,N]=\{0,1,\dots,N\}$, $c_\alpha\in[0,C_\alpha]=\{0,1,\dots,C_\alpha\}$, $P_\alpha=\{p_\alpha(1),\dots,p_\alpha(r_\alpha)\}$, $-\infty<p_\alpha(1)<\dots<p_\alpha(r_\alpha)<\infty$, $p_\alpha(\beta)\in Z$, $\beta=\overline{1,r_\alpha}$, $\alpha=\overline{1,2}$, where Z is set of integer number; $y[n,c_1,c_2]=(y_1[n,c_1,c_2],\dots,y_k[n,c_1,c_2])$,

$\upsilon_{i,\overline{\eta},\overline{w}}[n,c_1,c_2]=(\upsilon_{1,i,\overline{\eta},\overline{w}}[n,c_1,c_2],\dots,\upsilon_{r,i,\overline{\eta},\overline{w}}[n,c_1,c_2])$;

Definition of $\overline{\eta}$, $\Lambda(i)$, $Q_0(\overline{\eta})$, $\Psi(\overline{\eta})$, L_1, L_2, $\Gamma(\gamma_1,\gamma_2,\overline{\overline{m}})$, $L_{\ell,1}(\gamma_1(\ell))$, $L_{\ell,2}(\gamma_2(\ell))$, γ_1, γ_2, $\overline{\overline{\overline{j}}}$, $\overline{\overline{\mu}}$, $\overline{\overline{m}}$, $Q''_\ell(\eta_\ell,\gamma_1(\ell),\gamma_2(\ell),\overline{m}_\ell)$ etc. given in [1]. Let

$$V_0(i,\overline{\eta},\overline{w},\overline{\overline{\overline{j}}},\overline{\overline{\mu}},\overline{\overline{\overline{\tau}}}_\xi)=$$

$$=\left\{\prod_{\ell\in Q_0(\overline{\eta})}\ \prod_{(\alpha_\ell,\beta_\ell,\sigma)\in Q''_\ell(\eta_\ell,\gamma_1(\ell),\gamma_2(\ell),\overline{m}_\ell)}\upsilon_{\ell,i,\overline{\eta},\overline{w}}[n-\tau_\ell^{(\xi)}(\alpha_\ell,\beta_\ell,\sigma),c_1+p_1(j_{\alpha_\ell}(\ell)),c_2+p_2(\mu_{\beta_\ell}(\ell))]\right\}. \tag{2}$$

$$V_1(i,\overline{\eta},\overline{w},\overline{\overline{\overline{j}}},\overline{\overline{\mu}})=(V_0(i,\overline{\eta},\overline{w},\overline{\overline{\overline{j}}},\overline{\overline{\mu}},\overline{\overline{\overline{\tau}}}_1)\ \dots\ V_0(i,\overline{\eta},\overline{w},\overline{\overline{\overline{j}}},\overline{\overline{\mu}},\overline{\overline{\overline{\tau}}}_{|\Gamma(\gamma_1,\gamma_2,\overline{\overline{m}})[\overline{w}]|}))\ , \tag{3}$$

$$V_2(i,\overline{\eta},\overline{w})=(V_1(i,\overline{\eta},\overline{w},\overline{\overline{\overline{j}}}_1,\overline{\overline{\mu}}_1),\dots,V_1(i,\overline{\eta},\overline{w},\overline{\overline{\overline{j}}}_1,\overline{\overline{\mu}}_{|L_2|}),\dots,V_1(i,\overline{\eta},\overline{w},\overline{\overline{\overline{j}}}_{|L_1|},\overline{\overline{\mu}}_{|L_2|}))\ , \tag{4}$$

$$V_3(i,\overline{\eta})=(V_2(i,\overline{\eta},\overline{w}_1),\dots,V_2(i,\overline{\eta},\overline{w}_{|\Psi(\overline{\eta})|}))\ ,$$

$$V_4(i)=(V_3(i,\overline{\eta}_1),\dots,V_3(i,\overline{\eta}_{|\Lambda(i)|}))\ , \tag{5}$$

$$V_5=(V_4(1)\ \dots\ V_4(S))\ ,\quad V=diag[V_5,\dots,V_5]\,. \tag{6}$$

In (2) $\tau_\ell^{(\xi)}(\alpha_\ell,\beta_\ell,\sigma)$ is ℓ-th components of ξ-th triple $\overline{\overline{\overline{\tau}}}_\xi$ from $\Gamma(\gamma_1,\gamma_2,\overline{\overline{m}})[\overline{w}]$. In matrix $V_0(i,\overline{\eta},\overline{w},\overline{\overline{\overline{j}}},\overline{\overline{\mu}},\overline{\overline{\overline{\tau}}}_\xi)$ each triple (n,c_1,c_2), $n\in[0,N]$, $c_1\in[0,C_1]$, $c_2\in[0,C_2]$, corresponds to arow, i.e. this matrix is with dimension's $(N+1)(C_1+1)(C_2+1)\times 1$. In (6) matrix V is block matrix $n-$th order. Matrix V isalsoanordinarymatrixofdimension's $(N+1)(C_1+1)(C_2+1)k\times M$, where $\alpha=\overline{1,M}$. Let's

$R_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}})$, $R_2(i,\overline{\eta},\overline{w})$, $R_3(i,\overline{\eta})$, $R_4(i)$ are number of columns respectively matrix's $V_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}})$, $V_2(i,\overline{\eta},\overline{w})$, $V_3(i,\overline{\eta})$, $V_4(i)$. Let for all $\ell \in Q_0(\overline{\eta})$, $\overline{w} \in \Psi(\overline{\eta})$, $\overline{\eta} \in \Lambda(i)$, $i \in \{1,\dots,S\}$, input sequences $\{\upsilon_{\ell,i,\overline{\eta},\overline{w}}[n,c_1,c_2]: n \in [0,N], c_1 \in [0,C_1], c_2 \in [0,C_2]\}$ such, that thematrix V, formed by (2) - (6) is orthogonally, i.e. the matrix V satisfies the conditions of orthogonallity $V^T \cdot V = diag[d_{1,1},\dots,d_{M,M}]$; $d_{\alpha,\alpha} > 0$, $\alpha = \overline{1,M}$, where $d_{\alpha,\alpha}$, $\alpha = \overline{1,M}$ are elements of the matrix $V^T \cdot V$. Then this sequence is called the orthogonal input sequence (OIS) of 3D–MNMDS (1).

Theorem 1. For, so that V satisfies the ortogonality conditions, is necessary and sufficient to for all $\overline{w} \in \Psi(\overline{\eta})$, $\overline{\eta} \in \Lambda(i)$, $i \in \{1,\dots,S\}$ satisfy the relation

$$V_2(i,\overline{\eta},\overline{w})^T V_2(i,\overline{\eta},\overline{w}) = diag\{d_{1,1}(2,\overline{\eta},\overline{w}),\dots,d_{R_2(i,\overline{\eta},\overline{w}),R_2(i,\overline{\eta},\overline{w})}(2,\overline{\eta},\overline{w})\},$$

$$d_{\gamma,\gamma}(2,\overline{\eta},\overline{w}) > 0, \gamma = \overline{1,R_2(i,\overline{\eta},\overline{w})},$$

where $d_{\gamma,\gamma}(2,\overline{\eta},\overline{w})$, $\gamma = \overline{1,R_2(i,\overline{\eta},\overline{w})}$, are elements of the matrix $V_2(i,\overline{\eta},\overline{w})^T \cdot V_2(i,\overline{\eta},\overline{w})$, but for all $\overline{w} \in \Psi(\overline{\eta})$, $\overline{\eta} \in \Lambda(i)$, $i \in \{1,\dots,S\}$, $\overline{w}' \in \Psi(\overline{\eta}')$, $\overline{\eta}' \in \Lambda(i')$, $i' \in \{1,\dots,S\}$, $(i,\overline{\eta},\overline{w}) \neq (i',\overline{\eta}',\overline{w}')$ carried to ratio $V_2(i,\overline{\eta},\overline{w})^T V_2(i',\overline{\eta}',\overline{w}') = 0.$

Theorem 2. Let's $\overline{w} \in \Psi(\overline{\eta})$, $\overline{\eta} \in \Lambda(i)$, $i \in \{1,\dots,S\}$. For own orthogonalityofsequences $\{\upsilon_{\ell,i,\overline{\eta},\overline{w}}[n,c_1,c_2]: n \in [0,N], c_1 \in [0,C_1], c_2 \in [0,C_2]\}$, $\ell \in \{1,\dots,r\}$, it is necessary and sufficient, that for all $(\overline{\overline{j}},\overline{\overline{\mu}}) \in L_1 \times L_2$ carried to

$$V_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}})^T \cdot V_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}}) = diag\{d_{1,1}(1),\dots,d_{R_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}}),R_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}})}(1)\},$$

$d_{\alpha,\alpha}(1)>0,\ \alpha=\overline{1,R_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}})}$, where $d_{\alpha,\alpha}(1)$, $\alpha=\alpha=\overline{1,R_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}})}$, are elements of the matrix $V_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}})^T\cdot V_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}})$, but for all $(\overline{\overline{j}},\overline{\overline{\mu}})\in L_1\times L_2$, $(\overline{\overline{j}}',\overline{\overline{\mu}}')\in L_1\times L_2$, $(\overline{\overline{j}},\overline{\overline{\mu}})\neq(\overline{\overline{j}}',\overline{\overline{\mu}}')$ carried to ratio $V_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}})^T\cdot V_1(i,\overline{\eta},\overline{w},\overline{\overline{j}}',\overline{\overline{\mu}}')=0$.

Theorem 3. Let: 1. Forall $\ell\in Q_0(\overline{\eta})$, $\overline{w}\in\Psi(\overline{\eta})$, $\overline{\eta}\in\Lambda(i)$, $i\in\{1,\dots,S\}$ sequences $\vartheta_{\ell,i,\overline{\eta},\overline{w}}[n,c_1,c_2]$ is $\{0,1\}$ – sequenceswith period's $T_{i,\overline{\eta},\overline{w}}+1$, $A_1(i,\overline{\eta},\overline{w})+1$ and $A_2(i,\overline{\eta},\overline{w})+1$ on n,c_1 and c_2 respectively, and $\overline{V}_2(i,\overline{\eta},\overline{w})^T\cdot\overline{V}_2(i,\overline{\eta},\overline{w})=diag\{d_{1,1}(2,\overline{\eta},\overline{w}),\dots,d_{R_2(i,\overline{\eta},\overline{w}),R_2(i,\overline{\eta},\overline{w})}(2,\overline{\eta},\overline{w})\}$,

$d_{\gamma,\gamma}(2,\overline{\eta},\overline{w})>0,\ \gamma=\overline{1,R_2(i,\overline{\eta},\overline{w})}$, where $d_{\gamma,\gamma}(2,\overline{\eta},\overline{w})$, $\gamma=\overline{1,R_2(i,\overline{\eta},\overline{w})}$ are elements of the matrix $\overline{V}_2(i,\overline{\eta},\overline{w})^T\cdot\overline{V}_2(i,\overline{\eta},\overline{w})$, but $\overline{V}_2(i,\overline{\eta},\overline{w})$ formed from sequences $\overline{\upsilon}_{\ell,i,\overline{\eta},\overline{w}}[n,c_1,c_2]$, $n\in[0,T_{i,\overline{\eta},\overline{w}}]$, $c_1\in[0,A_1(i,\overline{\eta},\overline{w})]$, $c_2\in[0,A_2(i,\overline{\eta},\overline{w})]$ by theformula**:**

$$\overline{V}_0(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}},\overline{\overline{\overline{\tau}}}_\xi)=$$

$$=\left\{\prod_{\ell\in Q_0(\overline{\eta})}\ \prod_{(\alpha_\ell,\beta_\ell,\sigma)\in Q''_\ell(\eta_\ell,\gamma_1(\ell),\gamma_2(\ell),\overline{\overline{m}}_\ell)}\overline{\upsilon}_{\ell,i,\overline{\eta},\overline{w}}[n-\tau_\ell^{(\xi)}(\alpha_\ell,\beta_\ell,\sigma),c_1+p_1(j_{\alpha_\ell}(\ell)),c_2+p_2(\mu_{\beta_\ell}(\ell))]\right\}.$$

$$\overline{V}_1(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}})=(\overline{V}_0(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}},\overline{\overline{\overline{\tau}}}_1)\ \dots\ \overline{V}_0(i,\overline{\eta},\overline{w},\overline{\overline{j}},\overline{\overline{\mu}},\overline{\overline{\overline{\tau}}}_{|\Gamma(\gamma_1,\gamma_2,\overline{\overline{m}})[\overline{w}]|}))$$

,

$$\overline{V}_2(i,\overline{\eta},\overline{w})=(\overline{V}_1(i,\overline{\eta},\overline{w},\overline{\overline{j}}_1,\overline{\overline{\mu}}_1),\dots,\overline{V}_1(i,\overline{\eta},\overline{w},\overline{\overline{j}}_1,\overline{\overline{\mu}}_{|L_2|}),\dots,\overline{V}_1(i,\overline{\eta},\overline{w},\overline{\overline{j}}_{|L_1|},\overline{\overline{\mu}}_{|L_2|}))$$

;

2. a) Forall $\ell \in Q_0(\overline{\eta}),\ \overline{w} \in \Psi(\overline{\eta}),\ \overline{\eta} \in \Lambda(i),\ i \in \{1,\dots,S\}$ and $(n, c_1, c_2) \in [0, T_{i,\overline{\eta},\overline{w}}] \times [0, A_1(i,\overline{\eta},\overline{w})] \times \times [0, A_1(i,\overline{\eta},\overline{w})] \times [0, A_2(i,\overline{\eta},\overline{w})]$ sequences $\upsilon'_{\ell,i,\overline{\eta},\overline{w}}[n, c_1, c_2]$ determined by the formula**:**

$$\upsilon'_{\ell,i,\overline{\eta},\overline{w}}[n,c_1,c_2] = \begin{cases} \overline{\upsilon}_{\ell,i,\overline{\eta},\overline{w}}[n,c_1,c_2], \text{ if } (n,c_1,c_2) \in F(i,\overline{\eta},\overline{w}) G_1(i,\overline{\eta},\overline{w}) \times G_2(i,\overline{\eta},\overline{w}), \\ 0 \qquad\qquad\qquad , \text{ if } (n,c_1,c_2) \notin F(i,\overline{\eta},\overline{w}) \times G_1(i,\overline{\eta},\overline{w}) \times G_2(i,\overline{\eta},\overline{w}), \end{cases}$$

where

$$F(i,\overline{\eta},\overline{w}) = [N_1(i,\overline{\eta},\overline{w}) - \tau(i,\overline{\eta},\overline{w}), N_1(i,\overline{\eta},\overline{w}) - \tau(i,\overline{\eta},\overline{w}) + T_{i,\overline{\eta},\overline{w}}] \subset [0, T'],$$

$$G_\alpha(i,\overline{\eta},\overline{w}) = [D_\alpha(i,\overline{\eta},\overline{w}), D_\alpha(i,\overline{\eta},\overline{w}) + A_\alpha(i,\overline{\eta},\overline{w})] \subset [0, C'_\alpha],\ \alpha = \overline{1,2},$$

$$\tau(i,\overline{\eta},\overline{w}) = \begin{cases} \max\{m_{\ell,1,1},\dots,m_{\ell,1,\gamma_2(\ell)},\dots,m_{\ell,\gamma_1(\ell),\gamma_2(\ell)}\} - 1, \text{ if } N_1(i,\overline{\eta},\overline{w}) > 0, \\ 0 \qquad\qquad\qquad , \text{ if } N_1(i,\overline{\eta},\overline{w}) = 0; \end{cases}$$

b) For all $\overline{w} \in \Psi(\overline{\eta}), \overline{\eta} \in \Lambda(i),\ i \in \{1,\dots,S\}$ integer numbers $N_1(i,\overline{\eta},\overline{w})$, $D(i,\overline{\eta},\overline{w})$, $D_2(i,\overline{\eta},\overline{w})$ and domain $[0,T'] \times [0, C'_1] \times [0, C'_2]$ such, that for any $\overline{w}' \in \Psi(\overline{\eta}'),\ \overline{\eta}' \in \Lambda(i'),\ i' \in \{1,\dots,S\}$, where $\langle i,\overline{\eta},\overline{w}\rangle \neq \langle i',\overline{\eta}',\overline{w}'\rangle$, are carried out the ratios $F(i,\overline{\eta},\overline{w}) \cap F(i',\overline{\eta}',\overline{w}') = \varnothing$ or $G_1(i,\overline{\eta},\overline{w}) \cap G_1(i',\overline{\eta}',\overline{w}') = \varnothing$ or $G_2(i,\overline{\eta},\overline{w}) \cap G_2(i',\overline{\eta}',\overline{w}') = \varnothing$;

3. For all $\ell \in Q_0(\overline{\eta}),\ \overline{w} \in \Psi(\overline{\eta}), \overline{\eta} \in \Lambda(i), i \in \{1,\dots,S\}$ sequences $\upsilon_{\ell,i,\overline{\eta},\overline{w}}[n, c_1, c_2]$ is the periodic continuation of sequence $\upsilon'_{\ell,i,\overline{\eta},\overline{w}}[n, c_1, c_2]$ from domain $[0,T'] \times [0, C'_1] \times [0, C'_2]$ to other parts domain $[0, N] \times [0, C_1] \times [0, C_2]$ with a period's $T_{i,\overline{\eta},\overline{w}} + 1$, $A_1(i,\overline{\eta},\overline{w}) + 1$ and $A_2(i,\overline{\eta},\overline{w}) + 1$ by n, c_1 and c_2 respectively. Then matrix is V orthogonally.

OIS is used in solving the problem of optimal synthesis of 3D - MNMDS. On the based above described conditions of orthogonality, it is possible to develop an algorithm for constructing the ORP for 3D –MNMDS (1).

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EXISTENCE, UNIQUENESS AND BOUNDEDNESS OF GLOBAL SOLUTIONS OF NON-STATIONARY DEGENERATE DIFFERENTIAL EQUATIONS

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We consider the initial value problem (IVP)

$$\frac{d}{dt}[A(t)x(t)]+B(t)x(t)=f(t,x(t)), \tag{1}$$

$x(t_0)=x_0$, where $t_0\geq t_+\geq 0$, $f:[t_+,\infty)\times\mathbb{R}^n\to\mathbb{R}^n$, $A,B:[t_+,\infty)\to L(\mathbb{R}^n)$ and the linear operators $A(t),B(t)$ may be degenerate (noninvertible). A differential equation (DE) of the type (1) with the degenerate operator $A(t)$ is called degenerate DE, differential-algebraic equation (DAE), descriptor system, algebraic-differential equation or system, singular system, etc. Degenerate

Des of the form (1) are used to describe mathematical models in control theory, radio electronics, economics, mechanics of multilink mechanisms, chemical kinetics, ecology and other fields. We do not consider the optimal control problem; therefore, there is no explicit dependence of f on a control input. However, we can consider the equation (1) as a control system (e.g., a closed-loop system), where $f(t,x)=g(t,x,u(t))$ and $u(t)$ is any admissible control such that f satisfies required conditions. We obtain conditions for the existence and uniqueness of global solutions, the Lagrange stability, and the Lagrange instability of the degenerate DE (1). A solution $x(t)$ of the IVP is called global if it exists on $[t_0,\infty)$, and Lagrange stable if it is global and bounded, i.e., $\sup\limits_{t\in[t_0,\infty)} \|x(t)\| < \infty$. The Lagrange stability of the degenerate DE (1) ensures that each its solution starting at the time moment t_0 exists on $[t_0,\infty)$ and is bounded.A solution of the IVP is called Lagrange unstable if it exists on some finite interval and is unbounded, i.e., it is blow-up in finite time. It is assumed that the operator pencil $\lambda A(t)+B(t)$ is regular for every $t \geq t_+$. The theorems on the Lagrange stability and instability of degenerate Des of the type (1) with stationary operators A, B were obtained in [1] (when $\lambda A+B$ is regular) and [2] (when $\lambda A+B$ is singular). In [3], two combined numerical methods for solving the degenerate DE with the stationary operators are obtained. The talk contains the results of the work partially supported by the NAS of Ukraine (project 0119U102376 “Qualitative, asymptotic and numerical analysis of various classes of differential equations and dynamical systems, their classification, and practical application”).

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THE QUALITATIVE PROPERTY OF SOLUTIONS LINEAR AND NON-LINEAR ELLIPTIC-PARABOLIC EQUATIONS

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Let Ω is a bounded open set in R^n and $Q_T = \Omega \times (0,T)$, $T > 0$ be a cylinder. We consider initial boundary value problem for linear and non-linear elliptic-parabolic equations in Q_T. This problem arise as mathematical models of various applied problems, for instance reaction-drift-diffusion processes of electrically charged species phase transition.

We study qualitative property, solvability these type problems, and some a priori estimates of solutions is obtained.

DEVELOPMENT THE EDUCATIONAL COURSE "ORTHOGONAL TRANSFORMATION METHODS"

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The article presents practical, theoretical and methodological features of the development of a distance course "Methods of orthogonal transformations"

Keywords: orthogonal transformations,students, mathematics, distance learning, matrix.

There are multidimensional quantities in engineering, mathematics, economics, computer science and other sciences, multidimensional quantities. This kind of information is presented in the form of matrices, which leads to the use of matrix algebra.

Orthogonal matrix transformations are used to solve a wide range of applied problems in various fields of science and technology, for example, in linear algebra for calculating inverse matrices and solving linear algebraic equations, in digital signal processing in satellite electronic radio systems, for digital image processing and many others, however, in university courses in linear algebra this topic is either not considered at all, or insufficient attention is paid to it.

However, the courses on the study of linear algebra are presented at many universities, for example, at the Ulyanovsk State Pedagogical University I. N. Ulyanova , course" Linear Algebra "for " Mathematics ", at the Ulyanovsk State University, course" Mathematics for economists "for the " Economics ", at the FSBEI of Higher Education in Kazan (Volga Federal University) course "Mathematical modeling" for the "Applied

Mathematics". Unfortunately, few hours are devoted to the study of the discipline and many hours are spent on independent work (50 percent or more of the total number of course hours). And the study of various types of orthogonal transformations does not pay attention at all.

LMS Moodle was chosen as a tool for designing a course in a distance learning system.

In terms of its capabilities, Moodle can be compared with the well-known commercial LMS, but at the same time it is distributed in open source code, which makes it possible to integrate new modules into it if necessary.

This distance educational course on the topic “Methods of orthogonal transformations” aims to teach students various methods of orthogonal transformations, that are actively used in solving systems of linear algebraic equations. The course is complementary to such sections as “Matrices”, “Linear Algebra”, etc.

It is assumed, that students starting to study this course already have the basic techniques of mathematical analysis, have studied vectors, have knowledge of analytical geometry and linear algebra, as well as basic programming skills in the Matlab software package. The educational course is divided into 3 modules, also it includes 12 lectures, test tasks and practical tasks, during which students consolidate their knowledge during the study of lectures.

The distribution of lectures by hours and hours of independent work to consolidate the received material are shown in table 1.

Table 1. Distribution of hours of lecture classes.

№	Theme	Lessons	
		Lecture	**Practice**
Module 1. Basic concepts and definitions of matrix theory			
1	The concept of a matrix. Matrix Types	1	2
2	Matrix Properties. Matrix rank	1	2
Module 2. Methods of orthogonal transformations			
4	Householder Orthogonalization	2	3
5	Givens Orthogonalization	2	3
6	Gram-Schmidt Orthogonalization	2	4
Module 3. Application of the result of orthogonal transformation			
8	Algorithm for solving systems after orthogonalization	2	3
9	Matrix inversion after orthogonalization	2	3

After module 1, students are given the opportunity to pass a closed test with 10 questions, aimed at consolidating the theory and applying knowledge in solving problems. After module 2, students are given 2 practical tasks for orthogonal transformations and one laboratory work in Matlab. To solve these practical problems, students need to apply not only the knowledge gained in the course, but also basic knowledge in the field of programming in the Matlab software package. After module 3, students do the control test.

We give the examples of the practical task and the laboratory work for students studying in this course.

This practical task follow after module 2 and it are mandatory for students of any specialties. The correct solution to each problem is estimated at 5 points.

The practical task to chapter 1.3:

For matrix

$$\mathrm{A} = \begin{pmatrix} 1 & 3 & 3 \\ -2 & 4 & -1 \\ -2 & 5 & 7 \end{pmatrix}$$

execute:

Using a QR – decomposition of matrix A solve the system of linear equations Ax = b,

where is the vector $\mathrm{b} = (-4{,}13, -9)^{\mathrm{T}}$.

Laboratory work to chapter 2:

Write a program that implements your version of orthogonal transformation (in Matlab). Check the correctness of the calculations "manually". Compare the results. To conclude.

The order of lectures in this educational course is not random; each lecture is based on some knowledge gained in previous topics. That is why all lectures must be studied strictly in the sequence of their following the course. Studying each topic, the student should familiarize himself with the recommendations in the description of each lecture, because all lectures have their own specifics and are designed for different levels of training. If it is difficult to study a topic, the student can resort to studying additional literature from the list given in each module, or the student can contact the teacher in the Moodle system and ask him questions that interest him. The same goes for the test and the practical tasks of the course. When studying each topic, a brief lecture notes should be compiled, which will provide faster assimilation of the material.

THE INVERSE PROBLEM OF NON-STATIONARY FLOW LIQUIDS IN A PIPE WITH A PERMEABLE WALL

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The process of unsteady flow of an incompressible viscous fluid in a horizontal pipe with a permeable wall, length l, diameter d, is considered. The mathematical model of this process is represented as the following system of partial differential equations

$$\frac{\partial u(x,t)}{\partial t}+u(x.t)\frac{\partial u(x,t)}{\partial x}=-\frac{1}{\rho}\frac{\partial p(x,t)}{\partial x}+v\frac{\partial^2 u}{\partial x^2}-\frac{\lambda|u(x,t)|}{2d}u(x,t), \tag{1}$$

$$-\frac{\partial u(x,t)}{\partial x}=\frac{4}{d}q(x,t), \tag{2}$$

where $p(x,t)$ is the fluid pressure in the pipe, $u(x,t)$ is the average velocity over the cross section of the pipe, $q(x,t)$ is the fluid outflow rate through the pipe wall, v is the kinematic viscosity of the fluid, ρ is the fluid density, λ is the hydraulic resistance coefficient.

It is assumed that the outflow of fluid through the permeable wall of the pipe occurs in accordance with Starling's law [1-3], according to which the rate of outflow of fluid is directly proportional to the difference between the fluid pressure inside the pipe and the pressure in the environment

$$q(x,t)=k(t)(p(x,t)-p_e(x,t)), \tag{3}$$

where $p_e(x,t)$ is the pressure of the external environment, $k(t)$ is the permeability coefficient of the pipe wall.

The system of equations (1) - (3) is transformed to the following equation with respect to $u(x,t)$

$$\frac{\partial u(x,t)}{\partial t}+u(x,t)\frac{\partial u(x,t)}{\partial x}=-\frac{1}{\rho}\frac{\partial p_e(x,t)}{\partial x}$$
$$+\frac{d}{4\rho k(t)}\frac{\partial^2 u(x,t)}{\partial x^2}+\nu\frac{\partial^2 u(x,t)}{\partial x^2}-\frac{\lambda\left|u(x,t)\right|}{2d}u(x,t) \quad (4)$$

For equation (4), the initial

$$u(x,0)=\phi(x) \quad (5)$$

and boundary conditions

$$u(0,t)=\theta(t), \quad (6)$$

$$u(l,t)=w(t). \quad (7)$$

In the framework of the obtained model, the inverse problem is posed of restoring the coefficient of permeability of the pipe wall $k(t)$ according to a given additional condition

$$p(l,t)=\gamma(t).$$

This condition by combining equations (2) and (3) is converted into an additional condition for equation (4)

$$-\frac{\partial u(l,t)}{\partial x}=\frac{4}{d}k(t)(\gamma(t)-p_e(l,t)). \quad (8)$$

The problem is to determine a pair of functions {} satisfying equation (4) and conditions (5) - (8). The posed problem (4) - (8) belongs to the class of coefficient inverse problems of mathematical physics [4, 5].

For the numerical solution of the problem (4)–(8), the difference grid is introduced

$$\overline{\omega}=\left\{(t_j,x_i):x_i=i\Delta x,\ t_j=j\Delta t,\ i=0,1,2,...n,\ j=0,1,2,...m\right\}$$

and we construct a discrete analog of the problem on this grid

$$\frac{u_i^j-u_i^{j-1}}{\Delta t}+u_i^{j-1}\frac{u_i^j-u_{i-1}^j}{\Delta x}=\nu\frac{u_{i+1}^j-2u_i^j+u_{i-1}^j}{\Delta x^2}+$$
$$+\frac{1}{k^j}\frac{d}{4\rho}\frac{u_{i+1}^{j-1}-2u_i^{j-1}+u_{i-1}^{j-1}}{\Delta x^2}-\frac{\lambda\left|u_i^{j-1}\right|}{2d}u_i^j-\frac{1}{\rho}\frac{p_{e\,i+1}^{\,j}-p_{e\,i-1}^{\,j}}{2\Delta x}$$

$$i = 1,\ 2,\ \ldots,\ n-1,$$

$$u_i^0 = \phi_i, \qquad i = 0,\ 2,\ \ldots,\ n,$$

$$u_0^j = \theta^j,$$

$$u_n^j = w^j,$$

$$-\frac{u_n^j - u_{n-1}^j}{\Delta x} = \frac{4}{d} k^j (\gamma^j - p_{en}^j),$$

где $u_i^j \approx u(x_i, t_j),\ p_{ei}^j \approx p_e(x_i, t_j), \quad k^j \approx k(t_j), \quad \gamma^j = \gamma(t_j),$ $w^j = w(t_j), \quad \theta^j = \theta(t_j), \quad \phi_i = \phi(x_i)$.

To solve the obtained system of difference equations, a special representation is proposed. The result is an explicit formula for determining the approximate value of the permeability coefficient of the wall at each discrete value of the time variable. On the basis of the proposed computational algorithm, numerical experiments were carried out for model problems.

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SOME RESULTS FOR WEIGHTED COMPOSITION VOLTERRA OPERATORS

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Recall that if $A : X \to X$ is a bounded linear operator on a Banach space X (i.e., $A \in \mathbf{B}(X)$), thenitscommutant is

$$\{A\}' := \{B \in \mathbf{B}(X) : BA = AB\}.$$

Theclosedsubspace $E \subset X$ is a hyper invariant subspace for A, if $BE \subset E$ forall $B \in \{A\}'$. The spectral multiplicity $\mu(A)$ of operator A is definedby

$$\mu(A) := \min\{\dim E : span\{A^n E : n \geq 0\} = X\},$$

where $span\{A^n E : n \geq 0\}$ meansclosure of the linearhull of theset $\{T^n x : x \in E \text{ and } n = 1,2,...\}$. Thesubspace $E \subset X$ witht hisproperty is called as a cyclicsubspace of theoperator A. Thevector $x \in X$ is called cyclic for A, if $span\{A^n x : n = 1,2,...\} = X$. In this case, A is called a cyclicoperator. It is clear that if A has a cyclicvector, then $\mu(A) = 1$. It is well known, for example, that $\mu(V) = 1$ and $\mu(M_x) = 1$, where V, $Vf(x) = \int_0^x f(t)dt$, is the Volterra integration operator on the space $L^p[0,1]$ $(1 \leq p < +\infty)$ and M_x,

$(M_x f)(x) = xf(x),$ is the multiplication operator on $L^p[0,1].$

It is necessary to remark that the concepts spectral multiplicity and cyclic vector are an important characteristics of operators. For instance, the notion of cyclic vector is important in connection with the general problem of existence of a non trivial in variant subspace, since an operator $A \in \mathbf{B}(X)$ has non on trivial invariant subspace if and only if everynon zero vector $x \in X$ is a cyclic vector for $A.$ Cyclic vectors are also useful in weighted polynomial approximation theory.

Recall also that the operator equation

$$AX = XB \qquad (1)$$

naturally arises in numerous issues of spectral theory of operators, representation theory, stability theory (Lyapunov's equation), etc. For example, if the set of solutions of equation (1) contains a boundedly invertible operator Y, then A and B are similar, $B = Y^{-1}AY$, and hence have many common spectral properties. If $B = \lambda A,\ \lambda \in \mathbb{C},$ following [1], one refers to λ as an extended eigenvalue of $A,$ and each bounded non zero solution of the equation $AX = \lambda XA,$ i.e., equation (1) with $B = \lambda A,$ is called an extended eigenvector of $A.$

Here we consider the weighted composition Volterra operators $V_{n,m}$ and $W_{n,m}$ defined on $L^p[0,1]$ by the formulas

$$V_{n,m} f(x) := x^n \int_0^{x^m} f(t)dt$$

and

$$W_{n,m} f(x) := \frac{1}{x^n} \int_0^{x^m} f(t)dt$$

for $n, m \geq 1,$ and study cyclic vectors, hyper invariant subspaces and extended eigenvalues of these operators. Their double integration analogy also will be considered.

Some other related questionsal so will be discussed.

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CONVERGENCE OF SPECTRAL EXPANSION IN ROOT-FUNCTIONS OF SCHRODINGER OPERATOR WITH MATRIX POTENTIAL

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We consider the Schrodinger operator

$$L\psi=-\frac{d^2\psi}{dx^2}+U(x)\psi\ ,\ x\in G=(0,1)$$

with matrix potential $U(x)$ all the elements $u_{ij}(x),\ i,j=1,\ 2,\dots,m$, of which are complex-valued functions summable on the interval $(0,1)$.

It is required that for the systems $\{\psi_k(x)\}_{k=1}^{\infty}$ and $\{\varphi_k(x)\}_{k=1}^{\infty}$ conditions A (V.A. Il'in conditions) be satisfied:

1) For each number $k=1,2.....$ the vector-functions $\psi_k(x)$ and $\varphi_k(x)$ possess components continuous together with the first derivatives on the segment $\overline{G}=[0,1]$, and for some complex number λ_k satisfy almost everywhere in $\overline{G}$ the equations $L\psi_k=\lambda_k\psi_k+\theta_k\psi_{k-1}$, $L^*\varphi_k=\overline{\lambda_k}\varphi_k+\theta_{k+1}\varphi_{k+1}$,where $L^*=-I\frac{d^2}{dx^2}+U^*$, U^* is a matrix adjoint to the matrix U , I is a unit matrix; θ_k is

a number equal to zero or a unit (in the last case the equality should be fulfilled $\lambda_k = \lambda_{k-1}$), moreover $\theta_1 = 0$;

2) there exist the constants C_0 and C_1 such that $|\operatorname{Im}\mu_k| \le C_0$, $k = 1, 2, ...;$ $\sum_{\tau \le \rho_k \le \tau+1} 1 \le C_1, \ \forall \tau \ge 0$ in which $\mu_k = \sqrt{\lambda_k}$, $\rho_k = \operatorname{Re}\mu_k \ge 0$;

3) even if one of two systems of vector-functions $\{\psi_k(x)\}_{k=1}^{\infty}$ and $\{\varphi_k(x)\}_{k=1}^{\infty}$ be complete $L_2^m(G)$;

4) there exist the constant C_2 such that for all numbers of k the inequality $\|\psi_k\|_{2,m} \|\varphi_k\|_{2,m} \le C_2$ be valid.

Theorem. *Let the biorthonormed pair* $\{\psi_k, \varphi_k\}_{k=1}^{\infty}$ *satisfy conditions* A, *the vector-function* $f(x) \in W_{1,m}^1(G)$ *satisfy the condition*

$$\left| \langle f, \varphi_k' \rangle \Big|_0^1 \right| \le C_3(f) \|\varphi_k\|_{2,m}, \ k = 1, \ 2, ...$$

and the number series $\sum_{k=2}^{\infty} k^{-1} \omega_{1,m}(f', k^{-1})$ converge.

Then the biorthogonal expansion of the vector-function $f(x)$ *in the system* $\{\psi_k(x)\}_{k=1}^{\infty}$ *converges absolutely and uniformly on the segment* $\overline{G} = [0,1]$.

ON THE NUMERICAL SOLUTION OF A SYSTEM OF LINEAR DIFFERENTIAL EQUATIONS WITH INTERVAL COEFFICIENTS

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We employ an approach that treats an interval problem as a set of real (classical) problems. In previous studies, we have investigated a system of linear differential equations with real coefficients, but with interval forcing terms and interval initial values. We have shown that the value of the solution at each time moment forms a convex polygon in the coordinate plane. The motivating question of the present study is to investigate if the same statement remains valid or not, when the coefficients are intervals. Numerical experiments show that the answer is negative. Namely, at a fixed time, the solution's value does not necessarily form a polygon. Moreover, it can form a non-convex region.

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DECOMPOSITION OF ONE LINEAR PROGRAMMING PROBLEM WITH COMPLEX STRUCTURES

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The thesis is devoted to the problem of linear programming with a special structure, where the main matrix is in a block-ladder form .

Mathematically, the problem statement is as follows:

$$(c, x_j) \to \max , \tag{1}$$

$$A_1 x_1 \le b, \qquad (2)$$

$$A_j x_j \le x_{j-1}, \ j \in [2; J], \qquad (3)$$

$$x_j \ge 0, \ j \in [1; J], \qquad (4)$$

The dimensions of the vector x_j and matrix A_j are consistent, b is vector with dimensions m_j, $j \in [1; J]$, χ and c are vectors with dimensoin $j \in [1; J]$ .An algorithm for solving the problem is created and a universal batch program is developed. At the same time, Danchik-Wolfe decomposition method was also used to solve the problem.

The algorithm was used to solve a practical problem, and a positive result was achieved.

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LOWEST-ENERGY CONTROL OF THE PARABOLIC SYSTEM UNDER POINT IMPACTS

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We consider a controlled process described by the function $u(t, x)$, which satisfies the equation

$$\frac{\partial u}{\partial t} = u^2 \frac{\partial^2 u}{\partial x^2} + f(t,x) + \sum_{i=1}^{m} \delta(t-t_i) p_i(x), \quad (1)$$

inside the domain $\overline{Q} = [0 \le x \le 1, 0 \le t \le T]$, and satisfies the initial and boundary conditions

$$u(0,x) = u^0(x), \quad (2)$$

$$\frac{\partial u(t,0)}{\partial x} = 0, \quad \frac{\partial u(t,1)}{\partial x} + \alpha u(t,1) = 0, \quad (3)$$

at the boundary of $\overline{Q}$, where $u^0(x) \in L_2(0,1)$ and $f(t,x) \in L_2(Q)$ are given functions, $0 < t_1 < t_2 < ... < t_m < T$ are fixed points in time, $\delta(t)$ is the Dirac's function, $p_i(x), i = \overline{1,m}$ are control function.

The class of admissible controls are all vector-functions $\overline{p}(t) = (p_1(x), p_2(x), ..., p_m(x))$ from $L_2^m(0,1)$.

Each concrete admissible control defines a unique generalized solution to the boundary-value problem (1)–(3) [1].

Let $\varphi(x) \in L_2(0,1)$ be a given function. The considered optimal control problem consists in finding a solution $u(t,x)$ of the problem (1)–(3) satisfying the condition

$$u(T,x) = \varphi(x) \quad (4)$$

and giving the smallest possible value to the functional

$$I[\overline{p}] = \|\overline{p}\|^2_{L_2^m(0,1)} = \int_0^1 \sum_{i=1}^{m} p_i^2(x) dx. \quad (5)$$

Using the Fourier method, the stated problem is formulated in an infinite-dimensional phase space.Applying the eigenvalue expansion of the problem

$$X''(x) + \lambda^2 X(x) = 0, \ 0 < x < 1, \ X'(0) = 0, \ X'(1) + \alpha X(1) = 0 \quad (6)$$

we obtain the following system of ordinary differential equations for the Fourier coefficients $u_n(t)$:

$$\dot{u}_n(t) = -a^2\lambda_n^2 u_n(t) + \sum_{i=1}^{m} \delta(t-t_i) p_{im} + f_n(t), \quad n = 1,2,... \quad (7)$$

with initial conditions

$$u_n(0) = u_n^0, \quad (8)$$

where $f_n(t)$ and u_n^0 are Fourier coefficients of the functions $f(t,x)$ and $u^0(x)$, respectively.

Then the functional (5) takes the form

$$I[\bar{p}] = \sum_{n=1}^{\infty} I_n[\bar{p}_n], \quad I_n[\bar{p}_n] = \sum_{i=1}^{m} p_{in}^2 . \quad (9)$$

Thus, the problem is reduced to the determination of the numerical parameters $\bar{p}_n = (p_{1n}, p_{2n}, ..., p_{mn}) \in E^m$ such that the corresponding solutions to the problem (7)–(8) would satisfy the conditions

$$u_n(T) = \varphi_n, \quad (10)$$

and at the same time for each fixed n, the functional $I_n[\bar{p}_n]$ would take its smallest possible value, where φ_n are Fourier coefficients of the function $\varphi(x)$.

Since, according to the Cauchy formula, the solution to problem (9)–(10) can be represented as

$$u_n(t) = u^0 e^{-a^2\lambda_n^2 t} + \int_0^t \left[f_n(\tau) + \sum_{i=1}^{m} \delta(\tau - t_i) p_{im} \right] e^{-a^2\lambda_n^2 (t-\tau)} d\tau ,$$

thenthe condition (10) can be written as

$$\sum_{i=1}^{m} p_{in} e^{-a^2\lambda_n^2 (T-t_i)} = \psi_n,$$

$$\psi_n = \varphi_n - u_n^0 e^{-a^2\lambda_n^2 T} - \int_0^T f_n(t) e^{-a^2\lambda_n^2 (T-t)} dt, \quad n = 1,2... \quad , (11)$$

or, designating by $\bar{e}_n = \left(e^{-a^2\lambda_n^2(T-t_1)}, \ldots, e^{-a^2\lambda_n^2(T-t_m)} \right)$ moment ratios in the form

$$\left(\bar{p}_n, \bar{e}_n\right) = \psi_n . \qquad (12)$$

Therefore, one hast of in davector $\bar{p}_n$ of minimum length, satisfying the condition (12).

Reasoning by the statement of Leva's theorem [2], we establish that if the considered optimal control problem has a solution, then it can be represented as

$$\bar{p}_n = \alpha_n \bar{e}_n \text{ или } p_{in} = \alpha_n e^{-a^2\lambda_n^2(T-t_1)}, \quad i = \overline{1,m}$$

where

$$\alpha_n = \frac{\psi_n}{(\bar{e}_n, \bar{e}_n)} = \frac{\psi_n}{\sum_{k=1}^{m} e^{-2a^2\lambda_n^2(T-t_k)}}, \quad n = 1,2,\ldots.$$

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ON THE EXISTENCE RESULTS FOR THE DOMAIN FUNCTIONALS OF THE EIGENVALUES OF THE PAULI OPERATOR

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Since the eigenvalues of the different (usually elliptic) operators describe various physical or mechanical parameters of the corresponding natural processes investigation of the problems involving eigenvalues presents a huge interest both from practical and theoretical points of view. It is well known that eigenvalues of the Schrodinger operator describe full energy of the quantum particle, biquadratic operator with corresponding boundary conditions - eigenfrequency of the free, clamped and pressed plates. There exists a lot of practical problems that require solution of the minimization problems with respect to the eigenvalues. Usually in such problems minimization procedure is carried out over the parameters that describe the materials of the plates, environment, boundary conditions etc. Indeed these parameters are described by functions. But there exist a large class of problems where minimization with respect to the geometrical characteristics (for example form of the plate) is required. These problems are called shape optimization problems. In the last three decades these problems have been intensively studied and a number of methods have been offered by different authors for their solution in various aspects. But till now the general method for solution of the different shape optimization problem is absent [2-4].

In this work we get existence result for the shape optimization problems for the eigenvalues of the Pauli operator.

Consider the following eigenvalue problem

$$P\varphi = \lambda\varphi\ , \qquad x \in D\ , \tag{1}$$

$$\varphi = 0, \qquad x \in S_D\ . \tag{2}$$

where P is the Pauli operator defined by the expression

$$P = P(a,v)\cdot J + \sigma B\ . \tag{3}$$

Here

$$J = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad P = (a,v) = (-i\nabla - a)^2 + V$$

; i is an imaginary unit; V is a smooth enough function; $\nabla = \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right\}$; $a = (a_1, a_2) \in R^2$ is a vector potential; B magnet field generated by the vector potential a

$$B = \frac{\partial}{\partial x} a_2 - \frac{\partial}{\partial y} a_1\ .$$

If to consider all these definitions one can write two dimensional Pauli in the form

$$P = \begin{pmatrix} (-i\nabla - a)^2 + a_2\frac{\partial}{\partial x} - a_1\frac{\partial}{\partial y} + V & 0 \\ 0 & (-i\nabla - a)^2 - a_2\frac{\partial}{\partial x} + a_1\frac{\partial}{\partial y} + V \end{pmatrix} =$$

$$= \begin{pmatrix} -\Delta + (2ia_1 + a_2)\frac{\partial}{\partial x} + (2ia_2 - a_1)\frac{\partial}{\partial y} + a^2 + V & 0 \\ 0 & -\Delta + (2ia_1 - a_2)\frac{\partial}{\partial x} + (2ia_2 + a_1)\frac{\partial}{\partial y} + a^2 + V \end{pmatrix}. \tag{4}$$

By K we denote the set of all bounded, convex domains Ω with smooth boundary Consider the problem

$$\min\left\{F(\lambda(\Omega)) : \Omega \in K, |\Omega| \le m\right\}. \tag{5}$$

Here $\lambda(\Omega)$ is the Dirichlet eigenvalues of operator (4) in the domain $\Omega \in K$, m is a given number.

The following theorem is proved for this problem.

Theorem. *Let* $F : R^n \to [0, +\infty]$ *be a semicontinuous and monotone function. Then optimization problem* (5) *has a solution on the set* K .

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APPLICATION OF FINITE INTEGRAL TRANSFORMATION METHOD TO THE SOLUTION OF MIXED PROBLEMS FOR PARABOLIC EQUATIONS WITH REVERSE FLOW OF TIME

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For simplicity of notation and reasoninshere we will consider the following model problem.

Problem statement.Find the classic solution of the equation

$$\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2},\ 0<x<1,\quad 0<t<T, \tag{1}$$

under boundary conditions

$$u(0,t)=u(1,t)=0 \text{ as } 0<t<T, \tag{2}$$

and satisfying the finite condition

$$\alpha\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=T}+\beta u(x,t)\big|_{t=T}=\psi(x),\quad 0<x<1, \tag{3}$$

where $a,\alpha,\beta,T-$ are the known numbers $\psi(x)-$ is a given function $u=u(x,t)-$ is the desired classic solution

Solution .

1^0.Let $a>0$ and $T>0.$

2^0. Let $\beta\neq\alpha k^2\pi^2$ for $k=1,2....$

3^0.Let

$$\int_0^1\psi(\xi)\sin k\pi\xi d\xi=\frac{\beta-\alpha\pi^2k^2}{e^{k^2\pi^2T}}\cdot\frac{\gamma_k}{k^2},\ k=1,2,...,$$

where γ_k are some numbers satisfying the inequality

$|\gamma_k|\leq const,$ for $k=1,2,.....$

Let problem (1)-(3) have a classic solution $u = u(x,t) \in C([0,1]\times[0,T])$. Assume

$$u(x,t)\big|_{t=0} = f(x), 0 < x < 1, \tag{4}$$

where $f(x)$ is an unknown function to be determined. At first, a priori assuming $f(x)$ to be known, applying the finite integral transformation [2]

$$K\varphi \equiv \int_0^t e^{-\lambda^2\tau}\varphi(\tau)d\tau, \tag{5}$$

to problem (1), (2), (4), we prove the following

Theorem 1. *Under constraints 1^0, if problem has a classic solution, then i) it is unique , ii) this solution is represented by the formula*

$$u(x,t) = \sum_{k=1}^{\infty} 2\sin k\pi x e^{-k^2\pi^2 t}\int_0^1 f(\xi)\sin k\pi\xi d\xi,\ 0 < x < 1,\ 0 < t \le T. \tag{6}$$

From (6) we have

$$\frac{\partial u(x,t)}{\partial t} = -\sum_{k=1}^{\infty} 2k^2\pi^2 \sin k\pi x e^{-k^2\pi^2 t}\int_0^1 f(\xi)\sin k\pi\xi d\xi. \tag{7}$$

Having substituted (6) and (7) in (3) we get

$$\sum_{k=1}^{\infty} 2(\beta - \alpha k^2\pi^2)\sin k\pi x e^{-k^2\pi^2 t}\int_0^1 f(\xi)\sin k\pi\xi d\xi = \psi(x),\ 0 < x < 1. \tag{8}$$

Expanding $\psi(x)$ in Fourier series [1], we have

$$\psi(x) = \sum_{k=1}^{\infty} 2\sin k\pi x\int_0^1 \psi(\xi)\sin k\pi\xi d\xi. \tag{9}$$

Substituting (9) in (8), we obtain

$$(\beta - \alpha k^2\pi^2)e^{-k^2\pi^2 T}\int_0^1 f(\xi)\sin k\pi\xi d\xi = \int_0^1 \psi(\xi)\sin k\pi\xi d\xi, k = 1,2....$$

Hence

$$\int_0^1 f(\xi)\sin k\pi\xi d\xi = \frac{e^{k^2\pi^2 T}}{\beta-\alpha\pi^2 k^2}\int_0^1 \psi(\xi)\sin k\pi\xi d\xi, k=1,2\ldots. \quad (10)$$

Thus, we established the following

Theorem 2. *Under constraints 1^0, 2^0 and 3^0 problem (1)-(3) has a unique classic solution and this solution is represented by the formula (6), where the desived function* $f(x)$ is represented by the formula

$$f(x) = \sum_{k=1}^{\infty} 2\sin k\pi x \int_0^1 f(\xi)\sin k\pi\xi d\xi,$$

the coefficients $\int_0^1 f(\xi)\sin k\pi\xi d\xi,$ are determined by formula (10).

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PROBLEMS OF APPROXIMATION IN LIZORKIN-TRIEBEL SPACES WITH DOMINANT MIXED DERIVATIVES

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In this abstract we study the approximation problems in the Lizorkin-Triebel $S^l_{p,\theta}F(G_\varphi)$ spaces with dominant mixed derivatives.

Definition. Denote by $S^l_{p,\theta}F(G_\varphi)$ the space of locally summable functions f on G having the generalized derivatives $D^{k^e} f$ with the finite norm

$$\|f\|_{S^l_{p,\theta}F(G_\varphi)} = \sum_{e\subseteq e_n} \left\| \left\{ \int_{0^e}^{h_0^e} \left[\frac{\Delta^{m^e}(\varphi(h), G_\varphi) D^{k^e} f(\cdot)}{\prod_{j\in e} [\varphi_j(h_j)]^{(l_j-k_j)}} \right]^\theta \prod_{j\in e} \frac{dh_j}{h_j} \right\}^{\frac{1}{\theta}} \right\|_p ,$$

where $G \subset R^n$, $1 < p \le \infty, 1 \le \theta \le \infty$, $l = (l_1, l_2, ..., l_n)$, $l_j \in (0, \infty)$ $(j \in e_n = \{1,2,...,n\})$, and let $l^e = (l_1^e, l_2^e, ..., l_n^e)$, $l_j^e = l_j$ $(j \in e \subseteq e_n)$, $l_j^e = 0$ $(j \in e_n \setminus e)$ and $\varphi(t) = (\varphi_1(t_1), ..., \varphi_n(t_n))$, $\varphi_j(t_j) > 0$, $(t_j > 0)$ is continuously differentiable functions; $\lim_{t\to+0} \varphi_j(t_j) = 0$, $\lim_{t\to+\infty} \varphi_j(t_j) = P_j \le \infty$;

$$G_{\varphi(t)}(x) = G \cap I_{\varphi(t)}(x) = G \cap \left\{ y : |y_j - x_j| < \frac{1}{2}\varphi_j(t_j), (j = 1,2,...,n) \right\}$$

The following theorem is proved.

Theorem. Let $1 < p < \infty$ and $f \in S^l_{p,\theta}F(G_\varphi)$. Then there exist the functions $h_s = h_s(x)$ (s=1, 2,…) infinitely differentiable finite in R^n such that

$$\lim_{s\to\infty} \|f - h_s\|_{S^l_{p,\theta}F(G_\varphi)} = 0.$$

ON BASICITY OF EIGENFUNCTIONS OF ONE DISCONTINUOUS SPECTRAL PROBLEM IN WEIGHTED LEBESGUE SPACES

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We consider the spectral problem for the discontinuous second order differential operator

$$l(y) = p_0(x)y'' + p_1(x)y' + p_2(x)y = \lambda y, \quad x \in \bigcup_{s=1}^{l}(x_{s-1}, x_s), \tag{1}$$

with the boundary conditions

$$\sum_{s=1}^{l}\sum_{j=0}^{k_\nu}\left(\alpha_{\nu sj}y^{(j)}(x_{s-1}+0) + \beta_{\nu sj}y^{(j)}(x_s - 0)\right) = 0, \; \nu = \overline{1,4}, \tag{2}$$

$$0 \le k_\nu \le 1, \qquad -\infty < a = x_0 < x_1 < x_2 < \ldots < x_l = b < +\infty,$$

$p_1(x),\ p_2(x) \in L_1(a,b)$, and the function $p_0(x)$ on each interval (x_{s-1}, x_s) has the form: $p_0(x) = p_{0s}(x)e^{i\varphi_s}$, $0 \le \varphi_s < 2\pi,\ s = \overline{1,l},$, where $p_{0s}(x) -$ is a positive absolutely continuous function on (x_{s-1}, x_s).

In the present work a definition of the regularity andstrongly regularity of boundary conditions is given, the asymptotic formulas for eigenvalues are obtained, a resolvent is constructed, the basis properties of eigen and associated functions arestudied in weighted Lebesgue spaces.

Let $\rho(x)=\prod_{s=1}^{l}|x-x_s|^{\alpha_s}$, be a weight function that satisfies the condition $-1/p<\alpha_k<1/q,\ k=\overline{1,m}$, where $1/p+1/q=1$. Let $L_{p,\rho}(a,b)$ be a weighted Lebesgue space with norm $\|f\|_{p,\rho}=\|f\rho\|_{L_p}$ and consider the system $\{y_n\}_{n\in N}$ in $L_{p,\rho}(a,b)$, where $y_n(x)$ are the eigenfunctions of the problem (1),(2). It is hold the following

Theorem. *The system $\{y_n\}_{n\in N}$ of eigen and associated functions of the regular problem (1), (2) form a basis with parentheses in the space $L_{p,\rho}(a,b)$, 1 <p<∞, and form a usual basis in this space if the boundary conditions are stronglyregular.*

Similar problems for discontinuous differential operators were studied in [1-3].

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BASIS PROPERTY OF THE SYSTEM OF EIGENFUNCTIONS CORRESPONDING TO A PROBLEM WITH A SPECTRAL PARAMETER IN THE BOUNDARY CONDITION

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Consider the following spectral problem

$$-y'' + q(x)y = \lambda y, \quad x \in (0,1), \tag{1}$$

$$\left.\begin{array}{l} y(0) = 0, \\ y'(0) = (a\lambda + b)y(1), \end{array}\right\} \tag{2}$$

where λ is a spectral parameter, $q(x) \in L_1(0,1)$ complex-valued function, a and b are arbitrary nonzero complex numbers. In this work the basis properties of the eigen and associated functionsof problem (1),(2) in spaces $L_p(0,1) \oplus C$ and $L_p(0,1), 1 < p < \infty$, are studied. Problem (1),(2) has a countable number of eigenvalues $\{\lambda_n\}_{n=o}^{\infty}$ that are asymptotically simple and have asymptotics

$$\lambda_n = \left(\pi n + O\left(\frac{1}{n}\right)\right)^2, n = 0,1,2,\dots .$$

Let the system $\{u_n(x)\}_{n=o}^{\infty}$ be the eigen and associated functions of problem (1),(2). Let us define an operator L in the space $L_p(0,1) \oplus C$ as follows

$$D(L)=\{\hat{u}=(u(x);au(1)),u(x)\in W_p^2(0,1),u(0)=0\},$$
$$L\hat{u}=(-u''(x);u'(0)-bu(1)) \quad for \quad \hat{u}\in D(L).$$

The eigenvalues of the operator L are

$$\hat{u}_n=(u_n(x),au_n(1)),\, n=0,1,2,\dots .$$

The following theorems are proved.

Theorem 1.*System* $\{\hat{u}_n\}_{n=o}^{\infty}$ *of eigen and associated vectors of the operator* L *forms a basis in the space* $L_p(0,1)\oplus C$, $1<p<\infty$, *equivalent to the system* $\{\hat{e}_n\}_{n=o}^{\infty}$, *where* $\hat{e}_0=(0,1),\ \hat{e}_n=(\sin \pi nx;0).$

Theorem 2.*Let* λ_{n_0} *be an arbitrary simple eigenvalue of problem (1),(2). Then the system* $\{u_n(x)\}_{n=o}^{\infty}$ $,\lambda_{n_0}, n\neq n_0$, *forms a basis in* $L_p(0,1), 1<p<\infty$, *equivalent to the system* $\{\sin \pi nx\}_{n=1}^{\infty}$.

Note that some special cases of problem (1),(2) analogous problems were studied in [1-3].

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ON NEW FAMILIES OF THE TETRANACCI IDENTITIES

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The tetranacci numbers are a more general version of Fibonacci numbers and start with four predetermined terms, each term afterwards being the sum of the preceding four terms.

The tetranacci sequence $\{t_n\}_{n\geq 0}$ is the sequence of integersdefined by the initial values $t_0 = t_1 = t_2 = 0$, $t_3 = 1$ and the recurrence

$$t_n = t_{n-1} + t_{n-2} + t_{n-3} + t_{n-4}, \qquad n \geq 4.$$

For Toeplitz-Hessenberg determinants entries of which are tetranacci numbers we use Trudi's formula (see [1–4] for more details) to obtain new identities involving sums of products of tetranacci numbers and multinomial coefficients.

Let F_n and T_n be the Fibonacci and tribonacci numbers, defined by recurrences

$$F_0 = 0, \quad F_1 = 1, F_n = F_{n-1} + F_{n-2}, \quad n \geq 2,$$

$$T_0 = T_1 = 0, \quad T_2 = 1, \qquad T_n = T_{n-1} + T_{n-2} + T_{n-3}, \quad n \geq 3,$$

respectively.

Let us denote $\sigma_n = s_1 + 2s_2 + \ldots + ns_n$, $|s| = s_1 + s_2 + \ldots + s_n$, and $p_n(s) = \dfrac{(s_1 + s_2 + \ldots + s_n)!}{s_1! s_2! \ldots s_n!}$.

Theorem. *The following formulas hold*

$$\sum_{\sigma_n = n} (-1)^{|s|+1} p_n(s) t_0^{s_1} t_1^{s_2} \ldots t_{n-1}^{s_n} = T_{n-2}, \qquad n \geq 2,$$

$$\sum_{\sigma_n=n}(-1)^{|s|}p_n(s)t_2^{s_1}t_3^{s_2}\dots t_{n+1}^{s_n}=\sum_{i=1}^{\lfloor n/2\rfloor}(-1)^iF_{n-2i+1},\qquad n\ge 1,$$

$$\sum_{\sigma_n=n}(-1)^{|s|+1}p_n(s)t_5^{s_1}t_7^{s_2}\dots t_{2n+3}^{s_n}=F_{n+2},\qquad n\ge 3,$$

$$\sum_{\sigma_n=n}(-1)^{|s|}p_n(s)t_4^{s_1}t_5^{s_2}\dots t_{n+3}^{s_n}=0,\qquad n\ge 5,$$

$$\sum_{\sigma_n=n}p_n(s)t_5^{s_1}t_6^{s_2}\dots t_{n+4}^{s_n}=\frac{(-1)^n+(-1)^{\lfloor 3n/2\rfloor}}{2},\qquad n\ge 2,$$

$$\sum_{\sigma_n=n}p_n(s)t_0^{s_1}t_1^{s_2}\dots t_{n-1}^{s_n}=\frac{2+(-1)^n}{(-1)^{\lfloor n/2\rfloor}10}+\frac{5(-1)^n+2^n}{30},\qquad n\ge 1,$$

$$\sum_{\sigma_n=n}p_n(s)t_1^{s_1}t_2^{s_2}\dots t_n^{s_n}=\sum_{i=0}^{\lfloor (n-3)/2\rfloor}\sum_{j=0}^{n-2i-3}\binom{n-3-i-j}{i}\binom{2i}{j},\qquad n\ge 1,$$

$$\sum_{\sigma_n=n}(-1)^{|s|+1}p_n(s)t_3^{s_1}t_4^{s_2}\dots t_{n+2}^{s_n}$$

$$=\sum_{i=0}^{\lfloor (n-1)/2\rfloor}\sum_{j=0}^{\lfloor (n-1)/2\rfloor}\binom{j}{2i+3j-n+1}\binom{2i+3j-n+1}{i},\qquad n\ge 1,$$

where the summation is over nonnegative integers s_i *satisfying Diophantine equation* $s_1+2s_2+\dots+ns_n=n$.

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THE SUFFICIENT CONDITION FOR THE REGULARITY OF BOUNDARY POINT WITH RESPECT TO THE DIRICHLET PROBLEM FOR THE PARABOLIC EQUATIONS SECOND ORDER

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Consider the Dirichlet problem

$$\sum_{i,k=1}^{n} a_{i,k}(t,x)u_{x_i x_k} - u_t(t,x) = 0, \quad u|_{\partial_p D} = f(t,x)$$

where $a_{i,k}(t,x) = a_{k,i}(t,x) \in H_\alpha(D)$, $f \in C(\partial_p D)$.

in the $D \subset R^{n+1}$ bounded domain for the parabolic equation.
Let $(t,x) = (t, x_1, \dots, x_n) \in R^{n+1}$ and $s > 0$, $\beta > 0$ are the fixed positive numbers. Denote the function type of Weierstrass kernel in R^{n+1} as

$$K_{s,\beta}(t,x) = \begin{cases} t^{-s} exp\left\{-\dfrac{|x|^2}{4\beta t}\right\}, t > 0 \\ 0, t \le 0 \end{cases}$$

and for $\lambda > 1$ and $m \in N = N \cup \{0\}$ the paraboloids

$$P_m = \{(t,x) \mid |x|^2 < -\lambda^m \cdot t, t < 0\}.$$

Let
$B_{m,k} = (\overline{P_{m+1} \backslash P_m} \cap \left[-t_k; -\frac{3}{4}t_k\right]$ for $m, k \in N$, and $B_{0,k} = \overline{P_{0,k}} \cap \left[-t_k; -\frac{3}{4}t_k\right]. t_1 > 0.$

Define the following cylinders $C_{m,k}$ with two parameters m, k

$$C_{m,k} = \{(t,x) \mid -t_k < t < 0,\ |x| < a\rho_{m,k}\},$$

where $a > 0$ - positive real number and $\rho_{m,k}^2 = \lambda^m \cdot t_k$ and denote by $S_{m,k}$ the lateral surface of the cylinder $C_{m,k}$.

Let 's call

$$T_{m,k} = C_{m,k} \backslash P_m$$

trapezoids of the compliment of paraboloids

$$P_m = \{(t,x) \mid |x|^2 < -\lambda^m \cdot t\}$$

with respect to the cylinders $C_{m,k}$ and denote by $T_{m,k}^{(j)}$, $j \in \{1,2,\dots,n_0(n)\}$, the minimal finite partition of $T_{m,k}$ such that for which the following inequality

$$|x - \xi| \le |\xi|$$

is fulfilled at $(t,x) \in T_{m,k}^{(j)}$ and $(\tau,\xi) \epsilon T_{m,k}^{(j)}$.

Lemma. There exist absolute constants $C_1 > 0$ and $C_2 > 0$, depending only on fixed numbers λ, a, s, β such that holds

$$\sup_{S_{m,k}} P_{B_{m,k}^{(j)}}(t.x) \le C_1 P_{B_{m,k}^{(j)}}(0.0),$$

and also for the finite partition and for all fixed $j \in \{1,\dots,n_0(n)\}$ such that

$$\inf_{T_{m,k+1}^{(j)}} P_{B_{m,k}^{(j)}}(t,x) \ge C_2\, P_{B_{m,k}^{(j)}}(0,0) .$$

moreover $C_2 > C_1$, where

$$P_B(t,x) = \int_B K_{s,\beta}(t-\tau, x-\xi)\mu(\tau,\xi)$$

the parabolic potential, with the generated kernel of $K_{s,\beta}(t,x)$, B is a Borel set and $\mu$is a Borel measure.

Theorem. In order to the boundary point to be regular with respect to the Dirichlet problem sufficiency is that

$\sum_{m,k=1}^{\infty} P_{\boldsymbol{B_{m,k}}}(0,0) = \infty.$

OPTIMIZATION OF ZONAL FEEDBACK PARAMETERS WHEN CONTROLLING THE ROD HEATING PROCESS

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Consider the problem of controlling rod heating process by means of point sources:

$$u_t = u_{xx} + f(x,t) + \sum_{i=1}^{l_{\mathrm{cont}}} v_i(t)\delta(x-\xi_i)$$
$$(x,t) \in (0,L) \times (t_0,t_f) \qquad (1)$$

whose locations are, in the general case, optimized.

Suppose that at the points $\eta = (\eta_1, \eta_2, \dots, \eta_{l_{\mathrm{obs}}})$, the coordinates of whichare, in the general case, also optimized), we take temperature measurements:

$$u(\eta,t) = [u(\eta_1,t); u(\eta_2,t); \dots; u(\eta_{l_{\mathrm{obs}}},t)] \qquad (2)$$

Let the range of temperature values of the rod under all possible control actionsbe given:

$$\underline{u} \le u(x,t) \le \overline{u} \qquad (3)$$

We divide this range into zones with the given values u^0, u^1,…, u^m:

$$[u^s, u^{s+1}], \qquad s = 0,1,2,\dots,m-1, \qquad u^0 = \underline{u}, \qquad u^m = \overline{u}$$

Moreover, the partitions are not necessarily uniform.

We introduce time intervals depending on the belonging of the current values of the state $u(x,t)$ at the measurement points η , i.e. $u(\eta,t)$, tothe corresponding zone.It is obvious that

$$(t_0,t_f)=\bigcup T^{\nu}=\bigcup_{\nu_1=0}^{m}\bigcup_{\nu_2=0}^{m}\dots\bigcup_{\nu_{l_{\text{obs}}}=0}^{m}T^{\nu_1\nu_2,\dots,\nu_{l_{\text{obs}}}}$$

where $\nu=(\nu_1,\nu_2,\dots,\nu_{l_{\text{obs}}})$ is a multi-index; i.e.for $t\in T^{\nu_1\nu_2,\dots,\nu_{l_{\text{obs}}}}\equiv T^{\nu}$ there takes place:

$u(\eta,t)\in U^{\nu_1,\nu_2,\dots,\nu_{l_{\text{obs}}}}\equiv U^{\nu}$

note that some T^{ν} can be empty sets, since the states in the corresponding zones may never be. In addition, the intervals T^{ν} can be multilinked, i.e. the process can get into any zone more than once, for example, r_{ν} times. In this case $T^{\nu}=\bigcup_{k=1}^{r_{\nu}}T_k^{\nu}$.

The values of the control sources are assigned depending on the temperature values at the measured points in the form of linear feedback:

$$v_i(t)=\sum_{j=1}^{l_{\text{obs}}}K_{ij}^{\nu}\left[u(\eta_j,t)-z_{ij}^{\nu}\right] \tag{4}$$

with optimizable capacities $z_{ij}^{\nu}=\text{const}$ and $K_{ij}^{\nu}=\text{const}$ for $u(\eta,t)\in U^{\nu}$.

Substituting (4) in (1), we obtain the equation:

$$u_t=u_{xx}+f(x,t)+\sum_{i=1}^{l_{\text{cont}}}\delta(x-\xi_i)\sum_{j=1}^{l_{\text{obs}}}K_{ij}^{\nu}\left[u(\eta_j,t)-z_{ij}^{\nu}\right] \tag{5}$$

where $x\in(0,L)$ and $t\in T^{\nu}$.

Equation (5) is a loaded equation.The constant synthesized coefficients $(z_{ij}^{\nu};K_{ij}^{\nu})$change their values when the state at the measurement points passes from one zone to another.Under these passages, the solution $u(x,t)$ is continuous in the time variable t .

The number of optimized parameters is, in the general case, equal to$(l_{\mathbf{cont}}+l_{\mathbf{obs}}+2l_{\mathbf{obs}}^{m})$. Forexample, for $l_{\mathbf{cont}}=l_{\mathbf{obs}}=\mathbf{2}$ and$m=\mathbf{5}$, thedimensionis68, which can be considered quite acceptable for control synthesis problems with distributed parameters.

Constraints imposed on controls, i.e.

$$\underline{v_i}\le v_i(t)\le\overline{v_i},\qquad i=1,2,\dots,l_{\mathbf{cont}},\qquad t\in[t_0,t_f],$$

can be taken into account with the use of penalty functions.

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RIESZ POTENTIAL AND ITS COMMUTATORS IN VARIABLE EXPONENT VANISHING GENERALIZED WEIGHTED MORREY SPACES

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The variable exponent generalized Morrey spaces $M^{p(\cdot),\varphi}(\Omega)$ over an open set $\Omega \subset R^n$ was introduced and the boundedness of the Hardy-Littlewood maximal operator, the Riesz potential, the singular integral operators and their commutators on these spaces was proved in [1] (bounded set) and [2] (unbounded set), respectively. The variable exponent generalized weighted Morrey spaces $M_{\omega}^{p(\cdot),\varphi}(\Omega)$ over an open set $\Omega \subset R^n$ was introduced and the boundedness of the Hardy-Littlewood maximal operator, the Riesz potential, the singular integral operators and their commutators on these spaces was proved in [3].

Let $\Omega \subset R^n$ be an open unbounded set. We consider generalized weighted Morrey spaces $M_{\omega}^{p(\cdot),\varphi}(\Omega)$ and vanishing generalized weighted Morrey spaces $VM_{\omega}^{p(\cdot),\varphi}(\Omega)$ with variable exponent $p(x)$ and a general function $\varphi(x,r)$ defining the Morrey-type norm. The main result of this study are to prove the boundedness of Riesz potential and its commutators on the spaces $VM_{\omega}^{p(\cdot),\varphi}(\Omega)$ (see [4]). This result generalizes several existing results for Riesz potential and its commutators on Morrey type spaces. Especially, it gives a unified result for generalized Morrey spaces and variable Morrey spaces which currently gained a lot of attentions from researchers in the theory of function spaces.

Keywords: variable exponent vanishing generalized weighted Morrey space, Riesz potential, commutator, BMO space.

This talk is based on joint research with J.J. Hasanov and X.A. Badalov.

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ON THE DETERMIONATION OF THE LOWEST COEFFICIENT OF ONE WEAKLY NONLINEAR WAVE EQUATION IN A MIXED PROBLEM

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In this paper, we consider the problem of determining the lowest coefficient of one weakly nonlinear wave equation. The problem reduces to the optimal control problem; in the new problem, the existence of an optimal control theorem is proved , the Frechet differentiability of the functional and the necessary optimality condition is derived.

Consider the problem of finding a pair $\{u(x,t)v(x)\}$ from the following relations

$$\frac{\partial^2 u}{\partial t^2} - \Delta u + vu = f(x,t,u),\ (x,t) \in Q, \tag{1}$$

$$u = 0,\ (x,t) \in S,\ u\big|_{t=0} = u_0(x),\ \frac{\partial u}{\partial t}\bigg|_{t=0} = u_1(x),\ x \in \Omega, \tag{2}$$

$$\int_0^T K(x,t)u(x,t)dt = \varphi(x),\ \ x \in \Omega, \tag{3}$$

where Δ -operator Laplace with respect to $x,\ f(x,t,u), u_0(x), u_1(x), K(x,t), \varphi(x)$ are given functions, $Q = \Omega \times (0,T)$ cylinder in $R^{n+1}(n \le 4),$ a Ω bounded region in R^n with a sufficiently smooth boundary $\Gamma, S = \Gamma \times (0,T)$ is the lateral surface of the cylinder $Q, T > 0$ is a given number.
Note that problem (1)-(3) is inverse to the direct problem (1), (2) for a given function $v(x)$. We reduce this problem to the following optimal control problem: in the class of functions

$$V = \{v(x) \in L_2(\Omega) / a \le v(x) \le b \ \ a.e.\ on\ \Omega\} \tag{4}$$

find the minimum of the functional

$$J_\alpha(v) = \frac{1}{2}\int_\Omega \left[\int_0^T K(x,t), u(x,t,v)dt - \varphi(x)\right]^2 dx + \frac{\alpha}{2}\int_\Omega |v(x)|^{2.} dx \tag{5}$$

under conditions (1), (2), where $\alpha > 0$ is a given number, a and b are some constants and $a < b,\ u(x,t;v)$ is a solution to the boundary value problem (1), (2) for $v = v(x) \in V,$ the function $v(x)$ is called the control, and the set V is called the class of admissible controls.

This problem will be called the problem (1),(2), (4), (5).

Let $\psi = \psi(x,t;v)$ be a generalized solution of the adjoint problem

$$\frac{\partial^2 \psi}{\partial t^2} - \Delta\psi + v\psi =$$

$$\frac{\partial f(x,t,u)}{\partial u}\psi - K(x,t)\left[\int_0^T K(x,t)u(x,t;v)dt - \varphi(x)\right], \quad (x,t) \in Q, \tag{6}$$

$$\psi = 0,\ (x,t) \in S,\ \psi\big|_{t=T} = 0,\ \frac{\partial\psi}{\partial t}\bigg|_{t=T},\ x \in \Omega. \tag{7}$$

Theorem 1. Let satisfies above conditions on the data of problem (l), (2), (4), (5).Then the functional (5) is continuously differentiable in terms of Frechet on V and its differential at the point $v \in V$ with increment $\delta v \in L_\infty(\Omega)$ is determined by the expression

$$\left\langle J_\alpha^{'}(v), \delta v \right\rangle = \int_\Omega \left[\alpha v - \int_0^T u\psi dt\right] \delta v(x)dx.$$

Theorem 2. Let the conditions of theorem 1 be fulfilled. Then for the optimality of the control $v_* = v_*(x) \in V$ in the problem (l), (2), (4), (5) it is necessary that the inequality

$$\int_\Omega \left[\alpha v_*(x) + \int_0^T u_*(x,t)\psi_*(x,t)dt\right](v(x) - v_*(x))dx \geq 0$$

holds for any control $v = v(x) \in V,$ where,

$$u_*(x,t) = u(x,y;v_*),\ \psi_*(x,t) = \psi(x,t;v_*)$$

solutions of problems (1), (2) and (6), (7) respectively for $v = v_*(x).$

OPTIMAL CONTROL PROBLEM FOR THE WEAK NONLINEAR EQUATION OF THIN PLATE WITH CONTROL AT THE COEFFICIENT OF LOWEST TERM

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Our aim to find the pair of functions $(u,\upsilon)\in W_2^{2,1}(Q)\times U_{ad}$ from the relations

$$\rho\frac{\partial^2 u}{\partial t^2}+\Delta(D\Delta u)+(1-\nu)\left(2\frac{\partial^2 D}{\partial x\partial y}\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 u}{\partial x^2}\right)+ \tag{1}$$

$$+\upsilon(x,y)u+|u|u=f(x,y,t),\ (x,y,t)\in Q,$$

$$u(x,y,0)=\varphi_0(x,y),\ \frac{\partial u(x,y,0)}{\partial t}=\varphi_1(x,y),\ (x,y)\in\Omega, \tag{2}$$

$$\begin{aligned} u(0,y,t)=0,\ \frac{\partial u(0,y,t)}{\partial x}=0,\ u(x,0,t)=0,\ \frac{\partial u(x,0,t)}{\partial y}=0,\\ u(a,y,t)=0,\ \frac{\partial u(a,y,t)}{\partial x}=0,\ u(x,b,t)=0,\ \frac{\partial u(x,b,t)}{\partial y}=0, \end{aligned} \tag{3}$$

$$\int_0^T K(x,y,t)u(x,y,t)dt=g(x,y),$$

where $(x,y)\in\Omega=\{(x,y):0<x<a,\,0<y<b\}$, $t\in(0,T)$, $Q=\Omega\times(0,T)$, a, b, T are given positive numbers, $\rho(x,y)$ is a dense of the mass at the point (x,y), $h(x,y)$ is the thickness of the plate in the point (x,y), $u(x,y,t)$ - is deflection of the plate in the point (x,y) at the moment t, Δ is Laplace operator with

respect to x, y, $D=\dfrac{Eh^3}{12(1-\nu^2)}$- cylindrical rigidity, ν $\left(0<\nu<\dfrac{1}{2}\right)$- Poisson's coefficient, $E>0$- Young's modulus.
$U_{ad}=\{\upsilon(x,y)\in L_2(\Omega):\mu_0\le\upsilon(x,y)\le\mu_1 a.\text{e.on}\Omega\}$,
$\varphi_0(x,y)\in \overset{o}{W}{}_2^2(\Omega)$, $\varphi_1(x,y)\in L_2(\Omega)$, $K(x,y,t)\in L_\infty(Q)$, $g(x,y)\in L_2(\Omega)$ are given functions, μ_0,μ_1 are given numbers.

Note that equation (1) is thin plate vibrations equation [see 1].

We study the generalized solution of the problem (1)-(3).

This problem we reduce to the following optimal control problem: to find the minimum of the functional in the set U_{ad}

$$J_\alpha(\upsilon)=\frac{1}{2}\int_\Omega\left[\int_0^T K(x,y,t)u(x,y,t)dt-g(x,y)\right]^2 dxdy+\frac{\alpha}{2}\int_\Omega \upsilon^2(x,y)dxdy \quad (4)$$

subject to (1)-(3), $\alpha>0$ is given number.

In this work obtained necessary condition of optimality in the form variation inequality in the considered problem (1)-(3), (4) [2].

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AN OPTIMAL CONTROL PROBLEM BY THE COEFFICIENT OF THE WAVE EQUATION WITH NON LOCAL CONDITION

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In the paper we consider a problem of definition of a pair of function $(u(x,t),\vartheta(x))\in W_2^1(Q)\times V$ from the conditions

$$\frac{\partial^2 u(x,t)}{\partial t^2}-\Delta u+\vartheta(x)u=f(x,t),\ (x,t)\in Q=\Omega\times(0,T), \quad (1)$$

$$u(x,0)=u_0(x),\frac{\partial u(x,0)}{\partial t}=u_1(x), x\in\Omega, \quad (2)$$

$$\left.\frac{\partial u}{\partial \nu}\right|_S=\int_\Omega K(x,y)u(y,t)dy,\ (x,t)\in S=\partial\Omega\times(0,T), \quad (3)$$

$$\int_0^T R(x,t)u(x,t)dt=\varphi(x), \quad (4)$$

$$V=\left\{\vartheta(x)\in W_2^1(\Omega): \nu\le\vartheta(x)\le\mu,\ \left|\frac{\partial\vartheta}{\partial x_i}\right|\le M, \text{almost everywhere on } \Omega\right\}, \quad (5)$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$, $T>0,\ M>0,\nu,\ \mu$ are the given numbers, $f\in L_2(Q)$, $u_0\in W_2^1(\Omega)$, $u_1\in L_2(\Omega)$, $R\in L_\infty(Q)$, $\varphi\in L_2(\Omega)$, $K\in L_\infty(\Omega\times\Omega)$ are the given functions.

For the problem (1)-(5) we associate the following appropriate optimal control problem: it is regured to minimize the functional

$$I(\vartheta)=\frac{1}{2}\int_{\Omega}\left[\int_0^T R(x,t)u(x,t;\vartheta)dt-\varphi(x)\right]^2 dx, \qquad (6)$$

under condition (1)-(3), (5), where $u=u(x,t)=u(x,t;\vartheta)$ is a solution of boundary value problem (1)-(3) corresponding to the function $\vartheta=\vartheta(x)\in V$.

In the paper we prove continuous Frechet differentiability of functional (6) and derive a necessary optimality condition in the from of variational inequality [1,2].

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HYERS-ULAM STABILITY OF 3-LIE HOMOMORPHISMS AND 3-LIE DERIVATIONS

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In this talk, we prove the Hyers-Ulam stability of 3-Lie homomorphisms in 3-Lie algebras for Cauchy-Jensen functional equation by using the fixed point method. We also prove the

Hyers-Ulam stability of 3-Lie derivations on 3-Lie algebras for Cauchy-Jensen functional equation by using the fixed point method.

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BOUNDEDNESS IN VARIABLE EXPONENT SPACES OF GENERALIZED POTENTIAL OPERATORS GIVEN ON NONHOMOGENEOUS SPACES

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We consider generalized potential operators with the kernel $\dfrac{a[\rho(x,y)]}{[\rho(x,y)]^N}$ on bounded measure metric space (X,μ,ρ) with measure \$\mu\$ satisfying the upper growth condition $\mu(B(x,r))\leq Ar^N$, $N\in(0,\infty)$, Under some natural assumptions on $a(r)$ in terms of almost monotonicity we prove that such potential operators are bounded from the variable exponent Lebesgue space $L^{p(\cdot)}(X,\mu)$ into a certain Musielak-Orlicz space

$L^{\Phi}(X,\mu)$ $a(r)$, if the corresponding maximal operator is bounded on $L^{p(\cdot)}(X,\mu)$.

COMMUTATORS OF SUBLINEAR OPERATORS WITH ROUGH KERNEL GENERATED BY CALDERON-ZYGMUND OPERATORS ON GENERALIZED WEIGHTED MORREY SPACES

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The classical Morrey spaces were introduced by Morrey (1938) to study the local behavior of solutions to second-order elliptic partial differential equations. Moreover, various Morrey spaces are defined in the process of study. Guliyev, Mizuhara and Nakai (1994) introduced generalized Morrey spaces $M_{p,\varphi}(R^n)$; Komori and Shirai (2009) defined weighted Morrey spaces $L^{p,k}(w)$; Guliyev (2012) gave a concept of the generalized weighted Morrey spaces $M_{p,\varphi}(w)$ which could be viewed as extension of both $M_{p,\varphi}$ and $L_{p,k}(w)$. In [1], the boundedness of the classical operators and their commutators in spaces $M_{p,\varphi}(w)$ was also studied.

Let $\Omega \in L_s(S^{n-1})$ with $1 < s \le \infty$ be homogeneous of degree zero. For a function b, suppose that $T_{\Omega,b}$ represents a linear or a sublinear operator, such that that for any $f \in L_1(R^n)$ with compact support and $x \notin \operatorname{supp} f$

$$|T_\Omega f(x)| \le c_0 \int_{R^n} |b(x)-b(y)| \frac{|\Omega(x-y)|}{|x-y|^n} |f(y)| dy, \quad (1)$$

where c_0 is independent of f and x.

Let $1 \le p < \infty$, φ be a positive measurable function on $R^n \times (0,\infty)$ and w be non-negative measurable function on R^n. The generalized weighted Morrey space $M_{p,\varphi}(w)$ is the set of all functions $f \in L^{\text{loc}}_{p,w}(R^n)$ with finite norm

$$\|f\|_{M_{p,w}(w)} = \sup_{x \in R^n, r>0} \varphi(x,r)^{-1} w(B(x,r))^{-\frac{1}{p}} \|f\|_{L_{p,w}(B(x,r))}.$$

The main goal of this work to study the boundedness of a large class of sublinear operators with rough kernel $T_{\Omega,b}$ on the generalized weighted Morrey spaces $M_{p,\varphi}(w)$ for with $q' \le p < \infty$, $p \ne 1$ and $w \in A_{p/q'}$ or $1 < p < q$ and $w^{1-p'} \in A_{p'/q'}$, where $\Omega \in L_s(S^{n-1})$ with $q > 1$ be homogeneous of degree zero (see [2]).

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ON THE BOUNDEDNESS OF COMMUTATORS DUNKL-TYPE FRACTIONAL INTEGRAL OPERATOR IN THE DUNKL-TYPE MODIFIED MORREY SPACES

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Let $\alpha > -1/2$ be a fixed number and μ_α be the weighted Lebesgue measure on $\mathbb{R}$, given by $d\mu_\alpha(x) := \left(2^{\alpha+1}\Gamma(\alpha+1)\right)^{-1} |x|^{2\alpha+1} dx.$

For every $1 \le p \le \infty$, we denote by $L_{p,\alpha}(\mathbb{R}) = L_p(\mathbb{R}, d\mu_\alpha)$ the spaces of complex-valued functions f, measurable on $\mathbb{R}$ such that

$$\|f\|_{p,\alpha} \equiv \|f\|_{L_{p,\alpha}} = \left(\int_{\mathbb{R}} |f(x)|^p d\mu_\alpha(x)\right)^{1/p} < \infty \quad \text{if} \quad p \in [1,\infty),$$

and $\|f\|_{\infty,\alpha} \equiv \|f\|_{L_\infty} = ess\sup\limits_{x\in\mathbb{R}} |f(x)| \quad \text{if} \quad p = \infty.$

For $1 \le p < \infty$ we denote by $WL_{p,\alpha}(\mathbb{R})$, the weak $L_{p,\alpha}(\mathbb{R})$ spaces defined as the set of locally integrable functions f with the finite norm

$$\|f\|_{WL_{p,\alpha}} = \sup_{r>0} r\left(\mu_\alpha\{x \in \mathbb{R} : |f(x)| > r\}\right)^{1/p}.$$

Let $B(x,t) = \{y \in \mathbb{R} : |y| \in]\max\{0, |x|-t\}, |x|+t[\,\}$ and $B_t \equiv B(0,t) =]-t,t[,\ t > 0$. Then $\mu_\alpha B_t = b_\alpha t^{2\alpha+2}$, where $b_\alpha = \left[2^{\alpha+1}(\alpha+1)\Gamma(\alpha+1)\right]^{-1}$.

We denote by $BMO_\alpha(\mathbb{R})$ (Dunkl-type BMO space) the set of locally integrable functions f with finite norm

$$\|f\|_{BMO_\alpha} = \sup_{r>0,\, x\in\mathbb{R}} \frac{1}{\mu_\alpha B_r} \int_{B_r} |\tau_x f(y) - f_{B_r}(x)| d\mu_\alpha(y) < \infty$$

or $\|f\|_{BMO_\alpha} = \inf_C \sup_{r>0,\, x\in \mathbb{R}} \frac{1}{\mu_\alpha B_r} \int_{B_r} |\tau_x f(y) - C| d\mu_\alpha(y)$, where

$$f_{B_r}(x) = \frac{1}{\mu_\alpha B_r} \int_{B_r} \tau_x f(y) d\mu_\alpha(y).$$

For all $x, y, z \in \mathbb{R}$, we put $W_\alpha(x,y,z) = (1 - \sigma_{x,y,z} + \sigma_{z,x,y} + \sigma_{z,y,x}) \Delta_\alpha(x,y,z)$, where

$$\sigma_{x,y,z} = \begin{cases} \frac{x^2 + y^2 - z^2}{2xy} & if \quad x, y \in \mathbb{R} \setminus \{0\} \\ 0 & otherwise \end{cases}.$$

In the sequel we consider the signed measure $\nu_{x,y}$, on $\mathbb{R}$, given by

$$\nu_{x,y} = \begin{cases} W_\alpha(x,y,z) d\mu_\alpha(z) & if \quad x, y \in \mathbb{R} \setminus \{0\}, \\ d\delta_x(z) & if \quad y = 0, \\ d\delta_y(z) & if \quad x = 0. \end{cases}$$

For $x, y \in \mathbb{R}$ and f a continuous function on $\mathbb{R}$, we put

$$\tau_x f(y) = \int_{\mathbb{R}} f(z) d\nu_{x,y}(z).$$

Definition. Let $1 \le p < \infty$, $0 \le \lambda \le 2\alpha + 2$, $[t]_1 = \min\{1, t\}$. We denote by $\tilde{M}_{p,\lambda,\alpha}(\mathbb{R})$ Dunkl-type modified Morrey space as the set of locally integrable functions $f(x)$, $x \in \mathbb{R}$, with the finite norm

$$\|f\|_{\tilde{M}_{p,\lambda,\alpha}} = \sup_{t>0,\, x\in \mathbb{R}} \left(t^{-\lambda} \int_{B_t} [\tau_x \mid f(y) \mid]^p d\mu_\alpha(y) \right)^{1/p}.$$

Given a measurable function b the operator $|b, I_{\beta,\alpha}|$ is defined by

$$|b,I_{\beta,\alpha}|\, f(x)=\int_{\mathbb{R}} |b(x)-b(y)|\,|y|^{\beta-2\alpha-2}\tau_y\,|f|(x)d\mu_\alpha(y), \quad 0<\alpha<2\alpha+2.$$

Theorem. Let $0<\alpha<2\alpha+2$, $0\le\lambda<2\alpha+2-\beta$, $\frac{\beta}{2\alpha+2}\le\frac{1}{p}-\frac{1}{q}\le\frac{\beta}{2\alpha+2-\lambda}$ and $1<p<\frac{2\alpha+2-\lambda}{\beta}$. Then the commutator $|b,I_{\beta,\alpha}|$ is bounded from $\widetilde{M}_{p,\lambda,\alpha}(\mathbb{R})$ to $\widetilde{M}_{q,\lambda,\alpha}(\mathbb{R})$ if and only if $b\in BMO_\alpha$.

ROUGH MULTILINEAR FRACTIONAL MAXIMAL OPERATOR ON GENERALIZED WEIGHTED MORREY SPACES

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The classical Morrey spaces were introduced by Morrey (1938) to study the local behavior of solutions to second-oder elliptic partial differential equations. Moreover, various Morrey spaces and defined in the process of study. Guliyev, Mizuhara and Nakai (1994) introduced generalized Morrey spaces $M_{p,\varphi}(\mathbb{R}^n)$;Komori and Shirai (2009) defined weighted Morrey spaces $L^{p,k}(w)$; Guliyev (2012) gave a concept of the generalized weighted Morrey spaces $M_{p,\varphi}(w)$ which could be viewed as extension of both $M_{p,\varphi}$ and $L_{p,k}(w)$. In [1], the boundedness of the classical operators and their commutators in spaces $M_{p,\varphi}(w)$ was also studied.

The generalized weighted Morrey space $M_{p,\varphi}(w)$ is the set of all functions $f \in L^{loc}_{p,w}(\mathbb{R}^n)$with finite norm

$$\|f\|_{M^{(w)}_{p,\varphi}} = \sup_{x\in R^n, r>0} \varphi(x,r)^{-1} w(B(x,r))^{-\frac{1}{p}} \|f\|_{L_{p.w}(B(x,r))}$$

Let us give the definition of the rough multilinear fractional maximal operator as follows:

$$M_{\Omega_1\alpha} f(x) = \sup_{r>0} \int_{B(x,r)} \frac{|\Omega(x-y)|}{|x-y|^{n-\alpha+m-1}} |R_m(A;x,y)| f(y)dy,$$

where $0<\alpha<n$, Ω is homogeneous of degree zero and

$$\Omega \in L_s(S^{n-1}), s>1, \; R_m(A:x,y)=$$

$$= A(x) - \sum_{|\gamma|<m} \frac{1}{\gamma!} D^\gamma A(y)(x-y)^\gamma$$

In this talk, we study the boundedness of multilinear fractional maximal operators with rough kernels $M^{A,m}_{\Omega,\alpha}$, which is generalization of the higher-order rough fractional maximal commutator on the generalized weighted Morrey spaces $M_{p,\varphi}(w)$. We find the sufficient conditions on the pair (φ_1,φ_2) with $w \in A_{p,q}$ which ensures the boundedness of the operators $M^{A}_{\Omega,\alpha}$ from $M_{p,\varphi_1}(w^p)$ to $M_{p,\varphi_2}(w^q)$for $1<p<q<\infty$ $(see[2])$.

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ORLICZCHARACTERIZATION OF MAXIMAL OPERATORS IN GENERALIZED WEIGHTED -MORREY SPACES

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The classical Morrey spaces were introduced by Morrey (1938) to study the local behavior of solutions to second-oder elliptic partial differential equations. Moreover, various Morrey spaces and defined in the process of study. Guliyev, Mizuhara and Nakai (1994) introduced generalized Morrey spaces $M_{p,\varphi}(\mathbb{R}^n)$; Komori and Shirai (2009) defined weighted Morrey spaces $L^{p,k}(w)$; Guliyev (2012) gave a concept of the generalized weighted Morrey spaces $M_{p,\varphi}(w)$ which could be viewed as extension of both $M_{p,\varphi}$ and $L_{p,k}(w)$. In [1], the boundedness of the classical operators and their commutators in spaces $M_{p,\varphi}(w)$was also studied.

The main goal of this talk to give necessary and sufficient conditions for the boundedness of the Hardy-Littlewood maximal operator and its commutators on generalized weighted Orlicz-Morrey spaces. The main advance in comparison with the existing results in that we manage to obtain conditions for the boundedness not in integral terms but in less restrictive terms of supremal operators and we do not need Δ_2-condition for the boundedness of the maximal operator (see [1]).

This contribution is based on recent joint works with V.S.Guliyev and F.Deringoz.

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ON THE EXISTENCE OF A GENERALIZEDSOLUTION OF THE INVERSE PROBLEM FOR EQUATION OF PARABOLIC TYPE

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For an equation of parabolic type, the following inverse problem is considered. By given functions $f(x,t,u), \varphi(x), \psi(x,t,u), h(x)$ it is required to determine a pair of functions $\{c(x), u(x,t)\}$ from the conditions:

$$u_t - \Delta u + c(x)u = f(x,t,u),\ (x,t) \in \Omega = D \times (0,T], \quad (1)$$

$$u(x,0) = \varphi(x),\ \ x \in \overline{D} = D \cup \partial D,\ \ D \subset R^n, \quad (2)$$

$$\frac{\partial u}{\partial \overline{N}} = \psi(x,t,u),\ \ (x,t) \in S = \partial D \times [0,T], \quad (3)$$

$$\int_0^T u(x,t)dt = h(x),\ x \in \overline{D},\ 0 < T = const. \quad (4)$$

We assume that the input datas of the problem satisfy the following conditions:

1^0. $f(x,t,p) \in C^{\alpha,\alpha/2}(\overline{\Omega}\times R^1)$, there is a constant $m_1 > 0$, such that for all $p_1, p_2 \in R^1$ and $(x,t)\in\overline{\Omega}$

$\left| f(x,t,p_1) - f(x,t,p_2)\right| \le m_1 |p_1 - p_2|$;

2^0. $\varphi(x) \in C^{2+\alpha}(\overline{D})$; 3^0. $h(x) \in C^{2+\alpha}(\overline{D})$;

4^0. $\psi(x,t,p) \in C^{\alpha,\alpha/2}(S\times R^1)$, there is a constant $m_1 > 0$, such that for all $p_1, p_2 \in R^1$ and $(x,t)\in S$

$|\psi(x,t,p_1) - \psi(x,t,p_2)| \le m_2 |p_1 - p_2|$.

Suppose that the input datas satisfy conditions 1^0 –4^0. Then the solution of problem (1) – (3) can be represented in the form

$$u(x,t) = \varphi(x) + \int_0^t \int_D \Gamma(x,t;\xi,\tau)[f(\xi,\tau,u) + \Delta\varphi(\xi) -$$

$$- c(\xi)H(\xi,t)]d\xi d\tau + \int_0^t \int_{\partial D} \Gamma(x,t;\xi,\tau)\rho(\xi,\tau)d\xi d\tau, \quad (5)$$

$$c(x) = \left[\varphi(x) + \Delta h(x) + \int_0^T f(x,t,u)dt - u(x,T)\right](h(x))^{-1},$$

$$(x,t) \subset \Omega,\ x \in \overline{D}. \quad (6)$$

Definition. A pair of functions $\{c(x), u(x,t)\}$ is called the integral (generalized) solution to problem (1) – (4) if:

1) $c(x) \in C(\overline{D})$; 2) $u(x,t) \in C^{2,1}(\Omega) \cap C^{1,0}(\overline{\Omega})$;

3) these functions are satisfied the relations of system (5) – (6).

The following theorem is proved:

Theorem. Let that conditions 1^0–4^0 are satisfied. Then there exists a $T^*(0 < T^* \le T)$, such that when $(x,t) \in \overline{D}\times[0,T^*]$

there exists a solution of the system of integral equations (5)–(6) and $c(x)\in C(\overline{D})$, $u(x,t)\in C\left(\overline{D}\times\left[0,T^*\right]\right)$.

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THE ESTIMATE OF NORM THE OPERATORS OF INTERMEDIATE DERIVATIVES WITH CONNECTION OF OPERATOR-DIFFERENTIAL EQUATIONS IN PARTIAL DERIVATIVES OF SECOND ORDER ON ALL SPACE R^n

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In present work korrectly solvabibity some class of operator-differential equation of second order on all space R^n is investiqated. For this is before of theorem about of isomorphism of head part of operator from space $W_2^4(R^n;H)$ on space $L_2(R^n;H)$proved.

We formulate theorem about estimate of norm operators of intermediate derivatives which play important role under get sufficiently condition for correctly solvability of equation described over coefficients giving equation.

Let $H-$separable Hilbert space, $C-$selfadjoint positive defined of operator in H. Well known, that domain of definition of the operator $C^{\gamma}(\gamma\le 0)$ is Hilbert space H_{γ} with scalar product $(x,y)_{\gamma}=\left(C^{\gamma}x,C^{\gamma}y\right)$. At $t=0$, take $H_0=H$.

We notation over $D\left(R^n;H\right)$ the set infinity differentiable vector-function with compact support in R^n. In linear set $D\left(R^n;H\right)$ we define the norm

$$\|u\|_{W_2^2(R^n;H)}=\left(\sum_{k=1}^{n}\left\|\frac{\partial u}{\partial x_k}\right\|^2_{L_2(R^n;H)}+\left\|C^2u\right\|^2_{L_2(R^n;H)}\right)^{1/2}$$

We complete the set $D\left(R^n;H\right)$ related this norm and define over $W_2^2\left(R^n;H\right)$.

In space H following the operator-differential equation considered

$$-\sum_{k=1}^{n}a_k\frac{\partial^2u}{\partial x_k^2}+\sum_{k=1}^{n}R_k\frac{\partial u}{\partial x_k}+Tu+C^2u=f(x),\ x\in R^n. \quad (1)$$

Definition 1. If at $f(x)\in L_2(R^n;H)$ exist of vector-function $u(x)\in W_2^2(R^n;H)$, which satisfy to equation (1) almost every in R^n then $u(x)$ is called requlurity solution of (1).

Definition 2. It at any $f(x)\in L_2(R^n;H)$ equation (1) have solution $u(x)\in W_2^2(R^n;H)$ which have estimate

$$\|u\|_{W_2^2(R^n;H)}\le const\|f\|_{L_2(R^n;H)}$$

then equation (1) is called correctly solvability.

Over L_0 we define the operator

$$L_0\left(\frac{\partial}{\partial x}\right)u = -\sum_{k=1}^{n} a_k \frac{\partial^2 u}{\partial x_k^2} + C^2 u,\ u(x) \in W_2^2(R^n; H)$$

We have

Theorem. Let $u(x) \in W_2^2(R^n; H)$.Then have following inequality

$$\left\| c\frac{\partial u}{\partial x_k} \right\|_{L_2(R^n;H)} \le c_k \left\| L_0\left(\frac{\partial}{\partial x}\right)u \right\|_{L_2(R^n;H)},\ k = 1,2,\ldots,n.$$

$$\left\| c^2 u \right\|_{L_2(R^n;H)} \le c_0 \left\| L_0\left(\frac{\partial}{\partial x}\right)u \right\|_{L_2(R^n;H)}.$$

where $c_0 = 1,\ c_k = a_k^{-\frac{1}{2}},\ k = 1,2,\ldots,n.$

This theorem have important role at sufficient condition for correctly solvability of equation (1).

FIRST LADY OF COMET– CAROLINE LUCREZIA GERSHELL (1750-1848)

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The article is dedicated to the life and work of CarolineLucrezia Herschel, her scientific heritage and role in the development of astronomy.

Keywords: astronomy, comet, nebulae, star catalog, observer astronomer, vocals.

In 2009, the European Union launched its telescope spaceward under the name of Herschel. It was much larger than first Hubble orbital telescope launched by NASA. If Hubble made us open our mouths in surprise, then Herschel would seem like a real miracle. "Herschel" is named after the greatest astronomer of the 18th century, William Herschel (1738-1822), an Englishman, who was born in Germany and changed his name to Williamwhen he was moving to the green fields of Britain. He became incredibly prominent owing to thanks to the discovery of the planet Uranus. William Herschel had a sister, Caroline Lucrezia Herschel, who was also engaged in astronomy like him.

In 1772, Caroline Lucrezia arrived in England at the invitation of her older brother William Herschel. They both made music, but besides music they had a hobby - astronomy. One day a book on astronomy fell into the hands of William Herschel. It was the book of Ferguson Astronomy (1750). William fell in love with the science of starry sky, and the love was mutual. When she was 32 years old, she became a student with her brother. Under the guidance of her brother, Caroline studied the basics of mathematics and then independently processed her and his observations.

Caroline began to manage her brother's affairs and carried out all the laborious trigonometric calculations before and after the observations. During her leisure time, Caroline Herschel independently observed the sky and she already discovered three new nebulae in 1783. In 1786, Caroline Herschel discovered a new comet - the first comet discovered by a woman, and behind this comet Caroline discovered another 8 new ones in 1786-1797. This was an outstanding achievement for any astronomer. And some began to call her "first lady of comets." Caroline began to be considered as an independent astronomer. Caroline completed a work on her own catalog of stars, where she described all her discoveries and clarified the famous results obtained by John Flemstead, the royal astronomer and contemporary of Newton. She cataloged 561 new stars. At that time, she became a respected astronomer and was invited to the royal court three times.

By 1828, she had completed a work on a catalog of 2,500 star nebulae observed by her brother. Her contribution to astronomy was recognized by the government of Great Britain. The Royal Astronomical Society of Great Britain awarded her with a gold medal and elected her as its honorary member in 1835. In the year of 1838, Caroline Herschel was elected as an honorary member of the Royal Irish Academy of Sciences. In 1846, the king of Prussia presented her with a gold medal for science.

Herschel named Lucrezius 281 asteroid in the honor of Caroline, discovered in 1888 and a crater on the moon with a diameter of 13 kilometers. Her name has been given on the map of the moon.

After her death, objects under NGC numbers such as 205,225,253,381,659,891,2349,2360,2548,6633,7380,7789 were listed in the new star catalog.

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GEOMETRIC DESCRIPTION OF A MECHANICAL-DIFFERENTIAL EQUATION FROM DIFFERENTIAL GEOMETRY VIEW POINT

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The aim of this talk is to present a qualitative and geometric description of a mechanical-differential equation without giving a direct solution. To understand the geometry of a phase space we use geometric and topological properties of its (co)-tangent bundle which are characterized by an algebraic equation. (Each differential equation in a phase space is an algebraic equation in the higher tangent bundles). Our goal is to investigate possible geometric relation(s) between the phase space and its (co)-tangent bundle. Let Ω be a system of differential equations of p independent and q dependent variables. The solution of this system is a p-dimensional submanifold of $\mathbb{R}^{p+q}$.

The higher tangent bundle of this submanifold satisfies an equation same as the one in Ω with the difference that instead of a differential equation we have an algebraic one. (Each differential equation of order n on M converts to an algebraic equation in tangent bundle of order n). Form advanced differential geometry we have the isomorphism between (higher) tangent bundles. Therefore, we may approach to our problem by investigating possible geometric relation(s) between the bundles TM,T^*M and $T(T^*M)$ with the manifold M. More precisely, one may rephrase the above problem as follows: if an algebraic equation on TM,T^*M and $T(T^*M)$ (obtained from a differential equation on M) has some special geometric properties, such as homogeneity, (local or generalized) symmetry, having critical or singular point or special curvature, then could M inherit the similar properties? Classical mechanics on a manifold M can be modeled as Hamiltonian mechanics on the tangent space or Lagrangian mechanics on the cotangent space. The same questions as above arise here: are there any geometric relation(s) between the model space and the base space (phase space)? If an equation on the model space has a special geometric character then does the base space (M) has the same character?[1]. The main key in understanding this geometric relation, is to know how tensors on M should lift to TM,T^*M and tensors on TM,T^*M should shrink to M. Many different forms of lifting such as vertical, horizontal and complete lift have been introduced. In this talk we review some of these liftings and shrinking. Toomanian and Ledger showed that if g (∇) is a semi-Riemannian metric (an affine connection) on M then the complete lift of g (∇) is a semi-Riemannian metric (an affine connection) on TM. Moreover, if M is homogenous then so is TM, and if M has a symmetry (local or generalized) of any kind (s-regular space) then, with complete lift, TM has the same kind of symmetry [1,3]. However, the converse is not always true. We show that if TM is homogenous (symmetric) then, with complete shrink, one can only say that M is local homogenous (symmetric) around non-singular points of the algebraic equation obtained from the corresponding differential equation.

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HOLDER CONTINUITY OF SOLUTIONS OF THE $p(x)-$ LAPLACE EQUATION, OF THE UNIFORMLY DEGENERATE ON A PART OF THE DOMAIN WITH RESPECT TO A SMALL PARAMETER

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Let $D\subset R^n, n\geq 2,$ be a domain such that the hypwerplane $\sum=\{x\in R^n, x_n=0\}$ divides it into two nonempty parts

$$D^{(1)}=D\cap\{x:x_n>0\} \text{and } D^{(2)}=D\cap\{x:x_n<0\}.$$

In the domain D consider the family of elliptic equations

$$L_\varepsilon u=\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(a_{ij}(x)\omega_\varepsilon(x)|\nabla u|^{p(x)-2}\frac{\partial u}{\partial x_i}\right)=0 \qquad (1)$$

with parameter $\varepsilon\in(0,1]$with a measurable exponent $p(x)$ such that

$$1<p_1\leq p(x)\leq p_2<\infty \qquad (2)$$

almost everywhere in D, and with the positive weight $\omega_\varepsilon(x)$, defined by the formula

$$\omega_\varepsilon(x)=\begin{cases}\varepsilon, & if \quad x\in D^{(1)}\\ 1, & if \quad x\in D^{(2)}.\end{cases} \tag{3}$$

The coefficients of the operator are measurable and symmetric, satisfy the uniform ellipticity condition

$$\alpha^{-1}|\xi|^2\le\sum_{i,j=1}^{n}a_{ij}(x)\xi_i\xi_j\le\alpha|\xi|^2, \tag{4}$$

To define what we call a solution of equation (1), let us introduce the function class

$$W_{loc}(D)=\left\{u:u\in W_{loc}^{1,1}(D),\ |\nabla u|^{p(x)}\in L_{loc}^{1}(D)\right\}$$

where $W_{loc}^{1,1}(D)-$ is the Sobolev space of functions locally integrable in D together with their first generalized derivatives. A solution of equation (1) will be understood as a function $u\in W_{loc}(D)$, satisfying the integral identity

$$\sum_{i,j=1}^{n}\int_D a_{vj}(x)|\nabla u|^{p(x)-2}\frac{\partial u}{\partial x_i}\frac{\partial\psi}{\partial x_i}dx=0 \tag{5}$$

for any functions $\psi\in C_0^\infty(D)$, which will be called test functions as usual. An important role is played by the question as to whether smooth functions are dense in the solution class $W_{loc}(D)$ thus introduced. It was shown in the paper [1] that if the logarithmic condition

$$|p(x)-p(y)|\le c\left(-\ln|x-y|\right)^{-1}\text{ for } x,y\in D,\ |x-y|<\frac{1}{2}, \tag{6}$$

is satisfied, then for an arbitrary function $u\in W_{loc}(D)$ there exists a sequence $\{u_j\}$, where $u_j\in C^\infty(D)$, such that the following relations hold in an arbitrary subdomain $\overline{D}'\subset D$

$$\lim_{j\to\infty}\|u_j-u\|_{W^{1,1}(D')}=0,$$

$$\lim_{j\to\infty}\int_{D'}\left|\nabla u_j\right|^{p(x)}dx=\int_{D'}\left|\nabla u\right|^{p(x)}dx,$$

Theorem. If conditions (2), (4) and (6), are satisfied, then there exists a constant $\alpha\in(0,1)$, independent of ε such that the family $\{u^{\varepsilon}(x)\}$ is compact in the space $c^{\alpha}(D')$ in an arbitrary subdomain $\overline{D}' \subset D$. This theorem was established by the present authors for the case of $p(x)=2$ in the paper [2].

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ABOUT ONE INVERSE PROBLEM FOR STURM-LIUOVILLE OPERATOR

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Consider the differential equation

$$-y'' + q(x)y = \lambda^2 y, \quad -\infty < x < \infty \qquad (1)$$

with discontinuity conditions at the points $\alpha_1, \alpha_2 \in R \quad (\alpha_1 < \alpha_2)$

$$y(\alpha_k - 0) = \alpha_k y(\alpha_k + 0), y'(\alpha_k - 0) = \alpha_k^{-1} y'(\alpha_k + 0), \qquad (2)$$

where $q(x)$ is a real-valued function that satisfies the condition

$$\int_{-\infty}^{\infty} (1 + |x|)\,|q(x)|dx < +\infty,$$

λ – is a complex parameter , α_k- are real numbers , where α_k>0, $\alpha_k \neq 1$ (k=1,2).

The inverse problem for the equation (1) with scattering data has been studied. The solution $e^{\pm}(x,\lambda)$ of the equation (1) satisfying the condition (2) and the following condition

$$\lim_{x\to\pm\infty} e^{\pm}(x,\lambda)e^{\mp i\lambda x} = 1$$

exits (Jost solution) and shown as

$$e^{\pm}(x,\lambda) = e_0^{\pm}(x,\lambda) \pm \int_x^{\pm\infty} K^{\pm}(x,t)\, e^{\pm i\lambda x}\, dt$$

where $e_0^{\pm}(x,\lambda)$- is Jost type solution of the equation (1), when $q(x) \equiv 0$. The kernel $K^{+}(x,t)$ and coefficients of the equation (1) are related with the identities

$$K^{+'}(x,x) = \begin{cases} -q(x), & x > a_2\,, \\ -\alpha_2^{+} q(x), & a_1 < x < a_2, \\ -\alpha_1^{+}\alpha_2^{+} q(x), & x < a_1 \end{cases}$$

where $\alpha_k^{\pm} = \frac{1}{2}(\alpha_k \pm \alpha_k^{-1})$.

The similar relation holds true for the kernel $K^{-}(x,t)$ and q(x) as well.

For simplicity let's assume that the eigenvalues of the equation (1) don't exist. Then the function $r^{+}(\lambda)$ can be taken as scattering data that defines the asymptotic of the right eigenfunction

$$u^{+}(x,\lambda) = r^{+}(\lambda)e^{i\lambda x} + e^{-i\lambda x} + o(1), \quad x \to +\infty.$$

The inverse problem consists of the following:

Define the function q(x) by $r^{+}(\lambda)$.

In order to solve the inverse problem derive the main equations

$$R_1^{+}(x,y) + K^{+}(x,y) + \int_x^{+\infty} K^{+}(x,t)R^{+}(t+y)dt - \frac{\alpha_2^{-}K^{+}(x,2\alpha_k - y)}{\alpha_2^{+}} +$$

$$+\sum_{k=1}^{\infty}(-1)^k \frac{1}{(\alpha_2^{+})^2}\left(\frac{\alpha_1^{-}}{\alpha_1^{+}}\right)^k \left(\frac{\alpha_2^{-}}{\alpha_2^{+}}\right)^{k-1} K^{+}(x,2\alpha_2 - 2k(\alpha_2-\alpha_1) - y) = 0, y > x$$

where

$$R^+(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(r^+(\lambda) - r_0^+(\lambda))\, e^{i\lambda x}\, d\lambda,$$

$$R_1^+(x,y) =$$

$$= \begin{cases} R^+(x+y)\ , & x > a_2, \\ \alpha_2^+ R^+(x+y) + \alpha_2^- R^+(2a_2 - x + y), & a_1 < x < a_2 \\ \alpha_1^+\left(\alpha_2^+ R^+(x+y) + \alpha_2^- R^+(2a_2 - x + y)\right) + & \\ +\alpha_1^-\left(\alpha_2^+ R^+(2a_1 - x + y) + \alpha_2^- R^+\left(2(a_2 - a_1) + x + y\right)\right), & x < a_1 \end{cases}$$

$$r_0^+(\lambda) = \frac{-(\alpha_1^+\alpha_2^- e^{2i\lambda a_2} + \alpha_1^-\alpha_2^+ e^{-2i\lambda a_1})}{\alpha_1^+\alpha_2^+ + \alpha_1^-\alpha_2^- e^{2i\lambda(a_2 - a_1)}}$$

ON CONVERGENCE RATE OF SPECTRAL EXPANSION IN EIGEN-FUNCTIONS OF FOURTH ORDER DIFFERENTIAL OPERATOR

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On the interval $G = (0,1)$ we consider the operator

$$L\psi = \psi^{(4)} + U_2(x)\psi^{(2)} + U_3(x)\psi^{(1)} + U_4(x)\psi$$

With matrix coefficients $U_\ell(x) = \left(u_{\ell ij}(x)\right)_{i,j=1}^m$, $\ell = \overline{2,4}$, where $u_{\ell ij}(x) \in L_1(G)$ are real functions $u_{\ell ij}(x) = u_{\ell ji}(x)$.

Under the eigen vector-function of the operator L responding to the eigen-value λ we will understand any identically non-zero vector-function $\psi(x) = (\psi_1(x), \psi_2(x), ..., \psi_m(x))^T$ satisfying equal almost everywhere in G the equation (see [1]) $L\psi + \lambda\psi = 0$.

Let $L_p^m(G), p\geq 1$ be a space m- component vector-functions $f(x)=\left(f_1(x), f_2(x),..., f_m(x)\right)^{\mathrm{T}}$ with the norm $\| f\|_{p,m}=\left\{\int\limits_G |f(x)|^p dx\right\}^{1/p}=\left\{\int\limits_G\left(\sum\limits_{i=1}^m |f_i(x)|^2\right)^{p/2} dx\right\}^{1/p}$.

Assume that $\{\psi_k(x)\}_{k=1}^\infty$ is a system completely orthonormed in $L_2^m(G)$ consisting of eigen vector-functions of the operator L. Denote by $\{\lambda_k\}_{k=1}^\infty$, $\lambda_k\leq 0$, the appropriate system of eigen values.

Denoting $\mu_k=\sqrt[4]{-\lambda_k}$ we consider a partial sum of orthogonal expansion of the vector-function $f(x)\in W_{1,m}^1(G)$ in the system $\{\psi_k(x)\}_{k=1}^\infty$

$$\sigma_\nu(x,f)=\sum_{\mu_k\leq\nu} f_k\psi_k(x),\quad \nu>0, \text{where}$$

$$f_k=(f,\psi_k)=\int\limits_0^1 < f(x),\psi_k(x)> dx=\int\limits_0^1\sum_{j=1}^m f_j(x)\psi_{kj}(x)dx,$$

$$\psi_k(x)=\left(\psi_{k1}(x),\psi_{k2}(x),...,\psi_{km}(x)\right)^{\mathrm{T}}.$$

Theorem. *Let the vector-function $f(x)\in W_{1,m}^1(G)$ satisfy the conditions $f(0)=f(1)=0$ and $\sum\limits_{n=2}^\infty n^{-1}\omega_{1,m}\left(f',n^{-1}\right)<\infty$.*

Then the expansion of the vector-function $f(x)$ in the system $\{\psi_k(x)\}_{k=1}^\infty$ converges absolutely and uniformly on $\overline{G}=[0,1]$.

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ON THE STUDY OF THE STRESS-STRAIN STATE OF THIN-WALLED SHELL STRUCTURES UNDER RADIATION EXPOSURE

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The development of nuclear energy has contributed to the formulation of tasks related to the reliability and durability of structural elements operating under irradiation conditions. This is due to the fact that these objects use structural elements that undergo a series of physicochemical transformations under the influence of irradiation. In this case, neutron fluxes have the greatest influence. They penetrate deep into the body and cause various structural changes in it. As a result of this effect, Young's modulus changes, strength indicators decrease, shear modules change, and volume changes appear. Moreover, all these changes at different points in the body can manifest themselves in different ways [1].

During operation, structural elements of nuclear power plants are exposed to external loads and volumetric deformation caused by deformation. Therefore, phenomena associated with a change in the mechanical properties of the irradiated body can have a significant effect on the performance of these structures.

Therefore, it is necessary to study the stress-strain state of these structures with a subsequent assessment of their stability parameters, that is, the state in which the loss of the bearing capacity of the structure occurs.

Among the many structures used at nuclear facilities, shell structures, which are characterized by high strength and rigidity, occupy a significant place. However, being thin-walled, such structures are especially susceptible to radiation exposure. Therefore, the tasks associated with the calculation and subsequent evaluation of the stability parameters of these structures during irradiation are of particular relevance.

Determining the stability parameters of structures operating under irradiation conditions is a rather difficult task. In this case, under the influence of external load and irradiation, the body is deformed and its deformation will be determined by the sum of two deformations: the deformation caused by fixing and loading, and the deformation associated with their radiation process. Moreover, if we take into account that the deformation caused by external forces satisfies Hooke's law, and also that both the mechanical properties of the material of the irradiated body and its volume expansion are a function of the coordinate x^k, the radiation dose D, and the physical parameters of body ϑ, then the covariant components of the strain tensor can be represented as follows:

$$e_{ij} = G_{ijkl}(x^k, D, \vartheta)\sigma^{kl} + \theta(x^k, D, \vartheta)g_{ij},$$

where G_{ijkl} is the tensor of elasticity, σ^{kl} are contravariant stress tensor components, θ is the volumetric deformation, g_{ij} is the metric tensor. For an unirradiated body, i.e. for $D = 0$ we have that $\theta(x^k, D, \vartheta) = 0$, and the tensors G_{ijkl} are the elastic constants of the unirradiated body.

In addition, when solving such problems, especially with respect to shell structures, it is necessary to take into account changes in their physical and mechanical properties over the entire thickness of the irradiated body and geometric nonlinearity [2].

Solving a problem of this kind by analytical methods is rather difficult, since in the considered case the solution of the corresponding equations is connected with the solution of a system of nonlinear defining equations and boundary value problems with variable coefficients.

Therefore, it is necessary to develop and apply approximate solution methods to such problems, in particular, variational methods. A variational principle of a mixed type can be proposed, where the variable values are the speeds of movement and the speeds of voltage, and the speed itself was determined as a derivative with respect to the radiation dose [3]. The task of the calculus of variations in this case is to build some three-dimensional functional.Along with the components of the above strain tensors, it would take into account the components of the strain tensor expressed in terms of the components of the displacement vector and finding its stationary value, taking into account the fulfillment of the equilibrium conditions and the requirements of boundary value problems.

The paper presents the construction of a functional based on a variational principle of a mixed type as applied to the study of the stress-strain state of shell structures in three-dimensional space under radiation exposure, a distinctive feature of which is that here the derivatives of the displacement vector and stress tensor with respect to the radiation dose are varied independently. It is proved that the stationary value of the presented functional is achieved for functions describing nonlinear equilibrium equations, physical relations of the irradiated body, and nonlinear boundary conditions. The determining system of equations obtained in this case is a system of quasilinear differential equations, the procedure for solving which by numerical methods on the computer is quite simple.

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SAMPLING THEOREM ON MATHEMATICAL PHYSICS PROBLEMS

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The Nyquist – Shannon – Kotelnikov sampling theorem [1]-[3] connects continuous and discrete signals. The theorem states that a discrete sequence of samples of a continuous signal $f(x)$ going one after another in T seconds, consisting of frequencies from 0 to f_c, $2f_cT=1$, carries all the information about a continuous signal. In other words: a continuous signal $f(x)$, the Fourier transform of which is equal to zero outside the segment $\left[-f_c, f_c\right]$, can be represented as an interpolation series:

$$f(x)=\sum_{k=-\infty}^{\infty} f(k\Delta)\operatorname{sinc}\left[\frac{\pi}{\Delta}(x-k\Delta)\right],\ 0<\Delta\leqslant T, \qquad (1)$$

where $\operatorname{sinc}(x)=\frac{\sin(x)}{x}$.

Based on the Kotelnikov formula (1), we obtain a solution to the Cauchy problem for the heat equation on the infinite axis in the following form:

$$u(t,x)=\sum_{k=-\infty}^{\infty}\frac{\exp\left(-(x-k\Delta)^2/4t\right)}{4\sqrt{\pi t}}\cdot\left(erf\left(\frac{\sqrt{t}}{\Delta}+i\frac{x-k\Delta}{2\sqrt{t}}\right)+\right.$$

$$\left.+erf\left(\frac{\sqrt{t}}{\Delta}+i\frac{x-k\Delta}{2\sqrt{t}}\right)\right)f(k\Delta). \qquad (2)$$

Similarly, we obtained a formula for solution the Dirichlet problem for the Laplace equation in the upper half-plane $y>0$:

$$u(x,y)=\frac{1}{\pi}\sum_{k=-\infty}^{\infty}\left(\frac{(x-k\Delta)\exp(-y/\Delta)\sin\left((x-k\Delta)/\Delta\right)}{(x-k\Delta)^2+y^2}-\right.$$

$$\left.-\frac{y\exp(-y/\Delta)\cos\left((x-k\Delta)/\Delta\right)-y}{(x-k\Delta)^2+y^2}\right)\cdot f(k\Delta). \qquad (3)$$

Our formulas (2) and (3) reveal that the sequence of samples $\{k\Delta\}$ of a continuous signal $f(x)$, consisting of harmonics with frequencies from 0 to f_c, carries all the information about the structure of nonstationary (2) and stationary (3) continuous fields. Formulas (2) - (3) determine the solution by the sum of a series, which will allow us to construct new computational algorithms.

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ON BESSEL PROPERTY OF ROOT VECTOR-FUNCTIONS OF 2M-TH ORDER DIRAC TYPE OPERATOR

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Let $L_p^{2m}(G),\ p \geq 1,\ m \geq 1$ be a space of $2m$-component vector-functions with the norm

$$\|f\|_{p,2m} = \left[\int_G \left(\sum_{j=1}^{2m} \left| f_j(x) \right|^2 \right)^{p/2} dx \right]^{1/p}$$

In the case $p = \infty$ $\|f\|_{\infty,2} = \sup vrai \left| f(x) \right|$. For $f(x) \in L_p^{2m}(G)$, $g(x) \in L_q^{2m}(G)$, where $p^{-1} + q^{-1} = 1$, $1 \leq p \leq \infty$ the scalar product $(f, g) = \int_G \sum_{j=1}^{2m} f_j(x) \overline{g_j(x)} dx$ is determined.

Let us consider $2m$-th order Dirac-type operator

$$Dy = B\frac{dy}{dx} + P(x)y,\ y(x) = \left(y_1(x), y_2(x), \ldots y_{2m}(x)\right)_1^T$$

where

$$B = \begin{pmatrix} 0 & 0 & \cdots & 0\ b_1 \\ 0 & 0 & \cdots & b_2\ 0 \\ \cdots & \cdots & \cdots & \cdots\cdots \\ b_{2m} & 0 & \cdots & \cdots 0 \end{pmatrix} \left(b_1 = b_2 = \ldots = b_m > 0;\ b_{m+1} = \ldots = b_{2m} < 0\right)$$

$P(x) = diag\left(p_1(x), p_2(x), \ldots p_{2m}(x)\right)$, moreover $p_i(x), i = \overline{1,2m}$ are complex-valued functions determined on arbitrary interval $G = (a, b)$ of real straightline.

Following [1], we will understand root functions of the operator D irrespective to the form of boundary conditions, more exactly, under the eigen-function of the operator D responding to the complex eigen-value λ, we will understand any identically non-zero complex-valued vector-function $\overset{0}{u}(x)$, absolutely continuous on any closed subinterval G and almost everywhere in G satisfies the equation $D\overset{0}{u} = \lambda \overset{0}{u}$.

Similarly, under the self-adjoint function $l, l \geq 1$ responding to the same λ and eigen-function $\overset{0}{u}(x)$, we will understand any complex-valued vector-function $\overset{l}{u}(x)$ that is absolutely continuous on any closed subinterval of the interval G and almost everywhere in G satisfies the equation $D\overset{l}{u} = \lambda \overset{l}{u} + \overset{l-1}{u}$.

Theorem: Let G be a finite interval, $p_i(x),\ i = \overline{1,2m}$ belong to the class $L_2(G)$ the length of any chain of root vectors be uniformly bounded and there exist a constant C such that

$$\left|\operatorname{Im}\lambda_k\right| \le C, \ \ k = 1,2,...$$

Then for the Bessel property of the root systems $\left[u_k(x)/\|u_k\|_{2,2m}\right]_{k=1}^{\infty}$ it is necessary and sufficient the existence of the constant M such that

$$\sum_{\left|\operatorname{Re}\lambda_k - \nu\right| \le 1} 1 \le M, \quad \forall \nu \in R$$

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INVESTIGATION OF THE DISTRIBUTION OF THE TIME OF THE SEMIMARKOV RANDOM WALK PROCESS WITH NEGATIVE DRIFT AND POSITIVE JUMPS WITHIN A BAND

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Let the sequance $\{\xi_k(\omega), \zeta_k(\omega)\}_{k\ge1}$ be given on the probability space $(\Omega, F, P(\cdot))$, where the random variables $\xi_k(\omega)$ and $\zeta_k(\omega)$, $k \ge 1$ are independent and identically distributed.

Let us construct the process:

$$X(t,\omega)=z-t+\sum_{i=0}^{k-1}\zeta_i(\omega), \quad if \quad \sum_{i=0}^{k-1}\zeta_i(\omega)\le t<\sum_{i=0}^{k}\xi_i(\omega),$$

where $\xi_0(\omega)=\zeta_0(\omega)=0$.

We denote:

$$R(t,a,\,0|z)=P\{\tau>t|z\},$$

$$\tilde{R}(\theta,a,\,0|z)=\int_{t=0}^{\infty}e^{-\theta t}\,R(t,a,\,0|z)dt.$$

It is obvious that

$$R(t,a,\,0|z)=P\left\{\inf_{0\le s\le t}X(s)>0;\ \sup_{0\le s\le t}X(s)<a\mid X(0)=z\right\}.$$

According to the formula of total probability, we can put it as

$$R(t,a,\,0|z)=P\left\{\inf_{0\le s\le t}X(s)>0;\ \sup_{0\le s\le t}X(s)<a;\ \xi_1>t\mid X(0)=z\right\}+$$
$$+P\left\{\inf_{0\le s\le t}X(s)>0;\ \sup_{0\le s\le t}X(s)<a;\ \xi_1<t\mid X(0)=z\right\}.$$

Then

$$R(t,a,\,0|z)=P\{z-t>0;\ \xi_1>t\}+\int_{s=0}^{t}\int_{y=0}^{a}P\{\xi_1\in ds;\ z-s>0;\ z-s+\zeta_1<a;\ z-s+\zeta_1\in dy\}\times$$
$$\times P\{\inf\nolimits_{0\le u\le t-s}X(u)>0;\ \sup\nolimits_{0\le u\le t-u}X(u)<a\mid X(0)=y\}.$$

From the last equation, we have:

$$R(t,a,\,0|z)=P\{z-t>0\}P\{\xi_1>t\}+$$
$$+\int_{s=0}^{t}\int_{y=0}^{a}P\{\xi_1\in ds\}P\{z-s>0\}dy\,P\{z-s-\zeta_1<y\}R(t-s;\,a;\,0|Y). \tag{1}$$

By applying the Laplace transformation with respect to t to both sides of the equation (1) we have:

$$\tilde{R}(t,a,\,0|z)=\int_{t=0}^{z}e^{-\theta t}\,P\{\xi_1>t\}dt+\int_{y=0}^{a}\tilde{R}(\theta,a,\,0|Y)\int_{t=0}^{\infty}e^{-\theta t}\,P\{\xi_1\in dt\}dy\,P\{\xi_1<y-z+t\},$$

or

$$\tilde{R}(t,a,\,0|z)=\int_{t=0}^{z}e^{-\theta t}\,P\{\xi_1>t\}dt+\int_{y=0}^{a}\tilde{R}(\theta,a,\,0|Y)\int_{t=\max(0;z-y)}e^{-\theta t}\,dP\{\xi_1<t\}dy\,P\{\xi_1<y-z+t\}.$$

From the last equation, we have:

$$\tilde{R}(t,a,\,0|z)=\int_{t=0}^{z}e^{-\theta t}\,P\{\xi_1>t\}dt+\int_{y=z}^{a}\tilde{R}(\theta,a,\,0|Y)\int_{t=0}^{\infty}e^{-\theta t}\,d_tP\{\xi_1<t\}dy\,P\{\xi_1<y-z+t\}+$$

$$+\int_{y=0}^{z}\tilde{R}(\theta,a,\,0|Y)\int_{t=z-y}^{\infty}e^{-\theta t}\,d_tP\{\xi_1<t\}dy\,P\{\xi_1<y-z+t\}.$$

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SOME WEIGHTED INEQUALITIES G - FRACTIONAL INTEGRALS

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In this abstract we reduce a some weighted inequality for G - fractional integrals $J_G^{\alpha,\lambda} f$ associated with Gegenbauer differential operator $G = (x^2 - 1)^{\lambda + \frac{1}{2}} \frac{d}{dx}$.

This results is an analog of Hening's type result. Further, the Stein-Weiss inequality for the G - fractional integrals is proved as an application of obtained results.(see,[1])

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TENTH ORDERED COMPACT FINITE DIFFERENCE SCHEMES FOR ONE DIMENSIONAL HELMHOLTZ EQUATIO USING NEUMANN BOUNDARY CONDITION

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This work is designed to derived Sixth, eight and tenth order compact finite difference schemes for one dimensional Helmholtz equation using Neumann boundary condition.

Numerical experiments was conducted to test the efficiency, accuracy and validity of the proposed shemes. Numerical results obtained from difference orders are compared and also with the exacts solution, Convergence and stability obtained and errors computed using L2 norms. Results shows that the tenth order of accuracy is better than order eight and sixth order while the sixth order of accuracy is better than the fourth order.

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THE PROBLEM OF BENDING AND UNFOLDING SURFACES IN L. EULER'S WORKS

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In the XVIII century the tasks implying the use of the calculus methods in the geometry were holding a high position. The calculus methods were created by Isaac Newton (1643–1727) and Gottfried Wilhelm Leibniz (1646-1716). These tasks lead to the inception of the differential geometry.

Leonard Euler (1707–1783) played an important role in the process of the creation of the differential geometry. This fact has been analysed by many mathematics historians. We performed the analysis of the works by Euler which were the part of the "Opera Omnia", published correspondence of the scientist as well as the materials found in his notebooks which are being kept in the Russian Academy of Sciences, St. Petersburg office. This analysis allowed us to prove the fact that Euler acquired the basic results in the stated area of research.

The issues of cartography, geodesy and mechanics led us to the necessity of the analysis of the spatial problems of Euler's differential geometry. The report will be devoted to the analysis of the issue of bending of one surface into another as Euler regarded it in his works. It is necessary to mention that bending is the deformation of surfaces when the length of an arc of a line drawn on the surface remains unchanged. The problem of bending of the surfaces is an important one because if one surface bends into another then their inner geometry is the same. The unfolding of a surface onto the two-dimensional subspace is a specific instance of bending.

Euler published his results of the analysis of the unfolding surfaces in his work "De solidis quorum superficiem in planum explicare licet" (1771, published in 1772).

It is worth mentioning that Euler acquired the results concerning the unfolding surfaces in 1767 five years before publishing. This is shown by his note in his notebook № 138 sheets 3об.–5об.

He also introduces the notion which now corresponds to the term bending of a surface onto another and sets the task to find the conditions of the bending in the same notebook, sheets 5об.–7.

The results Euler did formulate became the basis for the researches of Gaspard Monge (1746–1818) and Carl Friedrich Gauss (1777—1855).

WEIGHTEDINEQUALITYFOR $B_{k,n}$ FRACTIONAL INTEGRAL

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Suppose that $\mathbb{R}^n$ is the n-dimensional Euclidean space , $x=(x_1,\dots,x_n)$, $\xi=(\xi_1,\dots,\xi_n)$ are vectors in $\mathbb{R}^n$, $x\cdot\xi = x_1\xi_1+\ldots+x_n\xi_n$, $|x|=(x\cdot x)^{1/2}$, $x=(x',x'')$, $x'=(x_1,\ldots,x_k)\in\mathbb{R}^k$, $x''=(x_{k+1},\ldots,x_n)\in\mathbb{R}^{n-k}$,

$$\mathbb{R}^n_{k,+}=\{x\in\mathbb{R}^n; x_1>0,\ldots,x_k>0\},$$

$$\mathbb{R}^k_{++}=\{x\in\mathbb{R}^k : x_1>0\ldots,x_k>0\},$$

$$E(x,r)=\{y\in\mathbb{R}^n_{k,+}; |x-y|<r\},$$

$$E(x',r)=\{y'\in\mathbb{R}^k_{++}: |x'-y'|<r\},\ \gamma=(\gamma_1,\ldots,\gamma_k),$$

$\gamma_1>0,\ldots,\gamma_k>0$, $|\gamma|=\gamma_1+\ldots+\gamma_k$, $(x')^\gamma = x_1^{\gamma_1}\cdot\ldots\cdot x_k^{\gamma_k}$.

An almost everywhere positive and locally integrable function $\omega:\mathbb{R}^n_{k,+}\to\mathbb{R}$ willbe called a weight. We shall denote by $L_{p,\omega,\gamma}(\mathbb{R}^n_{k,+})$ the set of all measurable functions f on $\mathbb{R}^n_{k,+}$ such that the norm

$$\|f\|_{L_{p,\omega,\gamma}} \equiv \|f\|_{p,\omega,\gamma} = \left(\int_{\mathbb{R}^n_{k,+}} |f(x)|^p\ \omega(x)(x')^\gamma\,dx\right)^{1/p},\ 1\le p<\infty.$$

The operator of generalized shift (*Bk,n*-shift operator) is defined by the followingway

$$T^y f(x) = C_{k,\gamma}\int_0^\pi \ldots \int_0^\pi f\left((x',y')_\alpha, x''-y''\right) d\nu(\alpha),$$

where $C_{k,\gamma} = \pi^{-\frac{k}{2}} \prod_{i=1}^{k} \frac{\Gamma\left(\frac{\gamma_i+1}{2}\right)}{\Gamma\left(\frac{\gamma_i}{2}\right)}, \quad (x',x'') \in \mathbb{R}^k \times \mathbb{R}^{n-k},$

$$(x',y')_\alpha = ((x_1,y_1)_{\alpha_1},\ldots,(x_k,y_k)_{\alpha_k}), (x_i,y_i)_{\alpha_i}$$

$$= \sqrt{x_i^2 - 2x_i y_i \cos\alpha_i + y_i^2},$$

$$1 \le i \le k, \quad d\nu(\alpha) = \prod_{i=1}^{k} \sin^{\gamma_i - 1}\alpha_i d\alpha_i, \; 1 \le k \le n.$$

Definition 1. *The weight function ωbelongs to the class* $A_{p,\gamma}(\mathbb{R}^k_{+,+})$ *for* $1 < p < \infty$, *if*

$$\sup_{x\in\mathbb{R}^k_{+,+}, r>0} \left|E(x',r)\right|_\gamma^{-1} \int_{E(x',r)} \omega(y')(y')^\gamma\, dy$$

$$'\left(\left|E(x',r)\right|_\gamma^{-1} \int_{E(x,r)} \omega^{-\frac{1}{p-1}}(y')(y')^\gamma\, dy'\right)^{p-1} < \infty$$

Consider the $B_{k,n}$-fractional integral

$$I^\alpha_{B_{k,n}} f(x) = \int_{\mathbb{R}^n_{k,+}} T^y \, |x|^{\alpha-n-|\gamma|} |\, f(x)| (y')^\gamma\, dy, \quad 0 < \alpha < n + |\gamma|.$$

Теорема 1.3.1.*Let* $1 < p < \frac{n+|\gamma|}{\alpha}$, $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{n+|\gamma|}$.

Then the inequality

$$\left(\int_{\mathrm{R}^n_{k,+}} \left| I^{\alpha}_{B_{k,n}} \left(f \omega^{\frac{\alpha}{n+|\gamma|}} \right)(x) \right|^q \omega(x')(x')^{\gamma} dx \right)^{\frac{1}{q}} \le$$

$$\le c \left(\int_{\mathrm{R}^n_{k,+}} | f(x) |^p \, \omega(x')(x')^{\gamma} dx \right)^{\frac{1}{p}}$$

holds for any $f \in L_{p,\omega,\gamma}(\mathrm{R}^n_{k,+})$ *with a constant* $c > 0$, *independent of* f *if and only, if*

$$\omega \in A_{\beta,\gamma}(\mathrm{R}^k_{+,+}), \qquad \beta = 1 + \frac{q}{p'}.$$

THE SECOND VARIATION IN A VARIATIONAL PROBLEM WITH DELAYED ARGUMENTS

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This paper considers the problem of calculus of variations of the form

$$J(x(\cdot)) = \int_{t_0}^{t_1} L(t, x(t), x(t-\tau), \dot{x}(t), \dot{x}(t-\tau)) dt \to \min_{x(\cdot)}, \quad (1)$$

$$\begin{aligned} x(t) &= \varphi(t), \;\; t \in [t_0 - \tau, t_0], \\ x(t) &= \psi(t), \;\; t \in [t_1 - \tau, t_1], \end{aligned} \qquad (2)$$

where t_0, t_1 and $\tau = const > 0$ are given points in R, moreover $t_1 - \tau > t_0$, and $\varphi(t):[t_0 - \tau, t_0] \to R^n$, $\psi(t):[t_1 - \tau, t_1] \to R^n$

are given twice continuously differentiable functions. Here $x(t) \in C^1([t_0, t_1], R^n)$ is the sought function. For a given function $L(\cdot)$, called an integrand, we assume that, it is twice continuously differentiable in terms of all its variables. The function $x(t) \in C^1([t_0, t_1], R^n)$ that satisfies boundary conditions (2), is called admissible.

First of all, using linearly varying property of an admissible function $\bar{x}(\cdot)$, it is proved that, the second variation in the problem (1), (2) takes the following form

$$\delta^2 J(\bar{x}(\cdot), \delta y(\cdot)) = \int_{t_0}^{t_1} [\delta\dot{y}^T(t)\bar{L}_{\dot{y}\dot{y}}(t)\delta\dot{y}(t) + 2\delta y^T(t)\bar{L}_{y\dot{y}}(t)\delta\dot{y}(t) +$$

$$+\delta y^T(t)\bar{L}_{yy}(t)\delta y(t)]dt\,,$$

where

$$y(t) = (x(t),\, x(t-\tau))^T,\ \bar{y}(t) = (\bar{x}(t), \bar{x}(t-\tau))^T$$

$$\dot{y}(t) = (\dot{x}(t),\, \dot{x}(t-\tau))^T,\ \dot{\bar{y}}(t) = (\dot{\bar{x}}(t), \dot{\bar{x}}(t-\tau))^T$$

$$y(t) - \bar{y}(t) =: \delta y(t) = (\delta x(t),\, \delta x(t-\tau))^T$$

$\bar{L}_{\dot{y}\dot{y}}(t) = L_{\dot{y}\dot{y}}(t, \bar{y}(t), \dot{\bar{y}}(t))^T$, $\bar{L}_{y\dot{y}}(t)$ and $\bar{L}_{yy}(t)$ can be defined analogously.

Then, from the necessary condition $\delta^2 J(\bar{x}(\cdot); \delta y(\cdot)) \geq 0$, $\forall \delta\, y(\cdot)$, the Legendre condition for a weak local minimum is obtained.

In the paper, we also separately investigate the case that the Legendre condition degenerates at least at one point $t^* \in [t_0, \bar{t}_1]$.

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LINEAR INVERSE PROBLEM FOR A THIRD ORDER HYPERBOLIC EQUATION WITH AN INTEGRAL CONDITION

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The last years there is a great interest to studying wave processes in media characterized by the availability of dispersion and absorption. In particular, study of acoustic waves in media where wave propagation is disturbed by the state of thermodynamical and mechanical equilibrium is among these issues [1].

The goal of the paper is to prove the uniqueness and existence of the solution of the inverse boundary value problem for a third order partial differential equation with an integral condition.

For the equation [1]

$$u_{ttt}(x,t)-u_{txx}(x,t)+u_{tt}(x,t)-\alpha u_{xx}(x,t)=a(t)g(x,t)+f(x,t) \quad (1)$$

in the domain $D_T = \{(x,t): 0 \le x \le 1,\ 0 \le t \le T\}$ we consider inverse boundary value problem with initial conditions

$$u(x,0)=\varphi_0(x),\quad u_t(x,0)=\varphi_1(x),\ u_{tt}(x,0)=\varphi_2(x) \quad (0 \le x \le 1), \quad (2)$$

the periodic condition

$$u(0,t)=u(1,t) \ \ (0 \le t \le T), \quad (3)$$

the integral condition

$$\int_0^1 u(x,t)dx=0 \ \ (0 \le t \le T) \quad (4)$$

with additional condition and

$$u(x_0,t)=h(t) \quad (0\le t\le T), \tag{5}$$

where $x_0\in(0,1)$ is a fixed number, $\alpha>0$ is a given number, $f(x,t), g(x,t), \varphi_i(x)(i=0,1,2)$, $h(t)$ are the given functions, and $u(x,t)$ and $a(t)$ are desired functions.

We introduce the denotation

$$\tilde{C}^{2,3}(D_T)=\{u(x,t): u(x,t)\in C^2(D_T), u_{txx}(x,t), u_{ttt}(x,t)\in C(D_T)\}.$$

Definition. *Under the classic solution of the inverse boundary value problem (1)-(5) we understand the pair* $\{u(x,t),a(t)\}$ *of functions* $u(x,t)$, $a(t)$, *if* $u(x,t)\in\tilde{C}^{2,3}(D_T)$, $a(t)\in C[0,T]$ *and the relations (1)-(5) are fulfilled in the ordinary sense.*

Assume that the data of problem (1) - (5) satisfy the following conditions:

1. $\varphi_i(x)\in C^2[0,1], \varphi_i'''(x)\in L_2(0,1)$, $\varphi_i(0)=\varphi_i(1)$,
$\varphi_i'(0)=\varphi_i'(1)=0$,
$\varphi_i''(0)=\varphi_i''(1), \int_0^1\varphi_i(x)dx=0 \ (i=0,1)\cdot$

2. $\varphi_2(x)\in C^1[0,1], \varphi_i''(x)\in L_2(0,1)$, $\varphi_2(0)=\varphi_2(1)$,
$\varphi_2(0)=\varphi_2(1)$, $\int_0^1\varphi_2(x)dx=0\cdot$

3. $f(x,t),\ f_x(x,t)\in C(D_T),\ f_{xx}(x,t)\in L_2(D_T)$ and
$f(0,t)=f(1,t), f_x(0,t)=f_x(1,t), \int_0^1 f(x,t)=0 \ (0\le t\le T)$.

4. $h(t)\in C^3[0,T], h(t)\ne 0 \ (0\le t\le T), \varphi_0(x_0)=h(0)$,
$\varphi_1(x_0)=h'(0)$, $\varphi_2(x_0)=h''(0)$.

Theorem. Let conditions 1-4 be fulfilled. Then for small values of T the problem (1)-(5) has a unique classic solution.

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INVERSE PROBLEM FOR PSEUDO HYPERBOLIC EQUATION OF THE FOURTH ORDER WITH AN INTEGRAL CONDITION

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In this paper, due to the [1], we proved the existence and uniqueness of the solution of the inverse boundary value problem for the pseudo hyperbolic equation of fourth order with integral condition.

Consider for the equation

$$u_{tt} - \alpha u_{ttxx} + \beta u_{xxxx} = a(t)u(x.t) + b(t)u_t(x,t) + f(x,t) \quad (1)$$

in the domain $D_T = \{(x,t): 0 \le x \le 1,\ 0 \le t \le T\}$ an inverse boundary problem with the initial conditions

$$u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x) \quad (0 \le x \le 1) \quad (2)$$

the boundary conditions

$$\begin{aligned} &u_x(0,t) = 0, \ \ u(1,t) = 0 \\ &u_{xxx}(0,1) = 0, \ \ u_{xx}(1,1) = 0 \end{aligned} \qquad (0 \le t \le T) \quad (3)$$

and with the additional conditions

$$u(0,t) = h_1(t), \quad (0 \le t \le T) \quad (4)$$

$$\int_0^1 u(x,t)dx = h_2(t), \quad (0 \le t \le T) \tag{5}$$

where $\alpha > 0,\ \beta > 0$ are the given numbers, $f(x,t)$, $\varphi(x)$, $\psi(x)$, $h_1(t)$, $h_2(t)$ are the given functions, and $u(x,t)$, $a(t)$, $b(t)$ are the required functions. The condition (5) is a non-local integral condition of first kind, i.e. the one not involving values of unknown functions at the domain's boundary points.

Definition. The classic solution of problem (1) – (5) is the pair $u(x,t), a(t), b(t)$ of the functions $u(x,t)$, $a(t)$ and $b(t)$ with the following properties:

1) the function $u(x,t)$ is continuous in D_T together with all its derivatives contained in equation (1);

2) the functions $a(t)$ and $b(t)$ are continuous on $[0,T]$;

3) all the conditions of (1) – (5) are satisfied in the ordinary sense.

First, the given problem is reduced to an equivalent problem in a certain sense. Then, using the Fourier method the equivalent problem is reduced to solving the system of integral equations. The existence and uniqueness of a solution to the system of integral equation is proved by the contraction mapping principle. This solution is also the unique solution to the equivalent problem. Finally, by equivalence, the theorem of existence and uniqueness of a classical solution to the given problem is proved.

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INVERSE SCATTERING PROBLEM FOR LINEAR SYSTEM OF FOUR-WAVE INTERACTION PROBLEM WITH EQUAL NUMBER OF INCIDENT AND SCATTERED WAVES

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The first order strict and non-strict hyperbolic systems on the half-axis in the case of equal number of incident and scattered waves are considered. The uniqueness in the inverse scattering problem (the problem of finding the potential with respect to scattering operator) is studied by utilizing it to Gelfand–Levitan–Marchenko type linear integral equations. The principal difficulty is to determine the sufficient quantity of scattering problems (on the semi-axis for the same hyperbolic system) ensuring the uniqueness of the inverse scattering problem, [1, 2].

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ON BASICITY OF THE SYSTEM OF EXPONENTS IN GRAND-LEBESGUE SPACES

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Let $L^{p)}(-\pi;\pi)$, $1<p<+\infty$, be a grand-Lebesgue space. For $\forall f\in L^{p)}(-\pi;\pi)$ and $\forall\delta>0$ we assume

$$T_\delta f(x)=\begin{cases} f(x+\delta), x+\delta\in[-\pi;\pi] \\ 0, x+\delta\in R\setminus[-\pi;\pi]\end{cases}.$$

We denote by $\tilde{G}^{p)}(-\pi;\pi)$ a linear manifold of function $f\in L^{p)}(-\pi;\pi)$ satisfying the condition $\|T_\delta f-f\|_{p)}\to 0$, $\delta\to 0$. Let $G^{p)}(-\pi;\pi)$ be a closure $\tilde{G}^{p)}(-\pi;\pi)$ in $L^{p)}(-\pi;\pi)$. Let $\omega=\{z\in C:|z|<1\}$ be a unit disk, and $\gamma=\{z\in C:|z|=1\}$ be a unit circumference. Denote the grand-Hardy space $H^+_{p)}$, $p>1$, of analytic functions f in ω satisfying the condition

$$\|f\|_{H^+_{p)}}=\sup_{0<r<1}\|f_r\|_{p)}<+\infty,$$

where $f_r(t)=f(re^{it})$. Let the function $f(z)$ be analytic outside unit disk ω and have finite order at infinitely remote point, i.e. let $f(z)$ have a Laurent expansion of the form

$f(z)=\sum_{k=-\infty}^{m} a_k z^k$, $z\to\infty$, in a neighborhood of the infinitely remote point. f belongs to the class ${}_mH^-_{p)}$, $p>1$, if $\overline{f_0\left(\frac{1}{\bar{z}}\right)}\in H^+_{p)}$, $p>1$, where $f_0(z)=\sum_{k=-\infty}^{-1} a_k z^k$.

It is clear that every function $f\in H^+_{p)}$, $p>1$, has non-tangential boundary values $f^+(e^{it})$ almost everywhere on γ as $r\to 1$. Denote by $L^{p)}_+$ the subspace of $L^{p)}$ generated by the restrictions of the functions from $H^+_{p)}$. The operator $J_+ : H^+_{p)}\to L^{p)}_+$, defined by the formula $J_+ f(\xi)=f^+(\xi)$, $\xi\in\gamma$, performs isomorphism of the spaces $L^{p)}_+$ and $H^+_{p)}$. Let $G^{p)}_+=G^{p)}\cap L^{p)}_+$. Obviously, $G^{p)}_+$ is a subspace of the space $L^{p)}_+$. Now let ${}_mL^{p)}_-$ be a subspace of $L^{p)}$, generated by the restrictions of the functions from ${}_mH^-_{p)}$. Denote ${}_mG^{p)}_-=G^{p)}\cap_m L^{p)}_-$.

The following theorem is true.

Theorem. *A system of exponents* $\{e^{\mathrm{int}}\}_{n\in Z_+}$ *(* $\{e^{-\mathrm{int}}\}_{n\in N}$ *) forms a basis for the space* $G^{p)}_+(-\pi;\pi)$, *(* ${}_{-1}G^{p)}_-(-\pi;\pi)$ *),* $1<p<+\infty$.

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SOLVABILITY OF RIEMANN PROBLEM IN GRAND-HARDY CLASSES

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Let $L^{p)}(-\pi;\pi)$, $1<p<+\infty$, be a grand-Lebesgue space. For $\forall\delta>0$, consider the shift operator

$$T_\delta f(x)=\begin{cases} f(x+\delta), x+\delta\in[-\pi;\pi] \\ 0, x+\delta\in R\setminus[-\pi;\pi] \end{cases}, \ \forall f\in L^{p)}(-\pi;\pi).$$

Denote by $G^{p)}(-\pi;\pi)$ a closure in $L^{p)}(-\pi;\pi)$ of the linear manifold of functions $f\in L^{p)}(-\pi;\pi)$ satisfying the condition $\|T_\delta f-f\|_{p)}\to 0$, $\delta\to 0$.

Let $\omega=\{z\in C:|z|<1\}$ be a unit disk, and $\gamma=\{z\in C:|z|=1\}$ be a unit circumference. Denote by $H^+_{p)}$,

$p>1$, the grand-Hardy space of analytic functions f in ω satisfying the condition

$$\|f\|_{H^+_{p)}} = \sup_{0<r<1}\|f_r\|_{p)} < +\infty ,$$

where $f_r(t) = f(re^{it})$. Every function $f \in H^+_{p)}$, $p>1$, has non-tangential boundary values $f^+(e^{it})$ almost everywhere on γ as $r \to 1$.

Let the function $f(z)$ be analytic outside unit disk ω and have finite order at infinitely remote point, i.e. let $f(z)$ have a Laurent expansion of the form $f(z) = \sum_{k=-\infty}^{m} a_k z^k$, $z \to \infty$, in a neighborhood of the infinitely remote point. f belongs to the class $_m H^-_{p)}$, $p>1$, if $\overline{f_0\left(\frac{1}{\overline{z}}\right)} \in H^+_{p)}$, $p>1$, where $f_0(z) = \sum_{k=-\infty}^{-1} a_k z^k$.

Denote by $L^{p)}_+$ the subspace of $L^{p)}$ generated by the restrictions of the functions from $H^+_{p)}$. The operator $J_+ : H^+_{p)} \to L^{p)}_+$, defined by the formula $J_+ f(\xi) = f^+(\xi)$, $\xi \in \gamma$, performs isomorphism of the spaces $L^{p)}_+$ and $H^+_{p)}$. Let $G^{p)}_+ = G^{p)} \cap L^{p)}_+$ and $GH^+_{p)} = J^{-1}_+(G^{p)}_+)$. Now let $_m L^{p)}_-$ be a subspace of $L^{p)}$, generated by the restrictions of the functions from $_m H^-_{p)}$. Denote $_m G^{p)}_- = G^{p)} \cap_m L^{p)}_-$ and

${}_m GH^-_{p)} = J_-^{-1}({}_m G_-^{p)})$, where the isomorphism $J_- :{}_m H^-_{p)} \to {}_m L_-^{p)}$ is defined by the formula $J_- f(\xi) = f^-(\xi)$, $\xi \in \gamma$.

Consider the following nonhomogeneous Riemann problem in the classes $GH^+_{p)} \times {}_m GH^-_{p)}$:

$$F^+(\tau) - G(\tau)F^-(\tau) = f(\tau),\ \tau \in \gamma, \tag{1}$$

where $G(\tau)$ and $f(\tau)$ are the given functions on the unit circumference γ. By the solution of this problem we mean any pair of functions $F^+(z)$ and $F^-(z)$ belonging to the classes $GH^+_{p)}$ and ${}_m GH^-_{p)}$, respectively, whose boundary values $F^\pm(\tau)$ on unit circumference γ satisfy (1) almost everywhere.

The following theorem is true.

Theorem. *Let the following conditions hold:*

1) $f \in G^{p)}(\gamma)$, $G^{\pm 1}(\tau) \in L_\infty(\gamma)$, $\theta(t) = \arg G(e^{it})$ *is piecewise Holder on* $[-\pi;\pi]$, $\theta(t) = \theta_0(t) + \theta_1(t)$, *where* $\theta_0(t)$ *is a continuous part of* $\theta(t)$, $\theta_1(t)$ *is a jump function of* $\theta(t)$ *at discontinuity points* $-\pi < s_1 < s_2 < \ldots\ldots < s_r < \pi$, *i.e.* $\theta_1(-\pi) = 0$, $\theta_1(t) = \sum_{k:t<s_k} h_k$, $t \in (-\pi;\pi]$, *where* $h_k = \theta(s_k + 0) - \theta(s_k - 0)$, $k = \overline{1,r}$.

2) the relations $-1 + \frac{1}{p} < \frac{h_k}{2\pi} < \frac{1}{p}$, $k = \overline{0,r}$, *are satisfied for the sequence* $\{h_k\}_0^r$, $h_0 = \theta(-\pi) - \theta(\pi)$.

Then the following assertions concerning the solvability of the problem (1) in the classes $GH^+_{p)} \times {}_m GH^-_{p)}$, $p > 1$, *are true:*

α) for $m \geq -1$, the problem (1) has a general solution of the form

$$F(z) = Z_\theta(z) P_k(z) + F_1(z),$$

where $Z_\theta(z)$ is a canonical solution of corresponding homogenous problem, $P_k(z)$ is a polynomial of degree $k \leq m$ ($P_{-1}(z) \equiv 0$), and $F_1(z)$ is a function defined by the formula

$$F_1(z) = \frac{Z_\theta(z)}{2\pi i} \int_\gamma \frac{f(\xi)[Z_\theta^+(\xi)]^{-1}}{\xi - z} d\xi, \; z \notin \gamma,$$

β) for $m < -1$, the problem (1) is solvable if and only if the function $f(\tau)$, $p > 1$, satisfies the orthogonality condition

$$\int_{-o}^{\pi} \frac{f(e^{it})}{Z_\theta^+(e^{it})} e^{ikt} dt = 0, \; k = \overline{1, -m-1}.$$

and the problem (1) has a unique solution $F(z) = F_1(z)$.

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VECTOR-VALUED GENERALIZATION OF CONTINUOUS FRAMES

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Let X and Z be Banach spaces with the norms $\|\cdot\|_X$ and $\|\cdot\|_Z$, respectively. X^* will be the space conjugate to X, and the value of the functional $x^* \in X^*$ at $x \in X$ will be denoted by (x, x^*). By $L(X,Z)$ we denote the Banach space of linear bounded operators $T : X \to Z$.

The concept below is a generalization of a continuous frame in a Banach space.

Definition. Let Ω be some set. The mapping $F : \Omega \to L(Z, X)$ is called a $c\tilde{X}$-frame in Z with respect to Ω if $F(\cdot)z \in \tilde{X}$, $\forall z \in Z$ and $\exists A, B > 0$ such that

$$A\|z\|_Z \le \|F(\cdot)z\|_{\tilde{X}} \le B\|z\|_Z,\ \forall z \in Z. \quad (1)$$

The constants A and B are called the lower and upper bounds of $c\tilde{X}$-frame, respectively. In case where the right-hand side inequality in (1) is true, $F : \Omega \to L(Z, X)$ is said to be $c\tilde{X}$-Besselian in Z with respect to Ω with the bound B.

The theorem below presents a criterion for $c\tilde{X}$ -frameness of a mapping.

Theorem 1. Letthe mapping $F:\Omega \to L(Z,X)$ be such that $F(\cdot)z \in \tilde{X}$, $\forall z \in Z$. Then F is a $c\tilde{X}$ -frame in Z only when there exists a bounded operator $T:\tilde{X}^* \to Z^*$ defined by the formula

$$(z, T\tilde{x}^*) = (F(\cdot)z, \tilde{x}^*),\ \forall \tilde{x}^* \in \tilde{X}^*,\ \forall z \in Z,$$

and $\operatorname{Im} T = Z^*$.

The theorem below concerns the stability of $c\tilde{X}$ -frames in Banach spaces.

Theorem 2. Let $F:\Omega \to L(Z,X)$ be a $c\tilde{X}$ -frame in Z with the bounds A and B. Assume that the mapping $G:\Omega \to L(Z,X)$ is such that $G(\cdot)z \in \tilde{X}$, there exist the numbers $\lambda, \beta, \mu \geq 0$ such that the following conditions hold:

1) $\max\left\{\lambda + \dfrac{\mu}{A}, \beta\right\} < 1;$

2) $\|F(\cdot)z - G(\cdot)z\|_{\tilde{X}} \leq \lambda \|F(\cdot)z\|_{\tilde{X}} + \beta \|G(\cdot)z\|_{\tilde{X}} + \mu \|z\|_Z .$

Then G is a $c\tilde{X}$ -frame in Z with the bounds $\dfrac{A(1-\lambda)-\mu}{1+\beta}$ and $\dfrac{B(1+\lambda)+\mu}{1-\beta}$.

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FRACTIONAL MAXIMAL OPERATOR ON GENERALIZED WEIGHTED MORREY SPACES

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The classical Morrey spaces were introduced by Morrey [1] to study the local behavior of solutions to second-order elliptic partial differential equations. Moreover, various Morrey spaces are defined in the process of study. Guliyev, Mizuhara and Nakai [2, 3, 4] introduced generalized Morrey spaces $M_{p,\varphi}\left(R^n\right)$. Komori and Shirai [5] defined weighted Morrey spaces $L_{p,k}(\omega)$. Guliyev [6] gave a concept of the generalized weighted Morrey spaces $M_{p,\varphi}\left(R^n,\omega\right)$ which could be viewed as extension of both $M_{p,\varphi}\left(R^n\right)$and $L_{p,k}(\omega)$. In [6], the boundedness of the classical operators and their commutators in spaces $M_{p,\varphi}\left(R^n,\omega\right)$was also studied.

We study the boundedness of the fractional maximal operators M_α on generalized weighted Morrey spaces $M_{p,\varphi}(\omega)$

. We find the sufficient conditions on the pair (φ_1, φ_2) with $\omega \in A_{p,q}(R^n)$ which ensures the boundedness of the operators M_α from one generalized weighted Morrey space $M_{p,\varphi_1}(\omega)$ to another $M_{q,\varphi_2}(\omega)$ for $1 < p < q < \infty$ and from the space $M_{1,\varphi_1}(\omega)$ to the weak space $WM_{1,\varphi_2}(\omega)$ for $1 < q < \infty$. In all cases the conditions for the boundedness of the operator μ_Ω is given in terms of supremal type inequalities on (φ_1, φ_2) and ω, which do not assume any assumption on monotonicity of $\varphi_1(x,r), \varphi_2(x,r)$ in r.

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UNILATERAL GLOBAL BIFURCATION FROM INFINITY IN NONLINEAR STURM-LIOUVILLE PROBLEMS WITH A SPECTRAL PARAMETER IN THE BOUNDARY CONDITION

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We consider the nonlinear Sturm-Liouville problem

$$-(py')'-qy=\lambda r(x)y+F(x,y,y',\lambda),\ x\in(0,\ \pi), \quad (1)$$

$$U_1(y)\equiv b_0 y(0)-d_0 p(0)y'(0)=0, \quad (2)$$

$$U_2(\lambda,y)\equiv(a_1\lambda+b_1)y(0)-(c_1\lambda+d_1)p(\pi)y'(\pi)=0, \quad (3)$$

where $\lambda\in R$ is an eigenvalue parameter, $p(x)\in C^1[0,\pi]$, $q(x)\in C[0,\pi]$, $r(x)\in C[0,\pi]$, $p(x),\ r(x)>0,\ x\in[0,\pi]$, b_0, d_0, a_1 b_1, c_1 and d_1 are real constants such that $|b_0|+|d_0|>0$, $a_1d_1-b_1c_1>0.$ The nonlinear term has the form $F=f+g$, where functions $f,g\in C\left([0,1]\times R^3\right)$ and satisfy the following conditions:there exists $M>0$ and sufficiently large $\tau_0>0$ such that

$$\left|\frac{f(x,u,s,\lambda)}{u}\right|\le M,\ \ (x,u,s,\lambda)\in[0,\ \pi]\times R^3,\ |u|+|s|\ge\tau_0;$$

for any bounded interval $\Lambda\subset R$

$$g(x,u,s,\lambda)=o(|u|+|s|)\ \text{ as}\ |u|+|s|\to\infty,$$

uniformly for $x\in[0,\ 1]$ and $\lambda\in\Lambda.$

Let $E=C^1[0,\ \pi]\cap\{y:U_1(y)=0\}$ be a Banach space with the norm $\|y\|_1=\|y\|_\infty+\|y'\|_\infty$, $\|y\|_\infty=\max\limits_{x\in[0,\ \pi]}|y(x)|$, and let S be the subset of E given by

$$S=\{y\in E:|y(x)|+|y'(x)|>0,\ \ x\in[0,\pi]\}.$$

For each $y\in S$ we define $\theta(y,\cdot)$ to be the continuous function on $[0,\ \pi]$ satisfying the conditions $\cot\theta(y,x)=p(x)y'(x)/y(x)$,

$\cot\theta(y,0)= b_0/d_0$.

Subsets $S_{k,\lambda}^{\nu}$, $k\in \mathrm{N}$, $\lambda\in R$, of S with fixed oscillation count we define as follows: $S_{k,\lambda}^{\nu} =\{ y\in S: \theta(y,\pi)=\gamma(\lambda)+k\pi,$ $y(x)>0$ for small $x>0\}$ [1]. Let $S_{k,\lambda}^{-} =- S_{k,\lambda}^{+}$ and $S_{k,\lambda} =S_{k,\lambda}^{+} \bigcup S_{k,\lambda}^{-}$. Moreover, we define the sets S_k^+, S_k^- and S_k as follows: $S_k^+ = \bigcup\limits_{\lambda\in R} S_{k,\lambda}^+$, $S_k^- = \bigcup\limits_{\lambda\in R} S_{k,\lambda}^-$ and $S_k = \bigcup\limits_{\lambda\in R} S_{k,\lambda}$. These sets are disjoint and open in E.

Theorem. *Let* $I_k =[\lambda_k - M/r_0 , \lambda_k + M/r_0]$, $k\in \mathrm{N}$, *where* λ_k *is a* k *-th eigenvalue of the linear problem (1)-(3) with* $g\equiv 0$ *and* $r_0 = \min\{ r(x), x\in[0, \pi]\}$. *Then for each* $k\in \mathrm{N}$ *there exit connected components* C_k^+ *and* C_k^- *of the set of solutions of problem (1)-(3) that contain* $I_k \times\{\infty\}$ *and for which at least one of the following holds: (i)* C_k^{ν}, $\nu\in\{+,-\}$, *meets* $I_m \times\{\infty\}$ *through* $R\times S_m^{\sigma}$ *for some* $(k,\nu)\neq(m,\sigma)$; *(ii)* C_k^{ν}, $\nu\in\{+,-\}$, *meets the set* $\{(\lambda,0) : \lambda\in R\}$ *for some* $\lambda\in R$; *(iii) the natural projection* $P_R(C_k^{\nu})$, $\nu\in\{+,-\}$, *of* C_k^{ν} *onto* $R\times\{0\}$ *is unbounded.*

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ON A QUADRATIC FORM AGREED WITH THE NORM IN ONE ALGEBRAIC EXTENSION

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Last several years they were arosed new methods of investigation in the theories of Diophantine Approximations and Diophantine Equations using computational calcula-tions. In 1981 (see [1]) one had introduced a new algorithm by using of which in $\mathbf{R}^n$ (n is a natural) could defined so called a reduced basis, if was given some other. This basis have some importante properties. The authors applied their method to the question on factorization of polynimials. Later this algorithm used for the complete solution of Diophantine equations using computational calculations. This algorithm used also in disproving the Mertens conjec-ture (see [2]).

The situation of algebraic extensions differs from the case of linear spaces. Despite that the algebraic extension has finite dimensional structure here the scalar product couldn't exist. In this article we introduce a quadratic form in the algebraic extension $\mathbf{Q}(\sqrt[3]{2})$ agreed with the norms of the elements of natural bases. Using associated bilinear form we define a scalar product.

Take the minimal polynomial for the number $\sqrt[3]{2}$ and consider the companion matrix to this polynomial:

$$A=\begin{pmatrix}0&0&2\\1&0&0\\0&1&0\end{pmatrix}.$$

Since the polynomial is irreducible then $\mathbf{Q}(\sqrt[3]{2})\cong\mathbf{Q}(A)$. Everi element $\alpha=c_0+c_1\sqrt[3]{2}+c_2\sqrt[3]{4}$ has a norm

$$N(\alpha)=\det(c_0E+c_1A+c_2A^2).$$

The quadratic form defined by this method has the matrix

$$A=\begin{pmatrix}1&2&4\\2&4&8\\4&18&16\end{pmatrix}$$

in the natural basis.

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ON FRAMES THAT ARE ITERATES OF A MULTIPLICATION OPERATOR

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Investigation of frame properties for families of elements obtained by iterates of operators in one of the central problems in dynamical sampling.

Note that investigation of basicity properties (completeness, Schauder basicity, frameness, etc.) of iterates of operators is problematic even in the case of well known "standard" operators. It has recently been shown that the iterates $\{T_\varphi^n f\}_{n=0}^{\infty}$ of the multiplication operator $T_\varphi f(t) = \varphi(t) f(t)$ cannot be a frame in $L_2(a,b)$ for any measurable function $\varphi(t)$ and square summable function $f(t)$. This fact shows in particular that a systems of the form $\{\varphi^n(t)\}_{n=0}^{\infty}$ cannot be a frame in $L_2(a,b)$ for any measurable function $\varphi(t)$. The classical exponential system shows that the situation changes drastically when one considers systems of the form $\{\varphi^n(t)\}_{n=-\infty}^{\infty}$ instead of $\{\varphi^n(t)\}_{n=0}^{\infty}$. This note is dedicated to the characterization of all frames of the form $\{\varphi^n(t)\}_{n=-\infty}^{\infty}$. It is shown in this note that this problem can be reduced to the following one:

Problem. Find (or describe a class of) all real-valued functions $\alpha(t)$ for which $\{e^{in\alpha(t)}\}_{n=-\infty}^{+\infty}$ is a frame in $L_2(a,b)$.

In this note we give a partial answer to this problem.

Lemma 1. If $\{x_n\}_{n\in N}$ is a frame, then it is bounded.

Lemma 2. Let $\Phi(t)$ be a measurable function on (a,b). Then $\{\Phi^n(t)\}_{n=0}^{\infty}$ is $L_2(a,b)$- bounded sequence if and only if $|\Phi(t)| \le 1$ a.e. on (a,b).

Lemma 3. Let $[p,q] \subset [0,2\pi]$. Then the classical exponential system $\{e^{\text{int}}\}_{-\infty}^{\infty}$ is a 1-tight frame in $L_2(p,q)$ space.

Theorem 1. Let $\varphi(t)$ be a measurable function on (a,b). If $\{\varphi^n(t)\}_{n=-\infty}^{+\infty}$ is a frame in $L_2(a,b)$, then $|\varphi(t)| = 1$ a.e. in

(a,b), i.e. the function $\varphi(t)$ is an exponential function of the form $\varphi(t)=e^{i\alpha(t)}$, where $\alpha(t)$ is real-valued function.

Theorem 2. Let a function $\alpha(x)$ defined on $[a,b]$ be an invertible function, inverse $\xi:[p,q]\to[a,b]$ of which satisfies the following conditions:

1) $\xi(t)$ is absolutely continuous, strictly increasing function on $[p,q]$;

2) $\xi(p)=a$ and $\xi(q)=b$;

3) there are constants $A,B>0$ such that $A\le\xi'(t)\le B$ for all $t\in[p,q]$;

4) $[p,q]\subset[0,2\pi]$.

Then the system $\left\{e^{in\alpha(t)}\right\}_{n=-\infty}^{+\infty}$ is a frame in $L_2(a,b)$.

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NEAREST-NEIGHBOUR INTERACTION WITH QUBIT TRANSFER IN FERMION SPIN CHAIN BASED ON DIFFERENCE EQUATIONS FOR RACAH POLYNOMIALS

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State transfer in a number of the known quantum systems is very important from their further successful application in solution of the advanced quantum information problems. Today, there are number of spin chain models with nearest-neighbour interaction and certain external magnetic field, where, qubit transfer can be achieved perfectly for case, when, N is an arbitrary [1,2]. However, we start from the point that what kind of spin chains of arbitrary N qubits with nearest-neighbour interaction and zero magnetic field one can construct, where, qubit transfer of high fidelity can be achieved [3,4]. Our aim is to construct spin chain, which has analytical solutions

Hamiltonian of spin chain of $(N+1)$ electrons coupled via the nearest-neighbour interaction is the following:

$$\hat{H} = \sum_{k=0}^{N-1} J_k (a_k^+ a_{k+1} + a_{k+1}^+ a_k).$$

Here, J_k is a parameter that exhibits nearest-neighbour interaction between two neighbor k and k+1 electrons.

Through assumption that the ‘state sender’ and the ‘state receiver’ are located at sites s and r of the spin chain under consideration, the transition amplitude of an excitation from site s to site r of the spin chain can be computed by the following time-dependent correlation function:

$$f_{r,s}(t) = \sum_{j=0}^{N} U_{rj} U_{sj} e^{-it\epsilon_j},$$

where, in case of the using pair of recurrence relations for Racah polynomials, U_{rs} being matrix elements of the unitary matrix U, are expressed via these polynomials and ϵ_j are eigenvalues of the single photon modes defined from the following Schrödinger equation:

$$\hat{H}\varphi_j = \epsilon_j \varphi_j, \quad \varphi_j = \sum_{k=0}^{N} U_{kj} |k\rangle.$$

In our case, exact expression of the eigenvalues ϵ_j is the following:

$$\epsilon_{m-k} = -2\sqrt{(\alpha + k + 1)(\beta + k)},$$
$$\epsilon_{m+k+1} = 2\sqrt{(\alpha + k + 1)(\beta + k)},$$
$$k = 0,1, \dots , m, \qquad m = \tfrac{1}{2}(N-1).$$

Assuming that transition from one end of the chain ($s = 0$) to the final end of the chain ($r = N = 2m + 1$), we can obtain more simplified explicit expression of the time-dependent correlation function, which becomes more simpler in case of the time to be equal to the $t \equiv T = \pi/2$:

$$f_{N,0}(\pi/2) = \sqrt{\frac{(\alpha-\delta+2)_m(\alpha+\delta+1)_m}{(\delta)_m(1-\delta)_m}}, \qquad \alpha = -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \dots.$$

One can easily check that expression of the correlation function, obtained in this work, never becomes equal to 1. Therefore, in case of the selection of nearest-neighbour coupling parametersJ_k, corresponding to pair of the recurrence relations for the Racah polynomials, one can assess only the case of the qubit transfer with the high fidelity. This allows observing the law of the breakdown of the perfect qubit transfer under the certain complication of the nearest-neighbour interaction parameter.

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PROBLEMS OF APPROXIMATION IN BESOV SPACES WITH DOMINANT MIXED DERIVATIVES

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In this abstract we study the approximation problems in the Besov $S^{l}_{p,\theta}B(G_\varphi)$ spaces with dominant mixed derivatives.

Definition. Denote by $S^{l}_{p,\theta}B(G_\varphi)$ the space of locally summable functions f on G having the generalized derivatives $D^{l^e} f$ with the finite norm

$$\|f\|_{S^{l}_{p,\theta}B(G_\varphi)} = \sum_{e\subseteq e_n}\left\{\int_{0^e}^{h_0^e}\left[\frac{\left\|\Delta^{m^e}(\varphi(h),G_\varphi)D^{k^e} f\right\|_p}{\prod\limits_{j\in e}\left[\varphi_j(h_j)\right]^{(l_j-k_j)}}\right]^{\theta}\prod_{j\in e}\frac{dh_j}{h_j}\right\}^{\frac{1}{\theta}},$$

where $G \subset R^n$, $1 \le p \le \infty$, $1 \le \theta \le \infty$, $l = (l_1, l_2, \ldots, l_n)$, $l_j \in (0, \infty)$ ($j \in e_n = \{1,2,\ldots,n\}$), and let $l^e = (l_1^e, l_2^e, \ldots, l_n^e)$, $l_j^e = l_j$ $(j \in e \subseteq e_n)$, $l_j^e = 0$ $(j \in e_n \setminus e)$ and $\varphi(t) = (\varphi_1(t_1), \ldots, \varphi_n(t_n))$, $\varphi_j(t_j) > 0$, $(t_j > 0)$ is continuously differentiable functions; $\lim\limits_{t \to +0} \varphi_j(t_j) = 0$, $\lim\limits_{t \to +\infty} \varphi_j(t_j) = P_j \le \infty$;

$$G_{\varphi(t)}(x) = G \cap I_{\varphi(t)}(x) = G \cap \left\{ y : \left| y_j - x_j \right| < \frac{1}{2} \varphi_j(t_j), (j = 1,2,\ldots,n) \right\}.$$

The following theorem is proved.

Theorem. Let $1 < p < \infty$ and $f \in S^l_{p,\theta} B(G_\varphi)$. Then there exist the functions $h_s = h_s(x)$ (s=1, 2,…) infinitely differentiable finite in R^n such that

$$\lim_{s \to \infty} \| f - h_s \|_{S^l_{p,\theta} B(G_\varphi)} = 0.$$

ON A BOUNDARY VALUE PROBLEM FOR THE EQUATION OF A VIBRATING ROD WITH SPECTRAL PARAMETER CONTAINED LINEARLY IN THE BOUNDARY CONDITIONS

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We consider the following eigenvalue problem

$$y^{(4)}(x) - (q(x)y'(x))' = \lambda\, y(x), \; 0 < x < 1, \tag{1}$$

$$y(0) = y'(0) = 0, \tag{2}$$

$$y''(1) = (a_1\lambda + b_1)\, y'(1), \tag{3}$$

$$Ty(1) = (a_2\lambda + b_2)\, y(1), \tag{4}$$

where $\lambda \in C$ is a spectral parameter, $q(x)$ is positive and absolutely continuous function on $[0, 1]$, $a_i, b_i, i = 1,2,$ are real constants such that $a_i > 0$ and $b_i < 0,\ i = 1,2.$

Problem (1)-(4) is associated with a specific problem of mechanics, namely, this problem arises when variables are separated in the boundary value problem describing bending vibrations of a homogeneous rod, in cross-sections of which the longitudinal force acts, the left end is fixed rigidly, the right end is fixed elastically and on this end the inertial mass is concentrated (see [1, p. 152-154]).

We introduce the boundary condition

$$y(1)\cos\delta - Ty(1)\sin\delta = 0, \tag{5}$$

where $\delta \in [\ \pi/2,\ \pi]$.

Alongside with the spectral problem (1)-(4) we consider the spectral problem (1)-(3), (5). Problem (1)-(3), (5) in a more general form was studied in [2], where, in particular, it is proved

that (see proof of [2, Theorem 2.2]) there exists an infinitely increasing sequence $\{\lambda_k(\delta)\}_{k=1}^{\infty}$ of real and simple eigenvalues of this problem such that $\lambda_k(\delta) > 0$ for $k \geq 2$.

Let $D_k = (\lambda_{k-1}(0), \lambda_k(0)),\ k \in \mathrm{N},$ where $\lambda_0 = -\infty$.

Theorem 1. *One of the following statements is true: (i) all eigenvalues of problem (1)-(4) are real; in this case,* D_1 *contains either two simple eigenvalues or one double eigenvalue, and* $D_k,\ k = 2, 3, \ldots,$ *contains one simple eigenvalue; (ii) all eigenvalues of problem (1)-(4) are real; in this case,* D_1 *contains* *no eigenvalues, and there exists a positive integer* $m_0 \geq 2$ *such that* D_{m_0} *contains either three simple eigenvalues, or one double eigenvalue and one simple eigenvalue, or one triple eigenvalue, and,* D_k, $k = 2, 3, \ldots,$ $k \neq m_0$, *contains one simple eigenvalue; (iii) problem (1)-(4) has one pair of nonreal complex conjugate eigenvalues; in this case,* D_1 *contains no eigenvalues, and* D_k, $k = 2, 3, \ldots,$ *contains one simple eigenvalue.*

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ON ONE PROBLEM OF MULTICRITERIA OPTIMIZATION BASED ON A STOCHASTIC NETWORK MODEL

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In the present paper, we consider the task of developing a criterion for the optimality of plans based on a stochastic network, taking into account the duration, cost and probability of the occurrence of work. An analysis of a large number of projects shows that the complex of interrelated work has to be presented in the form of a stochastic network model "AND / OR". In this model, vertices are jobs, arcs are logical connectives between jobs. Vertices are divided into disjunctive, conjunctive and unary [1]. Works are characterized by normative estimates of the duration, cost and probability [2]. If the original network is based on the “work-communication” principle, then it is advisable to convert it into “event-work" type model. In the resulting model, each arc (i,j) will correspond to some work, where i is the number of the beginning, j is the number of the end of the arc [3]. Alternative solutions (plans) are “AND” subnet with “event-work” network. These are random events that are described by sets of corresponding arcs: $R_1 = \{(i,j)_1\}$, $R_2 = \{(i,j)_2\}, \dots,$ $R_n = \{(i,j)_n\}$.

Each of R_i is associated with some set of works: $R_1', R_2', \dots, R_n'$. The sets R_i' are prototypes of R_i , and do not reflect the sequence of performance of works, which are described by the sets R_i . However, this pair of sets fully discloses all possible solutions.

The set of all such alternative plans is denoted by $\{G_1, G_2, \dots, G_n\}$. The parameters $P(G_i), T(G_i), S(G_i)$ are respectively the probability expressed as a percentage, the duration and cost of the plan G_i [4].

Classic problems of finding of the optimal plan:

1) by criterion P: $P(G_{i_0}) = \max_i P(G_i)$; 2) by criterion T: $T(G_{i_0}) = \min_i T(G_i)$; 3) by criterion S :

$$S(G_{i_0}) = \min_i S(G_i) .$$

We introduce a generalized criterion for identifying the "best" solution:

$$\min_i \{T(G_i) + S(G_i) - P(G_i)\} \tag{1}$$

To construct recursive formulas, we introduce the parameter Q_i, which includes all the arcs for the plan, optimal by criterion (1) with the initial vertex i, and the final vertex k. Thus, Q_1 provides the structure of the desired optimal plan.

Below, we present an algorithm for finding the "optimal solution" by criterion (1), which is implemented starting from the terminal vertex to minorant of the "event-work" type network. In these formulas, $P(i)$, $T(i)$, $S(i)$ are discrete functions of i.

1) The following formulas apply to the terminal vertex "k" :

$$Q_k = \varnothing,$$

$$P(k) = 1$$

$$T(k) = 0$$

$$S(k) = 0$$

2) If a vertex with number i, which has child vertices $i_1, i_2 \ldots, i_m$ is disjunctive, then:

$$\min_j \{s(i,i_j)+S(i_j)+t(i,i_j)+T(i_j)-p(i,i_j)P(i_j)\}\equiv$$
$$s(i,i_0)+S(i_0)+t(i,i_0)+T(i_0)-p(i,i_0)P(i_0)\}$$

$$P(i)=p(i,i_0)P(i_0)$$

$$T(i)=t(i,i_0)+T(i_0)$$

$$S(i)=s(i,i_0)+S(i_0)$$

$$Q_i = (i,i_0)\bigcup Q_{i_0}$$

3) If the vertex i is not disjunctive, i.e. with $m=1$ it is unary, , when $m>1$ it is conjunctive, then

$$Q_i = \bigcup_{j=1}^{m}(i,i_j)\bigcup_{j=1}^{m}Q_{i_j}$$

$$T(i)=max_j\{t(i,i_j)+T(i_j)\}, \quad P(i)=\prod_j P(i_j),$$
$$S(i)=\sum_{(i,j)\in Q_i'} s(i,j),$$

where $Q_i'=\{(i,j)\}$ is the prototype of Q_i and analogue of the set R_i'.

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INTEGRAL MODELING OF GAS WELL OPERATION TAKING INTO ACCOUNT RESERVOIR DEFORMABILITY

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Introduction Determination of bottom hole pressure and field of reservoir pressure distribution by wellhead conditions in gas wells with abnormally high reservoir pressure is of practical and scientific importance. A strict solution to this problem is to account for the interaction of the reservoir-well system. In this case, it is necessary to consider a system of equations describing the joint flow of gas in the reservoir and the wellbore [1-2].

In this work, an integral model of unsteady filtration and gas flow in a pipe is constructed taking into account the deformability of the reservoir and the resulting differential equations are solved.

The differential equation of the piezoconductivity of a plane-radial gas flow has the form [3-5].

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\chi(P) r \frac{\partial P^2}{\partial r}\right) = \frac{\partial P^2}{\partial t}, \qquad r_c \le r \le R_k, \quad r > 0 \tag{1}$$

where
$$\chi(P) = \frac{k(P) P_k}{\mu m_0}.$$

The boundary and initial conditions will be as follows:

$$P^2\Big|_{r=R_k} = P_k^2 \;, \tag{2}$$

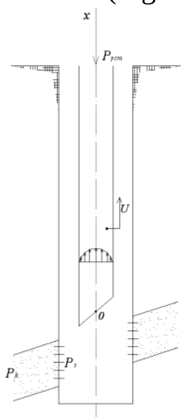
$$P_c^2\Big|_{r=r_c} = P_c^2(t) \;, \tag{3}$$

$$P^2(r,0) = P_k^2 + \frac{Q_0 \mu P_{at}}{\pi k h_1} \ln\left(\frac{r}{R_k}\right) \;. \tag{4}$$

The deformation of the reservoir depends on the gas pressure. As the pressure changes, the permeability of the formation also changes. In the first approximation, we take a linear change in the permeability of the reservoir depending on pressure [6-7].

$$\chi(P) = \chi_1 + \frac{\chi_1 - \chi_0}{P_k - P_0}(P - P_k)\,. \tag{5}$$

Next, consider the movement of gas in the pipe. The equation of gas motion in a pipe has the form (Fig. 1) [8].



(Fig. 1)

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} - 2h \frac{\partial u}{\partial t} - g \,, \tag{6}$$

The boundary and initial conditions will be as follows:

$$\left.\frac{\partial u}{\partial x}\right|_{x=l} = 0, \quad t \geq 0 \,, \tag{7}$$

$$u\big|_{x=0} = 0, \qquad t \geq 0 \,, \tag{8}$$

$$f \left.\frac{\partial u}{\partial t}\right|_{t=0} = Q_0 \,, \tag{9}$$

$$u(0, x) = -\frac{\rho_\Gamma g x^2}{2E}, \quad 0 \leq x \leq l \,. \tag{10}$$

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DYNAMICS OF VARIABLE SECTION ELASTIC BAR INITIAL –BOUNDARY VALUE PROBLEM

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The given class is considered in practice when studying dynamics in atmosphere of axially-symmetric rotation bodies deformable in the motion process. For solving the problem it is necessary to derive the equation of motion of the bar in inertial system of coordinates and also the equation of its elastic vibrations.

In the paper the initial-boundary value problem for of a variable section elastic bar was formulated and solved on the basis of vector mechanics of deformable systems . By deriving equations, a number of assumptions were made:

-the bar body deforms as in elastic inhomogeneous bar, and when calculating its flexural deformations we procced from the corresponding material resistance equations ;

-the bar's particles lying at the initial moment in its any cross-section, in the process of motion of the bar do not change their mutual arrangement. In conformity to the bar, this assumption coincides with the "flat sections hypothesis" oftenly used in the course of resistance of materials. This assertion also excludes possibility of taking into account real deformations of structural elements fastened to the bearing body , and deformation of fastening nodes.

-the bar possesses sufficient margin of safety

-torsional and longitudinal vibrations of the body are not taken account (therefore, in the text under elastic vibrations we understand lateral vibrations of the bar);

-we ignore influence of shear deformations and inertia of rotational displacements of cross-sections since their influence becomes perceptible only for bars with lateral sizes commeasurable with their length;

- when calculating vibrations damping we proced from the hypothesis of " viscous" friction.

For obtaining the equations of motion, we consider the results of imposition of the following motions: translational motion with a pole, rotational with respect to a pole elastic vibrations with respect to non-deformed state.

Differential equations describing the motion of the bar in a vertical plane are obtained based on the theorems of mechanics on the amount of motion and momentum of the amount of motion and also Hamilton-Ostrogradsky variational principle. Initial conditions are determined for the formulated equations of motion.

For calculating the bar motion parameters as an absolute body, the solution of the system of first order differential equations is sought . For obtaining the numerical method , the fourth order Runge- Kutta method is offered.

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SOLUTION OF A NONLOCAL PROBLEM FOR A LINEAR DIFFERENTIAL HYPERBOLIC EQUATIONS

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1. Statement of the problem. The following problem for a differential equation of hyperbolic type is studied in this paper: find continuous in a closed domain $\overline{D}=\{0\le x\le l,\ 0\le t\le T\}$ a function $u=u(x,t)$, that satisfies the equation

$$\frac{\partial^2 u(x,t)}{\partial t^2}=a^2\frac{\partial^2 u(x,t)}{\partial x^2}+bu(x,t)+f(x,t),\ 0<x<l,\ 0<t\le T, \quad (1)$$

boundary conditions

$$\alpha_0\frac{\partial u(0,t)}{\partial x}+\alpha_1\frac{\partial u(l,t)}{\partial x}+\alpha_2 u(0,t)+\alpha_3 u(l,t)=\mu_1(t),$$
$$0\le t\le T \quad (2)$$
$$\int_0^l c(x)u(x,t)dx=\mu_2(t),$$

and initial conditions

$$u(x,0)=\varphi_1(x),\ \frac{\partial u(x,0)}{\partial t}=\varphi_2(x),\quad 0\le x\le l. \qquad (3)$$

Here $f(x,t),\ c(x),\ \mu_1(t),\mu_2(t),\varphi_1(x),\ \varphi_2(x)$ are the known continuous functions of their arguments, $a,\ b,\alpha_0,\alpha_1,\alpha_2,\alpha_3-$ given real numbers. Note that the second condition in (2) is a nonlocal condition that presents a certain difficulty in solving this problem.

It is assumed that problem (1) - (3) has a unique solution that has the derivatives are needed during presentation.

2.Reducing the solution of the problem (1) - (3) to the solution of the problem with local boundary conditions.

Let's consider the second boundary condition in (2) and differentiate it twice by t :

$$\int_0^l c(x)\frac{\partial^2 u(x,t)}{\partial t^2}dx = \mu_2''(t).$$

If we replace the expression $\dfrac{\partial^2 u(x,t)}{\partial t^2}$ under the integral sign by the right-hand side of equation (1) and apply the formula of integration by parts to the integral $\int_0^l c(x)\dfrac{\partial^2 u(x,t)}{\partial x^2}dx$, then at $c(x)=c_1x+c_2$ or $c(x)=c_1\cos kx+c_2\sin kx$, or $c(x)=c_1e^{kx}+c_2e^{-kx}$, where $c_1, c_2 -$ arbitrary constants, we obtain a boundary condition of the form

$$\beta_0\frac{\partial u(0,t)}{\partial x}+\beta_1\frac{\partial u(l,t)}{\partial x}+\beta_2 u(0,t)+\beta_3 u(l,t)=\bar{\mu}_2(t), \quad (4)$$

where $\beta_0, \beta_1, \beta_2, \beta_3 -$ real numbers, $\bar{\mu}_2(t)-$ known continuous function.

From the first boundary condition in (2) and condition (4), eliminating at first $\dfrac{\partial u(l,t)}{\partial x}$, then $\dfrac{\partial u(0,t)}{\partial x}$, it is easy to obtain boundary conditions of the form

$$\begin{aligned}&\frac{\partial u(0,t)}{\partial x}+\gamma_1 u(0,t)+\gamma_2 u(l,t)=\overline{\mu}(t),\\&\frac{\partial u(l,t)}{\partial x}+\delta_1 u(0,t)+\delta_2 u(l,t)=\widetilde{\mu}(t).\end{aligned}\qquad 0\le t\le T. \quad (5)$$

Thus, finding a solution of the problem (1) - (3) reduced to finding a solution of the problem (1), (5), (3).

3. The difference problem. On the grid $\omega_{h\tau}=\{(x_n,t_j),x_n=nh,t_j=j\tau,n=0,1,\dots,N,\ j=0,1,\dots,j_0,\ Nh=l,\ j_0\tau=T\}$, defined in domain $\overline{D}$, the problem (1), (5), (3) is matched the following difference problem:

$$\begin{aligned}&y^j_{\bar{t}t,n}=a^2\Lambda\left(\sigma y_n^{j-1}+(1-2\sigma)y_n^j+\sigma y_n^{j+1}\right)+by_n^j+f_n^j,\\&n=1,2,\dots,N-1,\ j=1,2,\dots,j_0-1,\end{aligned}\qquad (6)$$

$$\begin{aligned}&-\frac{h}{2a^2}\left(y^j_{\bar{t}t,0}-by_0^j\right)+\sigma y^{j+1}_{x,0}+(1-2\sigma)y^j_{x,0}+\\&+\sigma y^{j-1}_{x,0}+\gamma_0 y_0^j+\gamma_1 y_N^j=\overline{\mu}^j-\frac{h}{2a^2}f(0,t_j),\\&\frac{h}{2a^2}\left(y^j_{\bar{t}t,N}-by_N^j\right)+\sigma y^{j+1}_{\bar{x},N}+(1-2\sigma)y^j_{\bar{x},N}+\\&\sigma y^{j-1}_{\bar{x},N}+\delta_0 y_0^j+\delta_1 y_N^j=\widetilde{\mu}^j+\frac{h}{2a^2}f(l,t_j),\end{aligned}\qquad (7)$$

$$y_n^0=\varphi_1(x_n),\ y^0_{t,n}=\overline{\varphi}_2(x_n). \qquad (8)$$

Here

$$f_n^j=f(x_n,t_j),\overline{\mu}^j=\overline{\mu}(t_j),\widetilde{\mu}^j=\widetilde{\mu}(t_j),$$

$$\overline{\varphi}_2(x_n)=\varphi_2(x_n)+\frac{\tau}{2}\left(a^2\varphi_1''(x)+b\varphi_1(x)+f(x_n,0)\right),$$

$\sigma -$ real parameter. Note that in these equalities we use the notation adopted in [1].

The difference problem (6) - (8) approximates problem (1), (5), (3) with accuracy $O(h^2+\tau^2),$ if equation (1) is satisfied both at the boundaries $x=0$ and $x=l$ of the region and the solution $u=u(x,t)$ is a sufficiently smooth function in the considering region $\overline{D}$.

The constructed difference problem (6) - (8) is reduced to the canonical form

$$By_{\overset{0}{t}}+\tau^2 Ry_{\bar{t}t}+Ay=\varphi(t),\ y(0)=g_0,\ y(\tau)=g_1 \qquad (9)$$

and the stability of this difference problem is investigated. Sufficient conditions for stability the difference problem (9) are obtained.

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TRANSFORMATION OPERATORS FOR THE STURM–LIOUVILLE OPERATORS WITH GROWING POTENTIALS

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In many aspects of the spectral theory an important role is played by so-called transformation operators (see [1] and the references therein). These operators arose from the general ideas of the theory of generalized shift operators created by Delsarte [2].

Let E is the space of complex-valued functions $f(x), 0 < x < \infty$, which are continuous and with a continuous second derivative and satisfying the conditions

$$f(x) \in C^{(2)}(0,\infty),\ \sup_{x\in(0,\infty)} \{|f(x)| + |f'(x)| + |f''(x)|\} < \infty .$$

The topology on E is defined via the uniform convergence of the functions and their first and second derivatives in each interval $[\varepsilon, \infty), \varepsilon > 0$.

We consider two operators

$$A = -\frac{d^2}{dx^2} + cx^\alpha + q(x),\ B = -\frac{d^2}{dx^2} + cx^\alpha ,$$

where $\alpha > 0$, c is complex number and complex-valued function $q(x)$ satisfy the conditions

$$q(x) \in C^{(1)}(0,\infty),\ \int_\varepsilon^\infty \left| x^{2\alpha+3} e^{\xi(x)} q(x) \right| dx < \infty,\ \varepsilon > 0 , \quad (1)$$

$$\omega(x) = \left(\frac{|c|}{\alpha+1} \right)^{\frac{1}{2}} (2x)^{\frac{\alpha+2}{2}} .$$

The present work is devoted to the question of the existence of a transformation operator for a pair of operators A and B with a boundary condition at infinity. A similar problem for the case $c = 1, \alpha = 1$ and $c = \pm 1, \alpha = 2$ was studied in the works $[12] - [14]$. Let $\sigma_0(x) = \int_x^\infty |q(t)| e^{\xi(t)} dt, \sigma_1(x) = \int_x^\infty \sigma_0(t) dt$.

Theorem. *If potential $q(x)$ satisfies condition (1), then the transformation operator X for a pair of operators A and B can be realized in the form*

$$Xf(x) = f(x) + \int_x^\infty K(x,t)\, f(t)dt.$$

It's kernel $K(x,t)$ is continuous function and satisfy the following conditions:

$$|K(x,t)| \le \frac{1}{2}\sigma_0\left(\frac{x+t}{2}\right)e^{\sigma_1\left(\frac{x+t}{2}\right)},$$

$$K(x,x) = \frac{1}{2}\int_x^\infty q(t)dt.$$

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DISTURBANCES ON THE SURFACE OF STRATIFIED FLOW OF TWO INCOMPRESSIBLE VISCOUS LIQUIDS IN NARROW CHANNEL.

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This paper explores the problem of mathematical modeling of the dynamics of the wave movements of various heterogeneous natural stratified mediums.

In a narrow channel $0 \le y \le h(x,t)$ the flow of a viscous incompressible fluid. Layer stratified fluid of variable depth $h(x,t)$ consists of fluid density ρ_1, viscosity μ_1, depth $h_1(x,t)$ \

and fluid density ρ_2 ($\rho_1 > \rho_2$), viscosity μ_2, depth $h(x,t) = h_1(x,t) + h_2(x,t)$.

Free surface and interface fluid in equilibrium in a gravity field is flat, i.e.

$$h_1(x,0) = h_{10}, \qquad h_1(x,0) + h_2(x,0) = h_{10} + h_{20} = h_0$$

The boundary conditions of the problem are

$$u_1(x,0,t) = 0, \; v_1(x,0,t) = 0$$

The coincidence of the velocity of the surface and the particles of the liquid and pressures in $y = h_1(x,t)$, $y = h_1(x,t) + h_2(x,t)$

$$\frac{\partial(h_1+h_2)}{\partial t}+u_2(x,h_1+h_2,t)\cdot\frac{\partial(h_1+h_2)}{\partial x}=v_2(x,h_1+h_2,t)$$

the lack of friction on the free surface of the channel and the coincidence of the pressure of liquid and gas where $y=h_1(x,t)+h_2(x,t)$

$$\frac{\partial u_2(x,h_1+h_2,t)}{\partial y}=0, \qquad p(x,h_1+h_2,t)=p_0$$

In contrast to the single-layer flow, in the case of a two-layer flow, we obtain a fourth-order dispersion equation that has solutions:

$$\left(\frac{k_*}{\omega}\right)_I^2=\frac{\sqrt{\mu a\varphi_0}}{\phi_0} \qquad \left(\frac{k_*}{\omega}\right)_{II}^2=-\frac{\sqrt{\mu a\varphi_0}}{\psi_0}$$

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MASTER'S PROGRAM «PROFESSIONAL ORIENTED MATHEMATICS EDUCATION»

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Master's program «Professionally-oriented teaching of mathematics» [1] has implemented in Saratov National Research State University since 2016. In connection with the approval of the new Federal State Educational Standard (FSES) for Higher Education in the direction of training 44.04.01 Pedagogical Education [2], it seems relevant to review existing educational programs for the training of mathematics teachers for a professional school. It's necessary bring this program into line with the approved FSES [2] and the Professional Standard (PS) «Teacher of vocational training, vocational education and continuing education» [3]. The report will present a theoretical justification, practical development and analysis of the results of testing the specified master's program. The program is aimed to preparing a graduate who is able to successfully work in the field of professional oriented mathematical education, on the basis competencies from the FSES [2] and labor functions 3.1, 3.4 and 3.8 from the PS [3]. The program is focused on the pedagogical, design, and research types of professional activity from the FSES

[2], which correlate with next labor activities of the functions in PS [3]. The pedagogical type of tasks corresponds to the functions: 3.1.1, 3.1.2, 3.8.1, 3.8 .2, 3.4.1, 3.4.2; the design type of tasks corresponds to the labor actions of functions 3.1.3, 3.8.3; the research type of tasks corresponds to the labor actions of functions 3.1, 3.4, 3.8. Master's program establishes next professional competencies (PC). PC-1. He/she able to carry out teaching courses, disciplines (modules) in mathematics according to vocational training programs (VTP), secondary vocational education (SVE), higher education (undergraduate level) (HE), and additional pre-vocational educational programs (APEP) into account the principle of professional orientation. He/she able use modern educational technologies corresponding to the personality and age characteristics of students, including students with specific education needs. PC-2. He/she able to organize research, design, educational, professional and other activities of students studying courses, disciplines (modules) in mathematics in the programs VTP, SVE, HE, APEP. PC-3. He/she able to develop educational and methodological support for the implementation of training courses, disciplines (modules) in mathematics according to the programs VTP, SVE, HE, APEP. PK-4. He/she possesses the skills of independent research in the field of mathematical education.

The preparation of future mathematics teachers for professional activities in the considered labor functions is structurally represented by three interconnected modules. The first (theoretical) training module consists of educational disciplines aimed to mastering the skills of designing and implementing professional oriented mathematical education in a specially organized educational and laboratory environment. The preparation of a mathematics teacher for professional activities of labor functions 3.1.1, 3.1.2, 3.4 and 3.8.1 from PS [3]. Such preparation ensures the formation of PC-1 and is described in article [4]. The subject base of the module is constituted by the disciplines: «Selected chapters of higher mathematics», etc. In the methodical section of this module are being studied «Theory and Methods of Teaching Mathematics in the Professional Education System» and others. The section of the theoretical module is devoted to design that corresponds to labor functions 3.1.3 and 3.8. 4, forms PC-3. This section is described in the paper [5] and is presented by the discipline «Pedagogical design in the field of professional activity» and others. The study of the theoretical module is supported by textbooks [6; 7 and others.]. The training of a mathematics teacher for professional activities in the labor function 3.8.2 ensure the formation of PC-2. This training is carried out by study of the discipline «Theory and Methods of Teaching Mathematics in the System of Professional Education» and is supplemented by practice. The labor function 3.4 is formed by study of the discipline «Educational activities of the teacher». Training and practices make up the second (practical) training module and form the practical readiness of the undergraduate to perform the duties of a mathematics teacher in a real educational organization [8]. The third module (research work) involves independent pedagogical research made by undergraduate which correspond to the formed functions [9]. Verification of the required competencies from FSES [2] and labor functions from PS [3] for students is carried out through an intermediate and final state certification [10]. It also takes into account the feedback of employers, expert assessments of the results of research work of students and self-esteem of graduates of the program.

The presented program is unique, interdisciplinary and competitive. At the same time, certain risks are possible during its implementation, both on the part of the university and on the part of the individual. To achieve reduction of those risks is possible only after the exact determination of the degree of outgoing threats. As the results of the experiment showed, such an organization of training of future mathematics teachers effectively contributes to the formation of their required competencies and labor functions. The level of residual knowledge of students is at least 80%. The average level of satisfaction with the quality of education in the master's program is 84,2%. We associate further prospects for the implementation of the program with a more intensive use of software educational resources as a means of supporting self-education of students.

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APPLIED MATHEMATICS EDUCATION: HISTORY AND MODERNITY

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The history of mathematics in Russia in the first half of the XIX century, was marked by major achievements that had a great influence on world science. One of the important prerequisites for the beginning of a new rise in mathematics were changes in the education system, which occurred at the beginning of the XIX century. Particular importance was the organization of new universities and the creation of physics and mathematics faculties [3]. Universities were opened in six cities: Moscow (since 1755 there was already a Moscow University); Tartu (1802); Vilnius (1803); Kharkiv (1805); Kazan (1805); St. Petersburg (1819). In 1832.the University in Vilnius was closed, and in 1834 the University in Kiev was established instead. In the XVIII century, a few specialists-mathematicians in Russia were trained at the Academy of Sciences. By the end of the XVIII century, Academic institutions had almost ceased operation, and in the beginning of XIX century was closed in connection with the overall reform of the public education system. At Moscow University, the first half-century of its existence, teaching mathematics did not go beyond elementary courses. In the military and naval schools taught the calculation of infinitely small and higher geometry, but the specialists of mathematics came out of such schools rarely. An important feature of the new system of educational institutions was its continuity: from the district school it was possible to go to the gymnasium, and gymnasium training was sufficient to start University classes. The other side of the reform was the new division of universities and the expansion of their programs.

The development of industry, economy, agriculture and other spheres of human activity has always needed the practical implementation of innovative applied research. The most important condition for the implementation of such projects is the availability of University training of highly professional, initiative specialists, including in the field of applied mathematics, who are able to independently develop and competently implement science-intensive, environmental technologies in practice. This circumstance largely explains the creation in the independent states (CIS) in the late 1960s — early 1970s. areas of training in applied mathematics. In the drafting of the first curriculum of the Department of applied mathematics and mechanics active involvement of academicians M. A. Lavrentyev, G. I. Marchuk, L. V. Ovsyannikov, N. N. , Yanenko, A. P. Ershov, V. N. The monks and they developed a position on the faculty of applied mathematics further has played a major role in the formation and development of many faculties and departments of applied mathematics in higher educational institutions of the CIS.
At present, the great role of applied mathematics in the development of world science and in the system of human knowledge is well known. The fundamental basis of applied mathematics was laid in the researches of H. Huygens, I. Newton, D. Bernoulli, L. Euler, A. C. Clairaut, J. L. D'alembert, J. B. J. Fourier, S. D. Poisson, M. V. Ostrogradsky, D. G. Stokes, O. Reynolds, N. E. Zhukovsky, A. N. Krylova, V. A. Steklov, S. A. Chaplygin and other scientists. Fundamental studies A. S. Alekseev, A. A. Andronov, A. A. Babaev, S. N. Bernstein, O. M. Belotserkovskii, E. P. Velikhov, V. Falkovich, A. I. Huseynov, N. M. Gunther, I. I .Ibragimov, M. V. Keldysh, A. N. Kolmogorov, S. P. Korolev, N. E. Cochin, N. N. Krasowski, M. A. Lavrent'ev, A. M. Lyapunov, O. E. H. Love, G. I. Marchuk, Yu. N. Pavlovsky, L. Prandtl, M. L. Rasulov, A. A. Samarskii, L.

I. Sedov, S. L. Sobolev, A. N. Tikhonova, Z. I. Khalilov, V. N. Chelomei, E. Schrödinger and other scientists led to the formation

of modern applied mathematics (see, for example, [1-3]), which includes a range of issues related to the use of mathematical methods and computer tools in the study of various physical processes and phenomena and their use in human practice.
Over the years of functioning of the areas of training in applied mathematics, formed the leading scientific schools in the CIS in various fundamental areas of applied mathematics, such as mathematical physics and spectral theory of differential equations, inverse and ill-posed problems, computational methods and mathematical modeling, nonlinear dynamic systems and control processes, synergetics, game theory and operations research, optimal control and system analysis, mathematical cybernetics and mathematical logic, probability theory and mathematical statistics, theoretical and applied programming and other areas of applied mathematics.

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ESTIMATIONS FOR ROOT VECTORS OF THE FUNCTIONS OF DIRAC TYPE OPERATOR

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In the paper we consider a Dirac type operator (a more general operator than the Dirac operator), establish estimations between L^2_∞ norms of two neighboring root vectors and between L^2_∞ and L^2_p, $1\le p<\infty$ norms of one and the same root vector of the given operator.

Let $L^2_p(G)$, $p>1$ be a space of two –component vector-functions with the norm

$$\|f\|_{p,2}=\left\{\int_G\left(|f_1(x)|^2+|f_2(x)|^2\right)^{p/2}dx\right\}^{1/p}.$$

In the case $p=\infty$, $\|f\|_{\infty,2}=\sup_{x\in G} vrai\,|f(x)|$. For $f(x)\in L^2_p(G)$, $g(x)\in L^2_q(G)$, where $\frac{1}{p}+\frac{1}{q}=1$, $1\le p\le\infty$, the scalar product $(f,g)=\int_G\sum_{j=1}^{2}f_j(x)\overline{g_j(x)}dx.$

Let us consider the Dirac type one-dimensional operator

$$Dy=B\frac{dy}{dx}+P(x)y,\quad y(x)=(y_1(x),y_2(x))^T,$$

where

$$B=\begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix},\ b_1>0,\ b_2<0,\ P(x)=diag\,(p_1(x),\ p_2(x)),$$

moreover $p_1(x)$ and $p_2(x)$ are complex-valued functions determined on an arbitrary finite interval $G=(a,b)$ of the real axis.

Following [1], under the eigen vector-function of the operator D, responding to the complex eigen value λ, we will understand any identically non-zero complex-valued vector-function $\overset{0}{u}(x)$, absolutely continuous on any closed subinterval G and almost everywhere in G satisfies $D\overset{0}{u}=\lambda\overset{0}{u}$.

Similarly, under the associated vector-function of order l, $l\geq 1$, responding to the same λ and eigen vector funtion $\overset{0}{u}(x)$, we will understand any complex-valued vector-function $\overset{l}{u}(x)$, absolutely continuous on any closed subinterval of the interval G and almost everywhere in G satisfies the equation $D\overset{l}{u}=\lambda\overset{l}{u}+\overset{l-1}{u}$.

Theorem . *Let the functions* $p_1(x)$, $p_2(x)$ *belong to the class* $L_1^{loc}(G)$. *Then for any compact* $K\subset G$ *there exist the constants* $C^i(K,l,b_1,b_2)$, $i=1,2$, $l=0,1,2,\ldots$, *independent on* λ, *such that the following estimations are valid;*

$$\left\|\overset{l-1}{u}\right\|_{L_\infty^2(K)}\leq C^1(K,l,b_1,b_2)(1+|\mathrm{Im}\lambda|)\left\|\overset{l}{u}\right\|_{L_\infty^2(K)}, \qquad (1)$$

$$\left\| \overset{l}{u} \right\|_{L^2_\infty(K)} \leq C^2(K,l,b_1,b_2)\left(1+\left|\operatorname{Im}\lambda\right|\right)^{\frac{1}{p}} \left\| \overset{l}{u} \right\|_{L^2_p(K)}, \quad 1 \leq p < \infty .$$

(2)

Remark . If G is a finite interval, $p_1(x)$ and $p_2(x)$ belong to the class $L_1(G)$, then estimations (1) and (2) are valid in the case $K = \overline{G}$ as well.

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ORGANIZATION OF WORK WITH GIFTED CHILDREN IN THE SYSTEM OF ADDITIONAL EDUCATION

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Key factors that influence mathematics education: achievements of mathematical and pedagogical science, informatization, digitalization and globalization. Among the achievements of pedagogical science that influenced the modern methodical system of teaching mathematics in school are the humanistic educational paradigm, the theory of developmental education and the concepts of creativity and talent. It was these developments that became the theoretical basis for the introduction of profile training, differentiation and individualization of training, provision of conditions for training and development of gifted children [1].

In this regard, of interest is the experience of working to support gifted schoolchildren in the Stavropol Territory. In the city of Stavropol, many sections and schools have been organized aimed at developing the intellectual abilities and skills of researchworks of schoolchildren. The Department of Applied Mathematics and Informatics of the Institute of Information Technologies and Telecommunications (IITT NCFU) takes an active part in the development of young talents. Teachers of the department are involved in the work of the school for gifted children "Poisk", the Small Academy of Sciences (IAS) at the Stavropol Palace of Children's Creativity (SDDT) and the Physics and Mathematics School based on IITT NCFU [2].

In 1991, the center “Search” was created, one of the best and most effective institutions in the system of additional education with gifted children. There is a full-time and part-time tuition. The center offers both group and individual lessons. In group classes, students study an in-depth program in mathematics and computer science. These classes help to learn new, different from school, mathematical methods and techniques, to see mathematical science in a form close to the university, as well as to learn the basics of programming and modern information technologies. The center "Search" also organized private lessons with experienced teachers. The Center conducts the Summer Mathematical School (LMS) and the Summer Computer School for high school students who graduated from grade 9 and grade 10.

The goal of the LMS program is to create conditions for high school students in the Stavropol Territory who expressed a desire to expand and deepen their knowledge of mathematics. The educational program of the Summer Mathematical School includes two workshops on solving problems of increased complexity: 1) a workshop on solving geometric problems (“Planimetry” for future tenth-graders, “Stereometry” for future eleventh-graders); 2) solving problems with parameters.

High qualification of teachers, advanced educational technologies, the level of the tasks under consideration allow pupils of summer schools to win at the All-Russian Olympiads of the first level.

The Small Academy of Sciences of the Stavropol Palace of Children's Art this year will celebrate its thirtieth anniversary. October 13, 1989 is the birthday of MAN SDDT. The Small Academy of Sciences was established on the basis of a new Palace of Pioneers and Schoolchildren in Stavropol. The building of the Palace was received as a gift from MS. Gorbachev (General Secretary of the Central Committee of the CPSU, then the President of the USSR). One of the initiators of the creation of the IAS is Candidate of Physical and Mathematical Sciences, Associate Professor of the Pedagogical Institute V.S. Igropulo, who for all the years of the existence of the Small Academy of Sciences is its supervisor. The purpose of the creation of MAN was the involvement of high school students in scientific and practical activities. The first scientific circles for students were created from 1987 - 1989. on the basis of the departments of theoretical physics, biology and mathematics of SSU; at the Department of Pharmacology, SSMA; at the Department of Industrial Electronics, SevKavGTU.At the first city scientific-practical conference of schoolchildren, held on April 12, 1989, there were three sections, fifteen reports were prepared. Until 2019, annually, in April, regional scientific-practical conferences of schoolchildren were held. Since the day of the first conference, the number of sections and presentations has increased significantly.

In 1991, the State Committee on Education was entrusted with the Minor Academy of Sciences to hold the All-Union Conference on Physics, Mathematics, Geography and Astronomy. After only two years of existence, the IAS held such a large-scale conference where representatives of each region of the USSR arrived.Since 2007, on the basis of the IAS SDDT, an annual competition of young researchers "Step into the Future" is held, in which representatives from all regions of the North Caucasus Federal District participate. In the Small Academy of Sciences, 2 math-cycle programs have been introduced, namely: "Mathematics for high school students", "Mathematics with an application package". Pupils receive new knowledge and are engaged in research of scientific problems. The results of the research students present at the annual conference of the IAS. The laureates of the Stavropol Conference represent their region at Russian and international scientific events of schoolchildren in Moscow, St. Petersburg, Obninsk, Korolev, Perm, Omsk, Nalchik, Dolgoprudny. Pupils of the IAS participated in conferences of the European Union (Portugal, 1998) and the London Youth Scientific Forum (1998), were candidates for international exhibitions held in the United States, England.

In 2012, IITT SKFU was created on the basis of the Faculty of Information Technologies and Telecommunications of SKFU and the departments of computer security, organization and technology of information protection, applied mathematics and computer science and applied computer science in SSU economics. The Physics and Mathematics School at IITT NCFU provides students of grades 9-11 with deep and solid knowledge, sufficient to enter a university. At the end of the school certificate is issued. Olympiads are organized every year for students of the physical and mathematical school. Diligent schooling helps to successfully pass a unified state exam and quickly adapt to university studies.

In many areas of the region there are branches of the Small Academy of Sciences and the Center "Search", which are taught by highly qualified teachers.Due to the high professionalism of teachers working with gifted children, the taste of initial victories and small accomplishments gives schoolchildren the “wings of luck” helping them to “fly up”.

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EFFECTS OF THE NANOPARTICLES AND MAGNETIC FIELD ON THE BLOOD FLOW IN STENOSIS ARTERY

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The mathematical model of unsteady blood flow in the stenosis artery subjected to imposed nanoparticles and magnetic field was derived and solved analytically. The effects of the

nanoparticles and magnetic field on the blood flow were simulated analyzed using Mathcad package. The result showed that the nanoparticles and magnetic field affected the flow field significantly which can be beneficial for some biomedical problems.

THE THEORY OF DUALITY TO PROBLEMS WITH HIGHER ORDER DIFFERENTIAL INCLUSIONS

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This paper on the whole concerns with the duality of Mayer problem for κ -th order differential inclusions (DFIs):

$$\text{infimum}\ \ \varphi(x(1), x'(1), \ldots, x^{(\kappa-1)}(1)),$$

$$(P_H) \qquad x^{(\kappa)}(t) \in F(x(t), x'(t), \ldots, x^{(\kappa-1)}(t), t), \text{ a.e. } t \in [0,1],$$

$$x(0) \in Q_0\ x'(0) \in Q_1, x''(0) \in Q_2, \ldots, x^{(\kappa-1)}(0) \in Q_{\kappa-1}.$$

Here $F(\cdot,t):(\mathbb{R}^n)^\kappa \rightrightarrows \mathbb{R}^n$ is convex set-valued mapping, φ is proper convex function and $Q_j \subseteq \mathbb{R}^n,\ j=0,...,\kappa-1$ are convex subsets, κ is an arbitrary fixed natural number. Thus, this work for constructing the dual problems to differential inclusions of any order can make a great contribution to the modern development of optimal control theory. To this end in the form of Euler-Lagrange type inclusions [2,3,5] and transversality conditions the sufficient optimality conditions are derived. The principal idea of obtaining optimal conditions is locally adjoint mappings [1,4,5]. It appears that the Euler-Lagrange type inclusions for both primary and dual problems are “duality relations”. To demonstrate this approach, some semilinear problems with κ-th order differential inclusions are considered. Also, the optimality conditions and the duality theorem in problems with second order polyhedral differential inclusions are proved. These problems show that maximization in the dual problems are realized over the set of solutions of the Euler-Lagrange type differential inclusions/equations. First, we note that the dual problem is always concave even if the primal is not convex. Second, the number of variables in the dual problem is equal to the number of constraints in the primal which is often less than the number of variables in the primal problem. Third, the maximum value achieved by the dual problem is often equal to the minimum of the primal. Here we treat dual results according to the dual operations of addition and infimal convolution of convex functions [2,6,7]. But the construction of the duality problem would lead us too far astray from the main themes of this paper and is therefore omitted. And in this sense the obtained results here are only the visible part of the "icebergs".

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HARDY TYPE INEQUALITIES WITH ADDITIONAL NONNEGATIVE TERMS

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Let Ω be an open proper subset of the Euclidean space $\mathbb{R}^n$. Denote by $C_0^1(\Omega)$ the family of continuously differentiable functions $f\colon \Omega \to \mathbb{R}$ with compact supports lying in Ω.

In this study we consider inequalities of Hardy type with weight functions depending on the distance function to the boundary of the domain Ω, i.e.

$$\delta = \delta(x) = \delta(x, \Omega) = dist(x, \partial\Omega).$$

In [1], Avkhadiev F.G. and Wirths K.-J. obtained Hardy type inequalities with an additional nonnegative-term for compactly supported smooth functions on open convex domains with finite inner radius

$$\delta_0 = \delta_0(\Omega) = \sup\{\delta(x, \Omega), : x \in \Omega\}.$$

More precisely, Avkhadiev F.G. and Wirths K.-J. obtained the following inequality

$$\int_\Omega |\nabla f|^2\,dx \geq \frac{1-\nu^2 m^2}{4}\int_\Omega \frac{|f|^2}{\delta^2}dx \;+\frac{c^2}{\delta_0^m}\int_\Omega \frac{|f|^2}{\delta^{2-m}}dx \qquad (1)$$

for any function $f \in C_0^1(\Omega)$, where $m > 0, 0 < \nu \leq 1/m$ and $c = c_\nu(m)$ is the constant satisfying the equation

$$J_\nu\left(\frac{2}{m}c\right) + 2cJ_\nu'\left(\frac{2}{m}c\right) = 0 \qquad (2)$$

for the Bessel function J_ν.

We obtain L_1 and L_p analogues of (1). We prove one dimensional Hardy type inequalities with additional nonnegative terms and by one dimensional inequalities we establish inequalities in convex domains with finite inner radius, see, for example, [1], [2].

For instance, if $m > 0, 0 < \nu \leq \frac{1}{m}$ and $c = c_\nu(m)$ is the constant satisfying (2), then the following inequality

$$\int_0^1 |f'(x)|dx \geq \frac{1-\nu m}{2}\int_0^1 \frac{|f(x)|}{x^2} + \frac{2c^2}{1+\nu m}\int_0^1 \frac{|f(x)|}{x^{2-m}}dx$$

is valid for all absolutely continuously function f such that $f(0) = 0$.

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A DECAY CONDITION FOR SMALL COEFFICIENTS OF ELLIPTIC EQUATIONS FOR UNIQUENESS IN CONE CONDITION UNBOUNDED DOMAINS

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In this abstract, we state a uniqueness assertion on the Dirichlet problem in the unbounded cone condition domains for the elliptic equations

$$\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right) + \sum_{i=1}^{n} b_i(x)\frac{\partial u}{\partial x_j} = 0, \qquad (1)$$

in the domain $D \subset \Re^n, n \geq 2$ satisfying the cone condition at infinity:

$$\frac{\left|Q_{2^m}^0 \cap D\right|}{\left|Q_{2^m}^0\right|} < \eta \ , \qquad m = m_0, m_0+1, m_0+2, \ldots \qquad (2)$$

with m_0 to be sufficiently large integer, η small positive number, and $|.|$ denotes the lebesgue measure of the corresponding set. Here the leading coefficients $a_{ij}(x)$ are satisfy the uniform ellipticity condition: there exist positive constants C_1, C_2 such that for any $\xi \in \Re^n, x \in D$ it holds

$$C_1 |\xi|^2 \leq \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \leq C_2 |\xi|^2 . \qquad (3)$$

Moreover, the small coefficients satisfy the

$$\left(\int_{2^m < |x| \leq 2^{m+1}} |b(x)|^p dx \right)^{1/p} 2^m \leq C_3 , \qquad (4)$$

by some $p > n$ and C_3 to be independent on m, where $|b(x)| = \left(\sum_{i=1}^{n} b_i^2(x) \right)^{1/2}$. Then the following assertion condition: for all $m = m_0, m_0 + 1, m_0 + 2, ...$ it hold takes place.

Theorem. Let $D \subset \Re^n$ be an unbounded domain satisfying (2). Let $u(x)$ be a solution of equation (1) in D receiving non-positive values on boundary ∂D of domain D and whose coefficients satisfy (3), (4).

Then either $u(x) \leq 0$ in D, or there is a constant $C_4 > 0$ depending on $C_1, C_2, C_3, C_4, n, \eta$ such that it holds

$$\liminf_{r \to \infty} \frac{M(r)}{r^{C_4}} > 0$$

This subject was studied by E.M.Landis [1] and A.A. Novruzov [2] and A. Aliyev[3] under the conditions for all $x \in D$:

$$\text{div } B \leq 0 \text{ and } B \cdot x \geq 0$$

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ON BEHAVIOR OF SOLUTION FOR NONLINEAR PROBLEM

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We consider the following problem

$$u_{tt} - \sum_{i=1}^{n} D_i\left(|D_i u|^{p-2} D_i u\right) - \alpha \Delta u_t + f(u) = 0,\ (x,t) \in \Omega x[0,T], \quad (1)$$

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x)\ \ x \in \Omega, \quad (2)$$

$$\sum_{i=1}^{n} \left(|D_i u|^{p-2} D_i u\right) \cos(x_i, \nu) + \alpha \frac{\partial u_t}{\partial n} = g(u),\ (x,t) \in \partial\Omega x[0,T], \quad (3)$$

where $\Omega \subset R^n$, $n \geq 2$ is a boundary domain with smooth boundary $\partial\Omega$, $u_0(x) \in W_2^1(\Omega)$, $u_1(x) \in L_2(\Omega)$ are given functions, $f(u)$ and

$g(u)$ are some nonlinear functions, α is positive number, $p \geq 2$,

$$D_i = \frac{\partial}{\partial x_i},\ i = 1,2,...,n,\ \frac{\partial}{\partial n} \text{ - the external normal in } \partial\Omega .$$

A lot of works (for example, see [1]- [5]) were devoted to the problems of solutions behavior for particular cases of question of type (1) with different conditions. In general, these works deal with nonlinearity presenting in the equation.

In this work, we study a blow up of solution for a problem (1)-(3), when boundary function has some smoothing properties.

Theorem. Let's for any $u \in R^1$ and for some $\alpha > 0$ satisfies following conditions

$$2(2\alpha+1)F(u) - uf(u) \geq 0,\ F(u) = \int_0^u f(s)ds,$$

$$ug(u) - 2(2\alpha+1)G(u) \geq 0,$$

$$\int_\Omega F(u_0)dx - \int_{\partial\Omega} G(u_0)ds + \frac{1}{p}\int_\Omega \sum_{i=1}^n |D_i u_o|^p dx \leq 0,$$

$(u_0, u_1) > 0$.

Then, if the problem (1)-(3) has a solution $u(x,t) \in W_2^1(0,T;W_2^2(\Omega)) \cap W_2^2(0,T;L_2(\Omega))$,then exists $t_o < \infty$ such that,

$$\lim_{t \to t_0}\left[\|u(x,t)\|^2 + \int_0^t \|\nabla u(x,\tau)\|^2 d\tau \right] = \infty$$

holds.

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FUNDAMENTAL SOLUTION OF GEOMETRIC MIDDLE BOUNDARY PROBLEM FOR ONE INTEGRO-DIFFERENTIAL EQUATION OF 3D BIANCHI

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In recent years, interest in three-dimensional local and non-local boundary value problems for the Bianchi equations [1-7] has increased significantly. This is due to their appearance in various tasks of an applied nature. Three dimensional Bianchi equation is used in models of vibration processes, and also has an important significance in the theory of approximation.

Consider the integro-differential equation 3D (three dimensional) Bianchi:

$$(V_{1,1,1}u)(x,y,z) \equiv u_{xyz}(x,y,z) +$$

$$+ \sum_{\substack{i,j,k=0 \\ i+j+k<3}}^{1} A_{i,j,k}(x,y,z) D_x^i D_y^j D_z^k u(x,y,z) +$$

$$+ \sum_{\substack{i,j,k=0 \\ i+j+k<3}}^{1} \int_{\sqrt{x_0 x_1}}^{x} \int_{\sqrt{y_0 y_1}}^{y} \int_{\sqrt{z_0 z_1}}^{z} K_{i,j,k}(\tau,\xi,\eta;x,y,z) D_x^i D_y^j D_z^k u(\tau,\xi,\eta)\, d\tau d\xi d\eta =$$

$$= \varphi_{1,1,1}(x,y,z),\ (x,y,z) \in G, \qquad (1)$$

with geometric middle boundary conditions in the non-classical form

$$\begin{cases} V_{0,0,0}u \equiv u(\sqrt{x_0 x_1}, \sqrt{y_0 y_1}, \sqrt{z_0 z_1}) = \varphi_{0,0,0} \\ (V_{1,0,0}u)(x) \equiv u_x(x, \sqrt{y_0 y_1}, \sqrt{z_0 z_1}) = \varphi_{1,0,0}(x), \\ (V_{0,1,0}u)(y) \equiv u_y(\sqrt{x_0 x_1}, y, \sqrt{z_0 z_1}) = \varphi_{0,1,0}(y), \\ (V_{0,0,1}u)(z) \equiv u_z(\sqrt{x_0 x_1}, \sqrt{y_0 y_1}, z) = \varphi_{0,0,1}(z), \\ (V_{1,1,0}u)(x,y) \equiv u_{xy}(x, y, \sqrt{z_0 z_1}) = \varphi_{1,1,0}(x,y), \\ (V_{0,1,1}u)(y,z) \equiv u_{yz}(\sqrt{x_0 x_1}, y, z) = \varphi_{0,1,1}(y,z), \\ (V_{1,0,1}u)(x,z) \equiv u_{xz}(x, \sqrt{y_0 y_1}, z) = \varphi_{1,0,1}(x,z), \end{cases} \qquad (2)$$

Here $u = u(x,y,z)$ is the desired function defined on G; $A_{i,j,k} = A_{i,j,k}(x,y,z)$ the given measurable functions on $G = G_1 \times G_2 \times G_3$, $K_{i,j,k}(\tau,\xi,\eta;x,y,z) \in L_\infty(G \times G)$, where

$G_1 = (x_0, x_1),\ G_2 = (y_0, y_1),\ G_3 = (z_0, z_1);\ \ \varphi_{i,j,k}(x, y, z)$ the given measurable function on G. In addition, it is assumed that $x_0 \geq 0,\ y_0 \geq 0,\ z_0 \geq 0.$

In this work the fundamental solution of geometric middle boundary problem(1)-(2) for integro-differential equation of 3D Bianchi (1) with nonsmooth coefficients is constructed.

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ARITHMETICAL MIDDLE BOUNDARY PROBLEM IN THE NON-CLASSICAL TREATMENT FOR ONE GENERALIZED ALLER EQUATION WITH NON-SMOOTH COEFFICIENTS

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In this work substantiated for a generalized Aller equation with non-smooth coefficients in the arithmetical middleboundary problem with non-classical boundary conditions is considered, which requires no matching conditions. Equivalence of these conditions boundary condition is substantiated classical, in the case if the solution of the problem in the anisotropic S. L. Sobolev's space is found.

Problem Statement. Consider generalized Aller equation

$$(V_{2,1}u)(x) \equiv D_1^2 D_2 u(x) + a_{1,1}(x) D_1 D_2 u(x) + a_{2,0}(x) D_1^2 u(x) + a_{1,0}(x) D_1 u(x) +$$
$$+ a_{0,1}(x) D_2 u(x) + a_{0,0}(x) u(x) = \varphi_{2,1}(x) \in L_p(G), \quad x = (x_1, x_2) \in G \tag{1}$$

Here $u(x)$ is a desired function determined on G; $a_{i_1,i_2} = a_{i_1,i_2}(x)$, $i_1 = \overline{0,2}$ $i_2 = \overline{0,1}$, $(a_{2,1}(x) \equiv 1)$ are the given measurable functions on $G = G_1 \times G_2$, where $G_1 = (x_1^0, h_1)$, $G_2 = (x_2^0, h_2)$; $\varphi_{2,1}(x)$ is a given measurable function on G ; The generalized Aller equation in various points of view was studied in the papers [1-9] and etc.

In the present work generalized Aller equation (1) is considered in the general case when the coefficients $a_{i_1,i_2}(x)$ are non-smooth functions satisfying only the following conditions:

$$a_{0,0}(x) \in L_p(G), a_{1,0}(x) \in L_p(G), a_{2,0}(x) \in L_{\infty,p}^{x_1,x_2}(G),$$

$$a_{0,1}(x) \in L_{p,\infty}^{x_1,x_2}(G), a_{1,1}(x) \in L_{p,\infty}^{x_1,x_2}(G).$$

Under these conditions, we'll look for the solution $u(x)$ of generalized Allerequation (1) in S.L. Sobolev anisotropic space $W_p^{(2,1)}(G) \equiv \{u(x) : D_1^{i_1} D_2^{i_2} u(x) \in L_p(G), i_1 = \overline{0,2}\ \ i_2 = \overline{0,1}\}$, where $1 \le p \le \infty$. $D_k^{\xi} = \partial^{\xi} / \partial x_k^{\xi}, k = \overline{1,2}$ - is a generalized differentiation operator in S.L. Sobolev sense, D_k^0 is an identity transformation operator. We'll define the norm in the space $W_p^{(2,1)}(G)$ by the equality

$$\|u\|_{W_p^{(2,1)}(G)} = \left\|D_1^2 D_2 u\right\|_{L_p(G)} + \left\|D_1^2 u\right\|_{L_p(G)} + \sum_{i_1=0}^{1} \sum_{i_2=0}^{1} \left\|D_1^{i_1} D_2^{i_2} u\right\|_{L_p(G)}.$$

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ABOUT THE INFLUENCE OF QUANTIFIER IN THE MODAL PROPOSITIONS

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In Tusi's treatise "Tajrid al-mantik", depending on the relationship between the subject and the predicate three types of modality are defined: necessaries, possibilities, impossibilities.

In addition, in the modalities, depending on how continues verdict - permanently or no permanently, also conditions on the subject the propositions may has difference qualities. Typically, the modalities of the propositions are fixed after verdict: "A is B - absolutely", "A is B -necessarily", "A is B – permanently (always)", "A is B – no permanently" so on. Also, existence or generality quantifiers may take part in such propositions. In this case, it is important to understand how and on which one the quantifier influences? For example, the proposition "some A-s is B - permanently" can be interpreted in different ways:

- some invariant and unchanged A-s are B always;

- always there is any A (may be different A's for different times) which is B.

Studing of contradiction of modal propositions and results of logical figures show that quantifiers influence on the unchanging or invariant elements of subject only.

Therefore,

- the proposition “some A-s is B - permanently” must be understand in following way: there are some concretely and unchanged elements of A, which are B every times. Contradiction of this proposition consist of two propositions – “every A is B - permanently” or “every A is not B - permanently”;

- the proposition “arbitrary A is B - permanently” means that, each element of A is B, all times;

- the proposition “some A is B – non-permanently” means that there are some unchanging elements of A, which are B only sometimes;

- the proposition “every A is B – non-permanent” means that every A is B, but only sometimes, non all times. And this proposal consists of two propositions – “some A is not B – every time” or “some A is B – every time”

ON THE EXPANSION FORMULA FOR A SINGULAR STURM-LIOUVILLE EQUATION WITH THE EIGENVALUE APPEARING NON-LINEARLY IN THE BOUNDARY CONDITION

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In this study it is considered on the semi-axis $[0,\infty)$ the boundary value problem generated by the Sturm-Liouville equation with the eigenvalue appearing non-linearly in the boundary condition. The scattering problem on the half line $[0,\infty)$ is investigated. The scattering data is defined and some of their properties are examined, the operator-theoretic formulation is given. It is proved that the boundary value problem has only a finite number of simple negative eigenvalues. The resolvent operator is constructed and the spectral expansion formula is obtained by using Titchmarsh's method [1]. This type of boundary problem arises from a varied assortment of physical problems and other applied problems such as the study of heat conduction in [2] and wave equation [3]. Spectral analysis of the problem on the half line is studied in [4].

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THE BEHAVIOUR OF GENERALIZED SOLUTIONS DEGENERATE PARABOLIC EQUATIONS IN NONREGULAR DOMAINS

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In cylindrical domains $Q_T = \Omega \times (0,T)$, $T > 0$ following equation

$$\frac{\partial u}{\partial t} - \sum_{|\alpha| \le 2m} (-1)^{\alpha} D^{\alpha} A_{\alpha}(x,t,u,Du,...,D^m u) = 0 \qquad (1)$$

considered. In parabolic boundary Q_T giving the Dirichlet condition. The domain Ω can be non-smooth or unbounded with non-compact boundary.

We get a priori estimates that analogies of Saint-Venant's principle. On basis this estimates behavior of solution to study. Also the finding uniqueness classes of solutions.

THE SOLUTION OF PROBLEM OF DETERMINATION OF STRESS-STRAIN STATE OF A CIRCULAR VISCOELASTIC BEAM WITH SEMI-CIRCULAR LONGITUDINAL NECK AT TORSION

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It is known that a circular shaft with a longitudinal semi-circular neck is an element of many constructions. The solution of the problem of torsion of such a shaft in the case of elastic material was first given by Hans H. [1]. The expressions for stress tensor components obtained when solving corresponding elastic problem, contain a material constant, shear modulus. This circumstance requires a special choice of a method for solving a problem of determination of stress-strain state of the considered shaft in the case when mechanical properties of its material are described by the relations of theory of viscoelasticity, second order Volterra integral equations

$$2G_0 e_{ij} = s_{ij} + \int_0^t \Gamma(t-\tau) s_{ij}(\tau) d\tau, \quad \theta = \sigma / k,$$

where G_0 is instant shear modulus, K is a volume deformation modulus, $\theta = 3\varepsilon$ is relative change of volume, ε is mean deformation, σ is mean stress, $e_{ij} = \varepsilon_{ij} - \varepsilon\delta_{ij}$ is deviator of deformations, ε_{ij} , δ_{ij} are Kronecker's symbols, $s_{ij} = \sigma_{ij} - \sigma\delta_{ij}$ is deviator of stresses σ_{ij} .

We give the statement of a problem of determination of stress and strain components that arise in a circular viscoelastic beam with a semi-circular neck at torsion. The problem is solved by A.A. Ilyushin's [2] approximation method using the solution of the corresponding problem of ideal elasticity. To this end we introduce a new parameter $\omega = 2G/3K$, where G is a shear modulus, K is a volume deformation modulus. In this connection the solution of ideal elasticity problem is written in the form allowing direct substitution of corresponding variables by Laplace-Carson transform. Passing in transforms from relations to preimages, by the approximation method we find the desired solution of the viscoelastic problem of the shaft under consideration.

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OPTIMAL PLACEMENT OF OFFHOSE OIL-GAS PLATFORMS WITH DATA IN THE FORM OF INTERVALS

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When operating offhose oil-gas fields, ceratin minimal number of platformas are built on the sea and the sed and in each platform a limited number of wells are drilled. This time is should be taken into account that each well should be drilled only from one platform. In this problem it is required to drill a well only from one platform with minimum expenses for drilling the sea weills and bulding platforms on the sea.

It should be noted that in these references, the data of the considered problems, i.e. drilling expenses in each platform and the expenses for placing the platforms are given as constants. It is clear that when constructing a mathematical model of such a problem, by placing these expenses writhin ceratin intervals we get a more real wal mathematical model. We acept the following denotation:

n is the number of wells in the field, m is the maximim number of platforms to be built , $c_{i,j}$ is the drilling expense of the j-th $\left(j=\overline{1,n}\right)$well to be drilled from the i-th $\left(i=\overline{1,m}\right)$ platform.

In this paper we sccept that $c_{ij} \in \left[\underline{c_{ij}}, \quad \overline{c_{ij}}\right], \quad a_{ij} \in \left[\underline{a_{ij}}, \quad \overline{a_{ij}}\right], P_i \in \left[\underline{P_i}, \quad \overline{P_i}\right]\left(i=\overline{1,m}, \quad j=\overline{1,n}\right)$. P_i - $(i=\overline{1,m})$ is the maximum, number of wells to be drilled from the i-th platform.

For constructing a mathematical model of the problem under consideratian we accept the unknowns as follows x_{ij} and y_i $\left(i=\overline{1,m},\ j=\overline{1,n}\right)$

$$\underline{x}_j = \begin{cases} 1, & \text{if the well number } j \text{ is drilled from the platform number } i \\ 0, & \text{in the constrary case } i=\overline{1,m}, \quad j=\overline{1,n} \end{cases}$$

$$y_i = \begin{cases} 1, & \text{if the platform is built in place number } i \\ 0, & \text{in the contrary case } (i=\overline{1,m}, \quad j=\overline{1,n} \end{cases}$$

Writhin these denotation, the total expense for drilling the wells from the platform is $\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\underline{c_{ij}}, \quad \overline{c_{ij}}\right]x_{ij}$. Total expence for buliding platforms is $\sum_{i=1}^{m}\left[\underline{a_i}, \quad \overline{a_i}\right]y_i$. On the other hand, from each i-th $\left(i=\overline{1,m}\right)$ platform at most entire number well on the interval $\left[\underline{P_i}, \quad \overline{P_i}\right]$ $(i=\overline{1,m})$ must be drilled. For that the condition $\sum_{j=1}^{n}x_{ij} \le \left[\underline{P_{i,}} \quad \overline{P_i}\right]y_i \ i=\overline{1,m}$ must be satisfied.

So, the mathematical model of the considered problem will be as follows:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\underline{c_{ij}}, \quad \overline{c_{ij}}\right]x_{ij} + \sum_{i=1}^{m}\left[\underline{a_i}, \quad \overline{a_i}\right]y_i \to \min \qquad (1)$$

$$\sum_{i=1}^{m}x_{ij} = 1, \quad j=\overline{1,n}, \qquad (2)$$

$$\sum_{j=1}^{n}x_{ij} \le \left[\underline{P_{i,}} \quad \overline{P_i}\right]y_i \ i=\overline{1,m} \qquad (3)$$

$$x_{ij} = 1 \vee 0,\ y_i = 1 \vee 0,\ i = \overline{1,m},\ j = \overline{1,n} \qquad (4)$$

Here condition (2) shows that each well should be drilled only from one plarform. Condition (3) is the number interval of wells to be drilled from each platform. We have worked out this approximate method for solving problem (1)-(4) . This method is based on optimistic and pessimatic startegies, and dixotomy (bisectiion) method was used.

ON APPROXIMATION THEOREM IN WEIGHTED LEBESGUE SPACES

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In this abstract we reduce a weighted Lebesgue modulus of continuity in weighted Lebesgue space and Popoviciu type theorem is studied.Let $x \in (0,\infty)$ and let $\rho(x) = 1 + x^2$. The weighted Lebesgue space $L_{p,\rho}(0,\infty)$ is the collection of all Lebesgue measurable functions, such that

$$\|f\|_{L_{p,\rho}(0,\infty)} = \left(\int_0^\infty \left(\frac{|f(x)|}{\rho(x)} \right)^p dx \right)^{\frac{1}{p}} < \infty, \quad (1 \le p < \infty).$$

We consider a new weighted modulus of continuity in Lebesgue spaces

$$\Omega_p(f;\delta)=\sup_{0<h\le\delta}\left\{\int_0^\infty\left(\frac{|f(x+h)-f(x)|}{(1+x^2)(1+h^2)}\right)^p dx\right\}^{\frac{1}{p}},\quad p\ge 1.$$

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CORROSIVE FAILURE OF THICK-WALLED PIPE UNDER THE ACTION OF TORQUE

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Corrosive-mechanical failure is one of the dangerous constructions. As is known such failure of metals and metal goods takes place under joint action of load and aggressive medium. Corrosive failure depends on many factors including mechanical stress, potential, temperature, time, etc. Determination of time to corrosion failure is one of the basic problems in studies of corrosion of metals. In the given paper [1] the case when in the corrosion process only stress undergoes a change in time is

considered. Considering corrosion as a process of accumulated damages of certain type, and also on the basis of analysis of experimental curves of corrosive strength, in [2], a theoretical formula that determines time to corrosion failure of metals under nonstationary change of stress in the corrosion process, is derived.

$$t_* = t_o(\sigma_o)\left[A_1 + A_2\frac{\sigma_b - \sigma_o}{\sigma_s - \sigma_o} + A_3\left(\frac{\sigma_b - \sigma_o}{\sigma_s - \sigma_o}\right)^2\right] \qquad (1)$$

Here t_* is time to corrosive failure of metals under nontationary change of stress, $\sigma = \sigma(t)$, where t is time; $\sigma_o = \sigma(o)$; σ_b is a stress under which detachment of pure metal happens; σ_o is some standard stress; A_1, A_2, A_3 are experimentally definable constants; $t_o = t_o(\sigma)$ is a universal function of the system “metal-corrosive medium”, time to corrosive failure of experimental sample under different constants of stress σ.

Using formula (1) time to corrosive failure at torsion of thick-walled pipe situated in corrosive medium, is determined.

The torsion problem is considered in physically linear, geometrically nonlinear statement whose solution is given in [3]. Time to corrosive failure of metallic thick-walled pipe at torsion under the action of torque acting on its ends and situated in corrosive medium, is determined.

The numerical calculations show that the corrosive process begins on the pipe surface.

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SECOND ORDER NECESSARY CONDITIONSFOR STRONG AND WEAK EXTREMA IN THE CALCULUS OF VARIATIONS

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In this paper, we consider the vector problem of calculus of variations

$$J(x(\cdot)) = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t))dt \to \min_{x(\cdot)}, \tag{1}$$

$$x(t_0) = x_0,\ x(t_1) = x_1, \tag{2}$$

where $x_0, x_1 \in R^n$, $t_0, t_1 \in R$ aregiven points.

For a given function $L(t, x, \dot{x}): [t_0, t_1] \times R^n \times R^n \to R$, called an integrand, we assume that, it is a twice continuously differentiable function with respect to a set of variables.

Functions $x(\cdot) \in KC^1([t_0, t_1], R^n)$, satisfying boundary conditions (2), are called admissible, where $KC^1([t_0, t_1], R^n)$ denotes the space of piecewise smooth functions.

It is known [1] that, if the integrand $L(t,x,\dot{x})$ is continuous in $Q\times R^n$, and the function $\bar{x}(\cdot)$ attains a strong minimum in problem (1), (2), then inequality

$$M(t,p,q,\xi;x(\cdot)):=pL\left(t,\bar{x}(t),\dot{\bar{x}}(t)+\frac{1}{p}\xi\right)+$$

$$+qL\left(t,\bar{x}(t),\dot{\bar{x}}(t)-\frac{1}{q}\xi\right)-(p+q)L(t,\bar{x}(t),\dot{\bar{x}}(t))\geq 0, \qquad (3)$$

$$\forall(t,\ p,\ q,\ \xi)\in I \text{ x } R_+ \text{ x } R_+ \text{ x } R^n,$$

holds, where Q is an open set, containing the graph $\{(t,\bar{x}(t)): t\in[t_0,t_1]\}$ of the functions $\bar{x}(\cdot)$, $I\subseteq[t_0,t_1]$ is the set of points of continuity of the functions $\dot{\bar{x}}(\cdot)$, and $R_+=(0,+\infty)$.

Introducing a new variation of theWeierstrass type in the paper, we study the case where condition (3) at the point $(\hat{t},\hat{p},\hat{q},\hat{\xi})\in I\times R_+\times R_+\times R^n\setminus\{0\}$ is equal to zero, i.e., condition (3) along with the Euler extremal $\bar{x}(\cdot)$ degenerates at point $(\hat{t},\hat{p},\hat{q},\hat{\xi})$. As a result, we obtain the second-order necessary conditions of the inequality type and for $\hat{t}\in \operatorname{int} I$ of the equality type for strong and weak extrema.

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THE PROBLEM OF OPTIMAL STABILIZATION OF THE HEAT DISTRIBUTION PROCESS IN RARE MEDIA

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Heat transfer in rare media during high-intensity non-stationary processes is described by the generalized Fourier law [1].Then, in the absence of internal sources, the relaxation model of the process is described by the equation:

$$\frac{\partial u}{\partial t}+\tau\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2},\quad (x,t)\in Q=(0,1)\times(0,T)\,, \tag{1}$$

with initial conditions

$$u(x,0)=\varphi_0(x),\quad \frac{\partial u(x,0)}{\partial t}=\varphi_1(t)\,, \tag{2}$$

and boundary conditions

$$\frac{\partial u(0,t)}{\partial x}=p(t),\quad u(1,t)=0\,, \tag{3}$$

where $\varphi_0(x)\in W_2^2(0,1)$, $\varphi_1(x)\in W_2^1(0,1)$ are given functions, and $p(t)\in L_2(0,T)$ is the control function.

It is required to find a feedback control so as to minimize the functional

$$J[p]=\int_0^\infty\left(\int_0^1 (u^2(x,t)+u_x^2(x,t))dx+p^2(t)\right)dt\,, \tag{4}$$

under the conditions (1)-(3).

Applying eigenfunction expansions of the problem

$$X''(x)+\lambda^2X(x)=0,\quad X(0)=0,\quad X'(0)=0\,, \tag{5}$$

we obtain an infinite system of ordinary differential equations for the Fourier coefficients:

$$\frac{d\bar{u}}{dt} = A\bar{u} + Bp, \quad \bar{u}(0) = \bar{\varphi}, \tag{6}$$

and functional (4) takes the form

$$J[p] = \int_0^\infty (\bar{u}D\bar{u} + p^2(t))dt, \tag{7}$$

where A, B, D, $\bar{u}$ and $\bar{\varphi}$ are matrices of infinite dimension of the form

$$A = \begin{pmatrix} A_1 & A_2 \\ \frac{A_3}{\tau} & \frac{A_4}{\tau} \end{pmatrix}, \; A_1 = 0, \quad A_2 = E, \quad A_4 = -E, \; A_3 = diay(-\lambda_n^2)_{n=1}^\infty,$$

$$B = (B_1,\, B_2), \; B_1 = 0, \quad B_2 = -\sqrt{2}(1,1,\ldots), \quad D = \begin{pmatrix} D_1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$D_1 = diay(1+\lambda_n^2)_{n=1}^\infty, \; \bar{u}(t) = (\bar{u}_1(t), \bar{u}_2(t)), \; \bar{u}_i(t) = (u_{in}(t))_{n=1}^\infty, \; i = 1,2$$

$$\bar{\varphi} = (\bar{\varphi}_0, \bar{\varphi}_1), \quad \bar{\varphi}_0 = (\varphi_{0n})_{n=1}^\infty, \; \bar{\varphi}_1 = (\varphi_{1n})_{n=1}^\infty.$$

Here $u_{1n}(t)$, $u_{2n}(t)$, φ_{0n} and φ_{1n} are Fourier coefficients of the functions $u(x,t)$, $u_x(x,t)$, $\varphi_0(x)$ and $\varphi_1(x)$, respectively.

For problem (6)-(7), the optimal synthesis can be sought in the following form (see [2])

$$p(t) = -B'K\bar{u}(t), \tag{8}$$

where the matrix K is a solution of a Riccati-type matrix algebraic equation

$$-KA - A'K + KBB'K - D = 0 \tag{9}$$

and has a block structure

$$K = \begin{pmatrix} K_{11} & \tau K_{12} \\ \tau K_{12} & \tau K_{22} \end{pmatrix}$$

To find an approximate positive definite solution of equation (9), each matrix $K_{ij} \quad (i, j = 1, 2)$ is sought in the form of a regular

series in powers of a small parameter (see [3])

$$K_{ij} = \sum_{n=0}^{\infty} \tau^n K_{ij}^{(n)} . \tag{10}$$

Substituting this expansion into equation (9) and equating the coefficients for the same powers of the parameter τ, we construct an iterative process of finding the members of the series (10). In particular, it is easy to obtain the first two terms of expansion (10) in an analytical form.

It is established that for the asymptotic stability of a closed system

$$\frac{d\overline{u}}{dt} = (A - B'K)\overline{u}(t), \quad \overline{u}(0) = \overline{\varphi} \tag{11}$$

it suffices to restrict ourselves to two terms of the series, i.e.

$$\overline{K}_{ij} = K_{ij}^{(0)} + \tau K_{ij}^{(1)} ,$$

and there is an estimate

$$\left| J - \overline{J} \right| < c\tau ,$$

where $\overline{J}$ is the approximate value of functional (7) on an suboptimal control.

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A PHRAGMEN–LINDELÖF TYPE THEOREM FOR THE ELLIPTIC EQUATIONS WITH SMALL COEFFICIENTS IN UNBOUNDED DOMAINS

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In this paper, we consider Phragmen-Lindelöf type result for the solutions of the Dirichlet problem in the unbounded domain for elliptic equation

$$\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right) + \sum_{i=1}^{n} b_i(x)\frac{\partial u}{\partial x_j} = 0. \qquad (1)$$

The domain $D \subset \Re^n, n \geq 2$ satisfies the condition at infinity: $\left|Q_{2^m}^0 \cap D\right| < \eta \left|Q_{2^m}^0\right|$ for $m = m_0, m_0+1, m_0+2, \ldots$. The leading coefficients $a_{ij}(x)$ satisfy the uniform ellipticity condition: $C_1|\xi|^2 \leq A(x)\xi\xi \leq C_2|\xi|^2$ and the small coefficients are such that : for all $m = m_0, m_0+1, m_0+2, \ldots$ it hold

$$\left(\int\limits_{2^m < |x| \le 2^{m+1}} |b(x)|^p dx \right)^{1/p} 2^{-m} \le C_3, \qquad (2)$$

and

$$\left(\int\limits_{2^m < |x| \le 2^{m+1}} |\operatorname{div} b(x)|^p dx \right)^{1/p} \le C_4 \qquad (3)$$

by some $p > \frac{n}{2}$ and C_3, C_4 to be independent on m. Then the following assertion takes place.

Theorem. *Let $D \subset \Re^n$ be an unbounded domain satisfying cone condition. Let $u(x)$ be a solution of equation (1) in D receiving non-positive values on boundary ∂D of domain D. The leading coefficients of (1) satisfy uniform ellipticity, the small term coefficients to the conditions (3), (4). Then either $u(x) \le 0$ in D, or there is a constant $C_4 > 0$ depending on $C_1, C_2, C_3, C_4, n, \eta$ such that it holds*

$$\liminf_{r \to \infty} \frac{M(r)}{r^{C_4}} > 0$$

This subject was studied by E.M.Landis [1] and A.A. Novruzov [2] and A. Aliyev [3] under the conditions for all $x \in D$:

div $B \le 0$ and $B \cdot x \ge 0$

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CHARACTERIZATION OF FRACTIONAL MAXIMAL COMMUTATORS ON ORLICZ SPACES IN THE DUNKL SETTING

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Harmonic analysis associated to the Dunkl transform and the Dunkl differential-difference operator gives rise to convolutions with a relevant generalized translation. On the R^d the Dunkl operators $\{D_{k,j}\}_{j=1}^{d}$ are the differential-difference operators associated with the reflection group Z_2^d on R^d.

Let $B(x,r) := \{y \in R^d : |x-y| < r\}$ denote the ball in R^d that centered in $x \in R^d$ and having radius $r > 0$. Then having $|B(0,r)|_k = \int\limits_{B(0,r)} h_k^2(x)dx$.

The *fractional maximal operator* $M_{b,\alpha,k}$, $0<\alpha<d+2\gamma_k$ associated with the Dunkl operator on R^d is given by

$$M_{b,\alpha,k} f(x)=\sup_{r>0}\left(\left|B(x,r)\right|_k\right)^{-1+\frac{\alpha}{d+2\gamma_k}}\int_{B(x,r)}\left|b(x)-b(y)\right|\left|f(y)\right|h_k^2(y)dy$$

$$,\ \forall x\in R^d .$$

It is well known that fractional maximal operators play an important role in harmonic analysis. In this work, in the framework of the Dunkl analysis in the setting R^d, we study the boundedness of the maximal commutator $M_{b,\alpha,k}$ and the commutator of the maximal operator, $\left[b,M_{\alpha,k}\right]$, on Orlicz spaces $L_\Phi\left(R^d,h_k^2(x)dx\right)$, when b belongs to the space $BMO\left(R^d,h_k^2(x)dx\right)$, by which some new characterizations of the space $BMO\left(R^d,h_k^2(x)dx\right)$ are given.

Theorem 1. *Let* Φ *be a Young function with* $\Phi\in\nabla_2$. *Then the condition* $b\in BMO\left(R^d,h_k^2(x)dx\right)$ *is necessary and sufficient for the boundedness of* $M_{b,\alpha,k}$ *on* $L_\Phi(R,dm_\nu)$.

Let $b^-(x)=0$, if $b(x)\geq 0$ and $b^-(x)=|b(x)|$, if $b(x)<0$, $b^+(x):=|b(x)|-b^-(x)$. From Theorem 1 we deduce the following conclusion.

Corollary 1 *Let* Φ *be a Young function with* $\Phi \in \nabla_2$. *Then the conditions* $b^+ \in BMO\left(R^d, h_k^2(x)dx\right)$ *and* $b^- \in L_\infty(R, dm_\nu)$ *are sufficient for the boundedness of* $[b, M_\nu]$ *on* $L_\Phi\left(R^d, h_k^2(x)dx\right)$.

ON THE SPECTRAL PROPERTIES OF THE STURM - LIOUVILLE OPERATOR WITH A BOUNDARY CONDITION QUADRATICALLY DEPENDENT ON THE SPECTRAL PARAMETER

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We consider the boundary value problem generated on the interval $[0,\pi]$ by the Sturm-Liouville equation

$$-y'' + q(x)y = \lambda^2 y \tag{1}$$

and the boundary conditions of the form

$$\begin{gathered} y(0) + \omega\, y(\pi) = 0, \\ \omega\, y'(0) + \left(m\lambda^2 + \alpha\lambda + \beta\right) y(\pi) + y'(\pi) = 0, \end{gathered} \tag{2}$$

where $q(x)$ is a real function belonging to the space $L_2[0,\pi]$, λ is a spectral parameter, and $\alpha,\ \beta,\ m,\ \omega$ are real numbers with $\alpha m \omega \neq 0$. Let the following condition be satisfied: for all functions $y(x) \in W_2^2[0,\pi]$, $y(x) \not\equiv 0$, 0 satisfying the boundary conditions (2), the inequality

$$\left(1+m|y(\pi)|^2\right)\left\{\beta|y(\pi)|^2+\int_0^{\pi}\left(|y'(x)|^2+q(x)|y(x)|^2\right)dx\right\}>0$$

(thisinequality is necessarily satisfied if $\beta \geq 0$, $m \geq 0$, $q(x) > 0$). It is proved that under this condition the eigenvalues of the boundary value problem (1) - (2) are real and nonzero. This task does not have associated functions to its eigenfunctions. In addition, a criterion for the multiplicity of the eigenvalue was found and an asymptotic formula for the eigenvalues was derived. Note that boundary value problems for ordinary differential equations with a parameter in the boundary conditions in various statements were studied in many articles (see [1–7] and the literature therein).

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A FRACTIONAL ORDER WEIGHTED SOBOLEV INEQUALITIES IN THE DOMAIN

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In this report, we study a sufficiency conditions for the fractional order weighted Sobolev's inequality

$$\left(\int_D |f(x)|^q v(x)dx\right)^{1/q} \le C\left(\int_D\int_D |f(x)-f(y)|^p \omega(x,y)dxdy\right)^{1/q}$$

on domain $D\subset\Re^n$ and a function $f\in C_0(D)$. In particularly, the following assertion is proved.

Theorem. Let ϕ be a positive measurable function in D such that $\frac{1}{\phi}\in A_\infty$ -the Muckenhoupt's class. Then the condition

$$\left(\sup_{x\in Q}\int_Q \phi(x-y)\,dy\right)\cdot\left(\frac{1}{|Q|^2}\int_Q \frac{1}{\phi(y)}dy\right)\le C$$

for any cube $Q=Q(x,r)$ with center in $x\in D$ edge $0<t<$ diam D is sufficient for the inequality

$$\int_D \frac{|f(x)|^2}{\phi(x)}dx \le C\int_D\int_D \frac{|f(x)-f(y)|^2}{|x-y|^n\phi(x-y)}dxdy$$

to hold for any continues function f in D vanishing on its boundary ∂D.

For this subject we cite [1,2].

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AN ANALOG OF THE METHOD DIVIDING THE LAGRANGE MULTIPLIER TO TERMS IN GOURSAT-DARBOUX STOCHASTIC SYSTEMS

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Let $(\Omega,F,P)-$ some probability space and on it a non-decreasing flow $\sigma-$algebra $\{F_{tx},(t,x)\in D=[t_0,t_1]\times[x_0,x_1]\}$.

$R(D)-$ space measurable in (t,x,ω) and $F_{tx}-$ agreed processes $z: D\times\Omega\to R^n$ such that $E\int_D \|z(t,x)\|^2 dxdt \le +\infty.$

$E-$ sign of expectation.

Let us consider the controlled stochastic differential equation of Darboux [1]:

$$z_{tx} = f(t,x,z,z_t,z_x,u) + g(t,x,z)\partial^2 W(t,x)/\partial t\partial x, \qquad (1)$$

with boundary conditions type Goursat

$$\begin{aligned} &z(t_0,x) = a(x), x\in[x_0,x_1], \\ &z(t,x_0) = b(t), t\in[t_0,t_1], a(x_0) = b(t_0). \end{aligned} \qquad (2)$$

Here $f(t,x,z,z_t,z_x,u)-$ given $n-$ dimensional vector function, continuous along set of variables together with partial derivatives with respect to z, z_t, z_x; $g(t,x,z)-$ given $n-$ dimensional vector function, continuous along set of variables together with partial derivatives with respect to z;

boundary $n-$dimensional function vector $a(x), b(t)-$ are given on $[x_0,x_1]$, $[t_0,t_1]$ respectively, satisfy the Lipschitz condition; $\partial^2 W/\partial t\partial x - n-$ dimensional two-parameter independent "white noise" on the plane.

Valid control class $U-$ is the class all measurable and bounded functions $u(t,x)$ with values in space R^r, i.e. $(u(t,x)\in L_\infty(D,U))$.

The goal of the optimal control problem is to minimize functional

$$S(u) = E\{\varphi(z(t_1,x_1))\}, \qquad (3)$$

defined on the solutions of problem (1) - (2) with admissible controls.

Here $\varphi(z)-$ given, continuously differentiable scalar function.

Our goal is, based on the method proposed in [2, 3], to derive the necessary first-order optimality conditions such as the Pontryagin maximum principle in the problem (1) - (3) under consideration.

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NECESSARY CONDITIONS FOR EXTREMUM IN NON-SMOOTH PROBLEMS OF VARIATIONAL CALCULUS

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In this paper, we consider the following vector problem of variational calculus

$$J(x(\cdot)) = \int_{t_0}^{t_1} L(t, x(t), \dot{x}(t)) \to \min_{x(\cdot)}, \tag{1}$$

$$x(t_0) = x_0, \;\; x(t_1) = x_1, x_0, x_1 \in R^n, \tag{2}$$

where R^n is n-dimensional Euclidean space and x_0, x_1, t_0, t_1 are the given points. The given function $L(\cdot):[t_0,t_1]\times R^n \times R^n \to R := (-\infty,+\infty)$,
called an integrant, is assumed to be continuous in each variable. The sought function $x(\cdot):[t_0,t_1] =: I \to R^n$ is a piecewise-

smooth vector-function. We denote the set of such functions by $KC^1(I,R^n)$. The functions $x(\cdot)\in KC^1(I,R^n)$ satisfying boundary conditions (2) are said to be admissible.

Let us remind (see for instance [1, p. 107]) some notion of classic variational calculus. The admissible function $\bar{x}(\cdot)$ is called a strong (weak) local minimum of problem (1), (2) if there exists a number $\bar{\delta} > 0$ ($\hat{\delta} > 0$) such that the in equality $J(x(\cdot)) \ge J(\bar{x}(\cdot))$ holds for all admissible functions $x(\cdot)$ which satisfies
$\|x(\cdot)-\bar{x}(\cdot)\|_{C(I,R^n)} \le \bar{\delta}$ $(\max\{\|x(\cdot)-\bar{x}(\cdot)\|_{C(I,R^n)}, \|\dot{x}(\cdot)-\dot{\bar{x}}(\cdot)\|_{L_\infty(I,R^n)}\}) \le \hat{\delta}.$

In this case, we say that the admissible function $\bar{x}(\cdot)$ is a strong (weak) local minimum of problem (1), (2) in $\bar{\delta}(\hat{\delta})$-neighborhood.

To obtain necessary conditions of classic variational calculus, as a rule, at least it is assumed that the integrant $L(\cdot)$ is continuously-differentiable in some domain U of R^{2n+1}. We call the variational problem non-smooth if the integrand of this problem is non-differentiable even if with respect to one of its arguments. Obtaining such results is important for problem (1),

(2) since they do not follow as corollaries from general theory of optimal control as it was done for example in [2, p. 33] and in the present paper.

In this paper we offer a new method that allows us to study strong and weak extrema in problem (1), (2).

The following theorem is proved as the basic result.

Let $\bar{x}(\cdot)$ be some admissible function in problem (1), (2) and $T_1 \subseteq I$ be the set of continuity points of the function $\dot{\bar{x}}(\cdot)$. We define the following function associated with to the integrant $L(t,x,\dot{x})$ and the function $\bar{x}(\cdot)$:

$$Q(t,\lambda,\xi;\bar{x}(\cdot)) = \lambda[L(t,\bar{x}(t),\dot{\bar{x}}(t)+\xi)-\bar{L}(t)]+$$

$$+(1-\lambda)[L(t,\bar{x}(t),\dot{\bar{x}}(t)+\frac{\lambda}{\lambda-1}\xi)-\bar{L}(t)],\ (t,\lambda,\xi)\in T_1\times[0,1)\times R^n,$$

where $\bar{L}(t) := L(t,\bar{x}(t),\dot{\bar{x}}(t))$.

Theorem. Let the integrand $L(t,x,\dot{x}): I\times R^n\times R^n \to R$ be continuous in each variable. Then:

a) if the function $\bar{x}(\cdot)$ is a strong local minimum of problem (1), (2), then the following inequality is satisfied:

$$Q(t,\lambda,\xi;\bar{x}(\cdot)) \ge 0,\ \ \forall (t,\lambda,\xi)\in T_1\times[0,1)\times R^n;$$

b) if $\bar{x}(\cdot)$ is a weak local minimum of problem (1), (2), then there exists a number $\delta > 0$ such that the following inequality is satisfied:

$$Q(t,\lambda,\xi;\bar{x}(\cdot)) \ge 0,\ \ \forall (t,\lambda,\xi)\in T_1\times[0,2^{-1}]\times B_\delta(0),$$

where the set $B_\delta(0)$ is a closed sphere of radius δ centered at the point $0\in R^n$.

Under the smoothness condition of the integrand $L(\cdot)$, we obtain different, more constructive corollaries including the Weierstrass conditions, from this theorem.

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EXISTENCE AND UNIQUENESS RESULTS FOR NONLINEAR IMPULSIVE DIFFERENTIAL WITH THREE-POINT AND INTEGRAL BOUNDARY CONDITIONS

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This thesis deals with the existence and uniqueness of the system of nonlinear impulsive differential equations of the type

$$\dot{x}(t) = f(t, x(t)) \text{ for } t \neq t_i, i = 1,2,\ldots,p, t \in [0,T], \tag{1}$$

subject to impulsive conditions

$$x(t_i^+) - x(t_i) = I_i(x(t_i)),\ i = 1,2,\ldots,p, t \in [0,T], \tag{2}$$

$$0 = t_0 < t_1 < \ldots < t_{p_1} < \tau < t_{p_{1+1}} < \ldots < t_p < t_{p+1} = T,$$

and three-point and integral boundary conditions

$$Ax(0) + Bx(\tau) + Cx(T) + \int_0^T n(t)x(t)dt = d, \tag{3}$$

where A, B, C are constant square matrices of order n such that

$$\det N \neq 0, N = (A + B + C + \int_0^T n(t)dt); f : [0,T] \times R^n \to R^n$$

$n : [0,T] \to R^{n \times n}$ and $I_i : R^n \to R^n$ are given functions;

$\Delta x(t_i) = x(t_i^+) - x(t_i^-)$, where

$$x(t_i^+) = \lim_{h \to 0+} x(t_i + h), \quad x(t_i^-) = \lim_{h \to 0+} x(t_i - h) = x(t_i)$$

Are the right- and left-hand limits of $x(t)$ at point $t = t_i$, respectively.

The aim of the thesis is to investigation a solution of system of the nonlinear impulsive differential equations with three-point and integral boundary conditions (1)-(3) . The Green function is constructed and we considered the original problem is reduced to the equivalent integral equations. Sufficient conditions are found for the existence and uniqueness of solutions to the boundary value problems for the first order non-linear system of the impulsive ordinary differential equations with three-point and integral boundary conditions. The Banach theorem about the fixed point is used to prove the uniqueness of a solution of the problem and Schafer's theorem about the fixed point is used to prove the existence of a solution of the problem considered [1].

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ON THE EXISTENCE OF THE SOLUTION OF A MIXED PROBLEM FOR A CLASS OF EQUATIONS WITH TYPICAL DEGENERATION

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In the papers [4], [5] it was shown that mixed problems both for I.G. Petrovsky well-posed equations can be ill-posed and for ill-posed equations they can be well-posed.

In this paper we study the existence and uniqueness of the solution of a mixed problem for a class of equations with complex-valued coefficients, behaving as parabolic one in spite of the fact that in the course of “time” they can pass from parabolic type to Schrodinger one or even into antiparabolic type. We study solvability of the mixed problem

$$M\left(t, \frac{\partial}{\partial t}\right)U = L\left(x, \frac{\partial}{\partial x}\right)U\,,\ 0 < t < T,\ 0 < x < 1 \quad (1)$$

$$U(0, x) = \varphi(x) \quad (2)$$

$$U(t,0) = U(t,1) = 0 \quad (3)$$

where $M\left(t, \frac{\partial}{\partial t}\right) = P_1(t)\frac{\partial}{\partial t} + P_0(t),\ L\left(x, \frac{\partial}{\partial x}\right) = a(x)\frac{\partial^2}{\partial x^2}$,

$P_1(t) = p_{11}(t) +, \ + ip_{12}(t)$ are complex-valued

functions, $a(x) > 0$,
$a(x) \in C[0,1]$, $P_j(t) \in C[0,1]$ $(j = 0,1)$, $P_1(t) \neq 0$.

The following conditions are supposed to be fulfilled:

1^0. $\operatorname{Re} \int_0^t \frac{d\tau}{P_1(\tau)} d\tau > 0, 0 < t < T$;

2^0. $a(x) > 0$, $0 < x < 1$

3^0. $\varphi(x) \in C^2[0, 1]$, $\varphi(0) = \varphi(1) = 0$.

Note that condition 1º allows to go beyond I.G. Petrovskyparabolicity (even well-posedness) of equation (1). Obviously, subject to condition 2º ,equation (1) is I.G. Petrovsky parabolic if

$$\operatorname{Re} P_1(t) > 0, \ 0 \leq t \leq T \ , \tag{4}$$

provided condition 1º*P_1 (t)*can be zero or negative in some part of(0, T] .

The following theorem is valid.

Theorem: Let conditions 1º, 2º, 3º be fulfilled. Then problem (1)-(3) has a unique classic solution $U(t,x) \in C^{1,2}((0,T] \times [0,1]) \cap C([0,T] \times [0,1])$and it is represented by the formula (for $t > 0$)

$$U(t,x) = -\sum_{k=1}^{\infty} \underset{\lambda_k}{res}\, e^{\int_0^t \frac{P_0(\tau)-\lambda^2}{P_1(\tau)} d\tau} \cdot \int_0^1 G(x,\xi,\lambda)\varphi(\xi)d\xi$$

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NONLINEAR INVERSE BOUNDARY-VALUE PROBLEM FOR THE LONGITUDINAL WAVE PROPAGATION EQUATION

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In some instances, the equation of propagation of longitudinal waves can arise in the theory of long waves, plasma physics, problems of hydrodynamics and so on [1-3].

We investigate an inverse problem of finding the functions $u(x,t)\in \tilde{C}^{(2,2)}(D_T)$ and $a(t),b(t)\in C[0,T]$, connected in the rectangular domain $D_T := \{(x,t): 0\le x\le 1,\ 0\le t\le T\}$ for the equation [1-3]

$$u_{tt}(x,t)-\alpha u_{ttxx}(x,t)-\beta u_{xx}(x,t)=$$
$$=a(t)u(x,t)+b(t)g(x,t)+f(x,t)\quad (x,t)\in D_T,$$

with the conditions

$$u(x,0) = \int_0^T P_1(x,t)u(x,t)dx + \varphi(x),$$

$$u_t(x,0) = \int_0^T P_2(x,t)u(x,t)dx + \psi(x),\ 0 \le x \le 1,$$

$$u_x(0,t) = u(1,t) = 0,\quad u(x_i,t) = h_i(t),\ \ i = 1,2,\ \ 0 \le t \le T,$$

where $\alpha, \beta > 0$, $x_i \in (0,1)$ $(i = 1,2;\, x_1 \ne x_2)$ are fixed numbers, $f(x,t),\ g(x,t),\ P_1(x,t),\ P_2(x,t),\ \varphi(x), \psi(x),\ h_i(t)$ $(i = 1,2)$ are given functions, and

$\tilde{C}^{(2,2)}(D_T) :=$

$:= \{u(x,t) : u(x,t) \in C^2(D_T),\ u_{ttx}(x,t),\ u_{txx}(x,t),\ u_{ttxx}(x,t) \in C(D_T)\}.$

In the present paper, a nonlinear inverse boundary-value problem for the equation of longitudinal wave propagation is considered. First, the original problem is reduced to an equivalent problem. Further, the existence and uniqueness of the solution of the equivalent problem are proved using a contraction mappings principle. Finally, using the equivalency, the existence and uniqueness of a solution of the considered problem are obtained.

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OSCILLATION PROPERTIES OF EIGENFUNCTIONS OF A SPECTRAL PROBLEM WITH SPECTRAL PARAMETER CONTAINED IN THE BOUNDARY CONDITIONS

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We consider the following eigenvalue problem

$$y^{(4)}(x) - (q(x)y'(x))' = \lambda y(x),\ 0 < x < 1, \tag{1}$$

$$y(0) = y'(0) = y''(1) - a_1\lambda y'(1) = Ty(1) - a_2\lambda y(1) = 0, \tag{2}$$

where $\lambda \in C$ is a spectral parameter, $Ty \equiv y''' - qy'$, $q(x)$ is positive absolutely continuous function on $[0, 1]$, a_2, a_2 are constants such that $a_1 a_2 \neq 0.$

The spectral properties of problem (1)-(2) in the case $a_1 > 0,\ a_2 < 0$ was studied in the paper [1]. In this note we investigated the oscillatory properties of this problem and their derivatives in the case $a_1 > 0,\ a_2 > 0.$

Theorem. Let $a_1 > 0,\ a_2 > 0.$ Then the eigenvalues of problem (1)-(2) are real, simple and form an infinitely increasing sequence $\lambda_1, \lambda_2, \ldots, \lambda_k, \ldots$ such that $\lambda_1 < 0$ and $\lambda_k > 0$ for $k \geq 2.$ Moreover, the corresponding eigenfunctions and their derivatives have the following oscillation properties:

(i) the eigenfunction $y_k(x)$, corresponding to the eigenvalue λ_k, for $k = 2$ has no zeros, for $k \geq 3$ has either $k-3$ or $k-2$ simple zeros in the interval $(0, 1)$;

(ii) the function $y_k'(x)$ for $k \geq 2$ has exactly $k-2$ simple zeros in $(0, 1)$;

(iii) the number of zeros of functions $y_1(x)$ and $y_1'(x)$ contained in (0, 1) are $\sum_{\varsigma_m \in (\lambda_1, 0)} i(\varsigma_m)$, where $i(\varsigma)$ is the oscillation index of the eigenvalue ς of problem

$$y^{(4)}(x) - (q(x)y'(x))' = \lambda y(x),\ 0 < x < 1,$$
$$y(0) = y'(0) = y''(0) = Ty(1) - a_2 \lambda y(1) = 0.$$

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A VARIATIONAL APPROACH TO SOLVING THE PROBLEM OF THE STABILITY OF A GENTLE ARCH

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In the work, a variational principle of a mixed type is developed for determining the stress-strain state of bodies composed of a finite number of elements. The effectiveness of the proposed method is illustrated by the example of solving the problem of snapping a linearly elastic piecewise-uneven in thickness gently sloping arch, articulated at its ends, which is subject to a uniformly distributed vertical load.

Consider a gently sloping arch, articulated at two ends, whose axis is formed by an arc of a sinusoid

$$\omega = \eta c_0 \sin\left(\frac{\pi z}{l}\right), \qquad (1)$$

where c_0 is the arrow to raise the arch, l is the span of the arch, η is the determining parameter, and z is the longitudinal coordinate. The cross section of the arch is assumed to be rectangular with height - $2h$ and width - b.

The arch carries a uniformly distributed vertical intensity load - q. Due to the law of flat sections, we write:

$$\dot{\varepsilon} = \dot{u}_{,z} + \omega_{,z}\dot{\omega}_{,z} - y\dot{\omega}_{,zz}. \tag{2}$$

To find the stationary values of the functional [1], we use the Rayleigh – Ritz method. We write the approximating functions as follows [2]:

$$\dot{\sigma} = E_1\left(\dot{\sigma}_0 + \dot{\sigma}_1^v\left(\frac{2y}{h}\right)\right),\ u = 0, \tag{3}$$

here $\sigma_1^v = \sigma_1 \sin(\pi z / l)$.

Given the formulas (1) - (3) in the expression of the functional, after some mathematical transformations, the functional will take the form:

$$J = bE_1\frac{\pi^2}{l}c_0^2 h\dot{\sigma}_0\eta\dot{\eta} + \frac{2}{3}bE_1\frac{\pi^2 h^2}{l}c_0\dot{\sigma}_1\dot{\eta} - \frac{bl}{2}E_1^2\dot{\sigma}_0^2\Phi_0 -$$
$$-\frac{bl}{2h^2}E_1^2\dot{\sigma}_1^2\Phi_2 + \frac{2l}{\pi}c_0\dot{\eta}.$$

Varying J with respect to $\dot{\eta}$, $\dot{\sigma}_0$, $\dot{\sigma}_1$ and integrating the obtained formula under initial conditions, we arrive at a system of equations. Combining these equations, we get one expression for determining the stress-strain state of the arch.

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MINIMAL SUBALGEBRAS GENERATED BY UNARY 2-FUNCTIONS IN ALGEBRA $P_k \times P_m$

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Let, $P_{k_i} =< P_{k_i}; \zeta, \tau, \Delta, \nabla, * >$ - be the Post algebra sonthe set $E_{k_i} = \{0,1,...,k_i - 1\}$ ($2 \le |E_{k_i}| < \infty$, where $|E_{k_i}|$ -is the power of the set E_{k_i}); $P_{k_i} = \bigcup_{n=1}^{\infty} P_{k_i}^{(n)}$ -is the set all multiplace functions on E_{k_i} $(i = 1,2,...,m)$; $P_{k_i}^{(n)}$ are all n-place functions acting on the set E_{k_i}. $\zeta, \tau, \Delta, \nabla, *$ are algebraic operations equivalent to the superposition ($f \in P_k$).

Web will assume that all E_{k_i} are different and do not intersect. Under the direct product of the Post algebra we will understand their product as graded algebra:

$$P_{k_1} \times ... \times P_{k_m} =< \bigcup_{n=1}^{\infty} P_{k_1}^{(n)} \times ... \times P_{k_m}^{(n)}; \zeta, \tau, \Delta, \nabla, * >$$

It is known that if f is a not identical unary operation for which $f^2 = f$ or $f^q = id_{\underline{k}}$ - is an idential operation (q is a prime number), then $[f]$ is a minimal subalgebra in P_k.

Lemma 1. Let the number of minimal subalgebras in algebras P_k and P_m generated by unary-functions f for which $f^2 = f$ equal $\tilde{k}$ and $\tilde{m}$ respectively, and the number of minimal subalgebras generated by the unary functions f_i $\left(\tilde{f}_j\right)$ for which

$$\begin{array}{ll} f_1^{q_1} = id_{\underline{k}}, & \tilde{f}_1^{p_1} = id_{\underline{m}} \\ \vdots & \vdots \\ f_i^{q_i} = id_{\underline{k}}, & \tilde{f}_j^{p_j} = id_{\underline{m}} \end{array}$$

be equal $\tilde{q}_i$ and $\tilde{p}_j$, respectively where q_i and p_j are prime numbers. Then in algebra $P_k \times P_m$ the number of minimal subalgebras generated by unary 2-functions f for which $f^2 = f$ equals

$$\left(\tilde{k}+1\right)\left(\tilde{m}+1\right)-1$$

The number of minimal subalgebras generated by unary 2-functions for which

$$f^{q_t} = id_{\underline{k}\times\underline{m}}$$

equals $\left(\tilde{q}_t+1\right)\left(\tilde{p}_s+1\right)-1$, where $1 \le t \le i;\ 1 \le s \le j,\ q_t = p_s$.

If for $t_1 \in \{1,...,i\}$ there does not exist $s_1 \in \{1,...,j\}$, such that $q_{t_1} = p_{s_1}$ and vice –versa forthere does not exits $s_2 \in \{1,...,j\}$ $t_2 \in \{1,...,i\}$, such that, $P_k \times P_m$, the numbers of minimal subalgebras of algebra generated by unary2-

functions for which $f^{q_{t_1}} = id_{\underline{k}\times\underline{m}}$ and $f^{p_{s_2}} = id_{\underline{k}\times\underline{m}}$, respectively, equal q_{t_1} and p_{s_2}.

The proof of the lemma is not difficult if we take into account that unary 2-functions $f(X)=(f_1(x_1), x_2)$ and $\tilde{f}(X)=(x_1, f_1(x_2))$ $(X=(x_1,x_2))$ are non –trivial functions, if $f_1(x_1)$ in P_k and $f_2(x_2)$ is non-trivial in P_m.

Theorem 1. *In algebra $P_2 \times P_2$ there exist 11 minimal subalgebras generated by unary 2-functions.*

Theorem 2. *In algebra $P_2 \times P_3$ there exist 37 minimal subalgebras generated by unary 2-functions.*

Theorem 3. *In algebra $P_3 \times P_3$ there exist 117 minimal subalgebras generated by unary 2-functions.*

OSCILLATIONS OF ANISOTROPIC RECTANGULAR PLATE LYING ON NON-HOMOGENEUS VISCOUS ELASTIC BASE

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This paper explores the problem of free oscillation of a continuously non-homogenous anisotropic plate lying on a non-homogenous viscous elastic base. The reaction of the base with the deflection is bound as follows [3, 4]:

$$R = \left(k_1(x,y) + k_2(x,y)\frac{\partial^2}{\partial t^2} \right) w(x,y,) \qquad (1)$$

There w-deflection, t-time, $k_1(x,y)$ and $k_2(x,y)$ continuous functions that characterize the base property.

The coordinate system is selected as follows axis X and Y is in the median plane Z - perpendicular to it. Material characteristics and density are functions of three spatial coordinates.

$$a_{ij} = a_{ij}^0 f_1(x,y) f_2(z); \quad \rho = \rho_0 \psi_1(x,y)\psi_2(z) \qquad (2)$$

There a_{ij}^0 and ρ_0 -correspond to homogeneous anisotropic material, $f_1(x,y)$ with its derivatives to the second order $f_2(z),\ \psi_1(x,y),\ \psi_2(z)$ themselves are continuous functions.

In the case when the material characteristics and density are continuous functions of the constructed coordinates of the equation of motion, it is written as follows:

$$L(W) + K_1(x,y)W + \left(K_2(x,y)\frac{\partial^2}{\partial x^2} + \bar{\rho}\psi_1(x,y)\frac{\partial^2}{\partial t^2} \right) W(x,y) = 0 \quad (3)$$

The solution (3) will be sought in the following form:

$$W(x,y,t) = V(x,y)e^{i\omega t} \qquad (4)$$

There, $V(x,y)$ - the boundary conditions must be satisfied, ω - frequency.

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DIFFERENTIAL DIAGNOSIS ON THE BASIS CLASSIFICATION OF THE BIOSYSTEMS

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When applied to the input of initial symptoms, medical diagnosis or from parameters, output parameters describe the diagnosis is made. Dependence on the input parameters and output parameters are used for the one, two or multi neural network. The most important first step is to remove the issue of what neural network input and output parameter. It is unthinkable that such a question is, at first glance, but the neural networks “from scratch” began in the dozens of mathematically feasible to set up or can be solved using neural networks architecture. In the second stage, the neural network should be taught, its weight must be chosen in such a way that the network to work properly.

Although the function of complex neural network study process depends on a lot of these procedures. Many learning algorithms have been developed specifically for architectures, allows you to assign a particular weight to the neural network. The experimental study is done based on the nature of the subject area. Clinical signs of experimental data are considered in the process of medical diagnosis. Majority of clinical symptoms is determined by the input parameters. The neural network training process is interactive as the input parameters are determined by the correlation between the final diagnosis. The optimal combination of weights connecting the neurons found in adjacent layers at the same time.

Expert evaluation of the neural network approach is particularly effective, because the function of the computer to process and both the number and the recognition function is used to generalize the brain. Neural network processing a large amount of factors wagon “good doctor” in any field, as it can make a diagnosis.

Diagnosis of poisoning by carbon monoxide and other toxic substances in the specificity of their input and output parameters, the dependence of the peer assessment provides a basis for the establishment of the neural network. For doing that, we need to define input and output parameters. Firstly determine the structure of the neural network (fig.1):

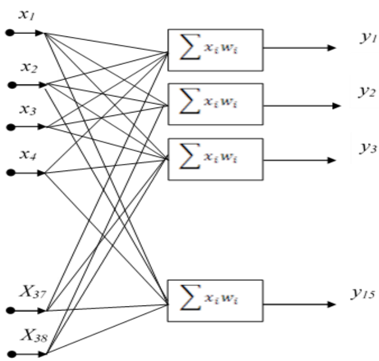

Fig. 1. A general description of the neural network with 15 hypothesis and 38 two layered symptoms.

The neural network creates as following:

$$y_j = \sum_{i=m,k,l,\dots}^{38} x_i^+ + \sum_{i=t,s,p,\dots}^{38} x_i^- + \alpha\left(\sum_{i=e,r,\dots,v}^{38} x_i^{yes/no}\right) \quad (1)$$

Where, x_i^+ - shows the mandatory presence of the parameter x_i in the corresponding hypothesis;

x_i^- - shows not the mandatory presence of the parameter x_i in the corresponding hypothesis;

$x_i^{yes/no}$ - shows the detection of insignificant parameter x_i in the corresponding hypothesis.

For that reason the evaluation will be:

$$x_i^{yes/no} = \begin{cases} x_i, if\ the\ symptom\ is\ definite, \\ 0, if\ the\ symptom\ is\ indefinite. \end{cases}$$

α – release of representatives that don’t included to sum up.

y_j – hypotheses, (j = 1,2,…,15);

x_i – parameters (i = 1,2,…,).

(1) In terms of the sum of the first and second hypothesis, we accept as a fact the existence of a zero, then

$$\alpha\left(\sum_{i=e,r,\dots,v}^{38} x_i^{yes/no}\right)$$

confirming the hypothesis that the expression of all the possible variations should be considered as a cluster. This cluster will be moved to two functions:

- All the α ≠ 0 incidence confirms the hypothesis;
- Training neural networks for the recognition of the majority of the α hypothesis set, the maximum value of the number of *(2n-1)*, the hypothesis, which is included in the *"yes/no"* equal to the number of *n*.

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ON THE GENERALIZATION OF SOME CLASSICAL FORMULAS FOR SUMS OF NUMBER SERIES

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The eigenvalues and eigenfunctions of certain operators generated by symmetric differential expressions with constant coefficients and self-adjoint boundary conditions in the space of Lebesgue squareintegrable functions on an interval are explicitly calculated, while the resolvents of these operators are integral operators with kernels for which the theorem on an eigenfunction expansion holds. In addition, each of these kernels is the Green's function of a self-adjoint boundary value problem, and the procedure for its construction is well known. Thus, the Green's functions of these problems can be expanded in series in terms of eigenfunctions. In this study, identities obtained by this method are used to calculate the sums of convergent number series and to represent the sums of certain power series in an intergral form.

In particular, the following is true. Let $P_n(x)$ be a polynomial of degree $n \geq 2$ with real coefficients. Consider a self-adjoint boundary value problem

$$\begin{cases} P_n\left(i\dfrac{d}{dx}\right)y = f \\ y^{(j)}(0) - y^{(j)}(2\pi) = 0, \qquad j = 0,\dots,n-1. \end{cases} \tag{1}$$

Теорема. *Let a polynomial with real coefficients* $p_m(x)$ *(*$m \geq 1$*) be such that* $p_m(k^2) \neq 0$ *for* $k = 0,1,..$ *and let*

$P_n(x) = p_m(x^2)$ *in problem (1). Then the Green's function* $G(x,t)$ *of this problem can be represented in the form*

$$G(x,t) = \frac{1}{2\pi p_m(0)} + \frac{1}{\pi}\sum_{k=1}^{+\infty}\frac{\cos k(x-t)}{p_m(k^2)}. \tag{2}$$

From equality (2), in turn, it follows that

$$\frac{1}{2p_m(0)} + \sum_{k=1}^{+\infty}\frac{1}{p_m(k^2)} = \pi G_0(0,0) \text{ and } \frac{1}{2p_m(0)} + \sum_{k=1}^{+\infty}\frac{(-1)^k}{p_m(k^2)} = \pi G_0(0,\pi). \tag{3}$$

Let $a \in (0,1)$ and $P_2(x) = x^2 - a^2$ in problem (1), then equalities (3) coincide with Euler's formulas fordecomposition of the functions $\pi ctg(a\pi)$ and $\pi / \sin(a\pi)$ into simple fractions. Thus, formulas (6) are, in a sense, generalizations of these decompositions (see more details [1]).

If we use the Green functions of other self-adjoint boundary value problems and Fourier series expansion of some elementary functions, then, in particular, we can obtain classical Euler formulas for the values of the Riemann ζ-function and the Dirichlet β-function at natural points and their generalization to an arbitrary admissible polynomial $p_m(x)$ (see [2]).

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QUANTUM MECHANICS THEORY AND QUANTUM COMPUTING BY NMR SPECTROSCOPY

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Quantum mechanics gradually arose from theories to explain observations which could not be reconciled with classical physics, such as Max Planck's solution in 1900 to the black-body radiation problem, and from the correspondence between energy and frequency in Albert Einstein's 1905 paper which explained the photoelectric effect. The modern theory is formulated in various specially developed mathematical formalisms. In one of them, a mathematical function, the wave function, provides information about the probability amplitude of position, momentum, and other physical properties of a particle. Important applications of quantum theory include quantum computing.

A quantum computer (QC) can operate in parallel on all its possible inputs at once, but the amount of information that can be extracted from the result is limited by the phenomenon of wave function collapse.

Researchers today are divided concerning whether or not it will ever be possible to build a QC of any significant size.

This paper describes a macroscopic analogue of a QC that can be implemented today, using commercially available NMR spectrometers and ordinary liquid samples. Such an NMR computer differs from a QC in that it uses the parallelism inherent in macroscopic ensembles to estimate expectation values, instead of the filtering and amplification concomitant upon wave function collapse. This enables it to efficiently solve a wider variety of problems, including NP-complete problems, although it may need an exponentially increasing sample size to do so.

The experimental issues involved in NMR computing, and in particular general methods for the preparation of pseudo-pure states, are being investigated. Despite its acknowledged limitations, it is possible that an NMR computer (or possibly some other type of EQC) will someday be built that can (with suitable algorithms) solve problems beyond the reach of conventional computers. Besides NP-complete problems, an NMR computer should be able to factorize integers by either a direct search procedure, or possibly by procedures based on Miller’s approach which scale better. Another likely application would be to simulate the statistical behavior of open quantum systems, as originally proposed for closed quantum systems using QCs by R. P. Feynman.

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DISCRETENESS OF THE SPECTRUM OF NEW TYPE BOUNDARY-VALUE PROBLEMS

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Abstract

The main goal of this study is to investigate some spectral properties of a new type boundary value problems, the main feature of which is the nature of the equation and boundary conditions imposed. We established some important spectral properties of the eigenvalues and corresponding eigenfunctions. Particularly we proved that the spectrum of the considered problem is discrete and derived some asymptotic formulas for the eigenvalues.

1. Statement of the problem

The classical boundary value problems (BVP's) arise as a mathematical modeling of many processes in various fields of natural science. But investigation of some processes in chemistry, aerodynamics, fluid mechanics, diffusion and etc. lead to various type nonclassical forms of BVP's (see, for example, [1,2,3,4]). In the present work we shall examine a new type of non-classical boundary value problems, consisting of a two-order differential equation

$$-y''(x)+q_1(x)y(x)+q_2(x)y(s(x))=\lambda y(x) \tag{1}$$

on $x\in[a,c_1)\cup(c_1,c_2)\cup(c_2,b]$, with boundary conditions at the end points $x=a$ and $x=b,$ given by

$$y(a)=y(b)=0 \tag{2}$$

and four transmission conditions at two interior points of interactions $x=c_1$ and $x=c_2,$ given by

$$\mathrm{y}(\mathrm{c}_1-0)=\alpha_1\,\mathrm{y}(\mathrm{c}_1+0) \quad ,\ \mathrm{y}(\mathrm{c}_2-0)=\alpha_2\,\mathrm{y}(\mathrm{c}_2+0) \tag{3}$$

$$\mathrm{y}'(\mathrm{c}_1-0)=\alpha_1^{'}\,\mathrm{y}'(\mathrm{c}_1+0) \quad ,\ \mathrm{y}'(\mathrm{c}_2-0)=\alpha_2^{'}\,\mathrm{y}'(\mathrm{c}_2+0) \tag{4}$$

where the functions $q_i(x)$ are continuous in the intervals (a,c_1) , (c_1,c_2) and (c_2,b) with the finite limits $q_i(a+0),q_i(c_1\pm0),q_i(c_2\pm0),q_i(b-0),i=1,2,\lambda$ is a complex spectral parameter, $\alpha_i,\alpha_i^{'}$ are real numbers with $\alpha_i.\alpha_i^{'}=1,i=1,2,s(x)$ is twice continuously differentiable function from [a,b] to [a,b].

3. Statement of the problem

For operator treatment of the considered problem let us define the linear operator $\mathrm{L}: \oplus\, \mathrm{L}_2 \to \oplus\, \mathrm{L}_2$ with the domain of definition

$$\mathrm{D(L)} = \left\{ \begin{array}{l} \mathrm{y}: \mathrm{y} \in \oplus \mathrm{W}_2^2, \mathrm{y(a)} = \mathrm{y(b)} = 0, \mathrm{y}(\mathrm{c_i} - 0) = \alpha_{\mathrm{i}} \mathrm{y}(\mathrm{c_i} + 0), \\ \mathrm{y}'(\mathrm{c_i} - 0) = \alpha_i^{'} \mathrm{y}'(\mathrm{c_i} + 0), \ \mathrm{i}=1,2 \end{array} \right\}$$

and action low

$$Ly := -y''(x) + q_1(x)y(x) + q_2(x)y(s(x))$$

where $\oplus \mathrm{L}_2 := \oplus_{i=1}^{3} \mathrm{L}_2(c_{i-1}, c_i)$, $\oplus \mathrm{W}_2^2 := \oplus_{i=1}^{3} \mathrm{W}_2^2(c_{i-1}, c_i)$, $c_0 = a$, $c_3 = b$.
Let $\mathrm{L}_0 \mathrm{y} := \mathrm{Ly} - \mathrm{y(s(x))}$.

Let us formulate the main results of this study.

Theorem 2.1. The linear differential operator L_0 is densely defined and symmetric in the Hilbert space $\oplus\, \mathrm{L}_2$.

Corollary 2.2. The eigenvalues of L_0 are real and the eigenfunctions corresponding to different eigenvalues are orthogonal.

Theorem 2.3. The linear differential operator L_0 has a precisely denumerable many eigenvalues $\lambda_{\mathrm{n}} = s_n^2, n = 1, 2, \ldots$ for which the asymptotic formula $s_{\mathrm{n}} = \dfrac{\pi n}{b-a} + O\left(\dfrac{1}{n}\right)$ holds.

Theorem 2.4. There are $\mathrm{K} > 0$, $\mathrm{C} > 0$ and $\delta > 0$ such that the estimate $\left\| \left(\lambda \mathrm{I} - \mathrm{L} \right)^{-1} \right\| \le C |\lambda|^{-1}$ holds for all λ with $\delta < \arg\lambda < 2\pi - \delta$, $|\lambda| > \mathrm{K}$.

Theorem 2.5. The spectrum of L is discrete.

Remark 2.6. The eigenvalues of L are not real in general.

Theorem 2.7. The spectrum of L consist of denumerable many eigenvalues $\tilde{\lambda}_n = \mu_n + i\,\tau_n$, n = 1,2,... for which the asymptotic formulas

$$\lim_{n\to\infty} n^{-2}\left|\mu_n - \frac{\pi^2 n^2}{(b-a)^2}\right| = 0 \text{ and } \lim_{n\to\infty} n^{-2}\left|\tilde{\lambda}_n - \mu_n\right| = 0$$

is valid.

Keywords : Boundary-value problems, discreteness, transmission conditions.

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ON BASICITY OF PERTURBED EXPONENTIAL SYSTEM WITHPIECEWISE LINEAR PHASE IN MORREY-TYPE SPACES

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An exponential system with piecewise linear phase depending on some parameters is considered in this work. Basis properties of this system (such as completeness, minimality and basicity) are studied in a subspace of Morrey space where continuous functions are dense. A sufficient condition for the completeness (minimality or basicity) of this system in the mentioned subspace is found.

First we define the Morrey space on the unit circle $\gamma = \{z \in C : |z| = 1\}$ on the complex plane C. Next, $\omega = \operatorname{int} \gamma$

will denote the unit ball in C. By $L_0(-\pi, \pi)$ we denote the linear space of all (Lebesgue-) measurable functions on $(-\pi, \pi)$. $L^{p,\alpha}(\gamma)$, $1 \le p < +\infty$, $0 \le \alpha \le 1$, will denote the normed space of all measurable functions $f(\cdot)$ on γ with the finite norm

$$\|f\|_{L^{p,\alpha}(\gamma)} = \sup_B \left(|B \cap \gamma|_\gamma^{\alpha-1} \int_{B\cap\gamma} |f(\xi)|^p |d\xi| \right)^{1/p} < +\infty ,$$

($|B \cap \gamma|_\gamma -$is the linear measure of intersection $B \cap \gamma$), where $\sup$ is taken over all balls centered at γ with an arbitrary positive radius. $L^{p,\alpha}(\gamma)$ is a Banach space with respect to this norm. We also define the space $L^{p,\alpha}(-\pi,\pi)$, $1\le p<+\infty$, $0\le\alpha\le1$, which consists of measurable functions $f(\cdot)$ on $(-\pi,\pi)$ with the finite norm

$$\|f\|_{L^{p,\alpha}(-\pi,\pi)}=\sup_{I\subset[-\pi,\pi]}\left(|I|^{\alpha-1}\int_I|f(t)|^p|dt|\right)^{1/p}<+\infty,$$

where sup is taken over all intervals $I\subset[-\pi,\pi]$.

Consider the subspace $M^{p,\alpha}$ of functions $f(\cdot)$ the shifts of which are continuous in $L^{p,\alpha}$, i.e.

$$\|f(\cdot+\delta)-f(\cdot)\|_{p,\alpha}\to0,\quad \delta\to0.$$

To determine the basicity of the exponential system

$$E_\beta=\left\{e^{i(nt-\beta|t|\,signn)}\right\}_{n\in Z},$$

inMorrey spaces $M^{p,\alpha}$, we used the method of boundary value problems.

The following main theorem is proved.

Theorem. *The system E_β forms a basis for the space $M^{p,\alpha}$, $1<p<+\infty$, for $\forall\beta\in C$.*

Attention should be paid to the fact that the corresponding result in case of the system $e_\beta\equiv\left\{e^{i(n+\beta signn)t}\right\}_{n\in Z}$, is quite different. Namely, the basicity of e_β requires inequality type restriction on β.

INVESTIGATION OF ANISOTROPIC FRACTIONAL MAXIMAL OPERATOR IN ANISOTROPIC MORREY-TYPE SPACES

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Let $f \in L_1^{loc}(R^n)$. The anisotropic maximal operator M_α^d is defined by

$$\left(M_\alpha^d\right)(x) = \sup_{t>0} |B(x,r)|^{-1+\frac{\alpha}{n}} \int_{B(x,r)} |f(y)| dy,$$

where $0 \le \alpha < n$, $|B(x,r)|$ is the Lebesgue measure of the anisotropic ball $B(x,r)$.

Definition. Let $0 < p, \theta \le \infty$ and let w be a non-negative measurable function on $(0,\infty)$. We denote by $LM_{p\theta,w,d}$ the local Morrey-type space, the space of all functions $f \in L_p^{loc}(R^n)$ with finite quasi-norms

$$|f|_{LM_{p\theta,w,d}(R^n)} \equiv \left\| w(r) \| f \|_{L_p(B(0,r))} \right\|_{L_\theta(0,\infty)}.$$

Theorem. Let $\frac{|d|}{p_1} \le \alpha < |d|$, $1 < p_1 \le p_2 < \infty$, $0 < \theta_1, \theta_2 \le \infty$, $w_1 \in {}^C\Omega_{\theta_1}$, $w_2 \in \Omega_{\theta_2}$. If $\theta_1 \le p_1$ and $p_2 \le \theta_2$ and

$$\sup_{B\subset R^n}\left\| M_\alpha^d\left(\chi_{B(x,r)}W_2^{\frac{p_1}{1-p_1}}\right)\right\|_{L_{p_2,w_2,d}(B)}\left\| W_1^{\frac{p_1}{1-p_1}}\right\|_{L_{p_1}}^{-1}$$

where for all $x\in R^n$

$$W_1=\|w_1\|_{L_{\theta_1}(0,\rho(x))},\quad W_2=\|w_2\|_{L_{\theta_2}(\rho(x),\infty)},$$

then M_α^d is bounded from ${}^C LM_{p_1\theta_1,w_1,d}$ to $LM_{p_2\theta_2,w_2,d}$.

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COERCIVE ESTIMATE FOR ABSTRACT ELLIPTIC EQUATIONS WITH PARAMETERS

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It is well known that the differential equations with parameters play important role in modeling of physical processes [1-3]. Convolution-differential operator equations (CDOEs) with small parameters have also significant applications to the developed theory to problems in mathematical physics. Note that, CDOEs have been investigated, e.g., in [4].

The main aim of this paper is to show the uniform separability properties of boundary value problems (BVPs) for the following CDOE
with parameters

$$Lu = -\varepsilon u''(t) + A_\lambda u(t) + \varepsilon^{1/2}(aA_1 * u')(t) + (A_0 * u)(t) = f(t),\ t \in (0,\infty) \tag{1}$$

$$L_1 u = \varepsilon^{(p+1)/(2p)}\alpha u'(0) + \varepsilon^{1/(2p)}\beta u(0) = f_0$$

where $u(t) = u(\varepsilon, t)$ is a solution of (1), $A, A_1 = A_1(t), A_0 = A_0(t)$ are linear operators in a Banach space E, $A_\lambda = A + \lambda I, a = a(t)$ is a scalar valued function on $(0,\infty)$, $f_0 \in E_p = (E(A), E)_{\theta,p}$, here $(E(A), E)_{\theta,p}$ denotes the real interpolation space between $E(A)$ and E, $p \in (1,\infty)$, $\theta = (1+p)/(2p), \alpha, \beta$ are complex numbers, ε is a small positive, and λ is a complex parameters, i.e. $\varepsilon \in (0,1)$.

Firstly consider the nonhomogeneous problem

$$\begin{cases} -\varepsilon u''(t) + A_\lambda u(t) = f(t) \\ L_1 u = f_0 . \end{cases}$$

Theorem. Assume the following conditions are satisfied,
1) E is a Banach space satisfying the uniformly multiplier condition for $p \in (1,\infty)$;
2) A is a R positive operator in E for $0 \le \varphi < \pi$.
3) $a(t) \in L_1(R_+),\ A_1(t)A^{-(1/2-\mu_1)} \in L_\infty(R_+; B(E))$, $A_0(t)A^{-(1-\mu_2)}$

$\in L_\infty(R_+;B(E))$ for $0<\mu_1<\frac{1}{2}$, $0<\mu_2<1$, and $-\beta\alpha^{-1}\in S_{\varphi_1}$, $0\le\varphi_1+\varphi<\pi$. Then for all $f\in L_p(R_+;E)$ and sufficiently large $|\lambda|>\lambda_0>0$ there exists a unique solution $u\in W_p^2(R_+;E(A),E)$ of the problem (1) and the following coercive uniform estimate holds

$$\sum_{i=0}^{2}\varepsilon^{i/2}|\lambda|^{1-i/2}\left\|u^{(i)}\right\|_{L_p(R_+;E)}+\|Au\|_{L_p(R_+;E)}\le c\|f\|_{L_p(R_+;E)}.$$

Let L_ε and $L_{0\varepsilon}$ are operators in $L_p(R_+;E)$ generated by (1) and (2) for
$\lambda=0$, respectively i.e., $L_\varepsilon u=L_{0\varepsilon}u+L_{1\varepsilon}u$, where
$L_{1\varepsilon}u=\varepsilon^{i/2}aA_1*u'+A_0*u$, $L_{0\varepsilon}u=-\varepsilon u''+A_\lambda u$,
$D(L_{0\varepsilon})=W_p^2(R_+;E(A),E,L_1)$.

It is easy to see that

$$\left\|L_{1\varepsilon}(L_{0\varepsilon}+\lambda)^{-1}\right\|<1. \tag{3}$$

In a similar way we obtain

$$(L_\varepsilon+\lambda)^{-1}=(L_{0\varepsilon}+\lambda)^{-1}\left[I+L_{1\varepsilon}(L_{0\varepsilon}+\lambda)^{-1}\right]^{-1}. \tag{4}$$

Due to perturbation theory of linear operators and the estimate (3) we obtain that the differential operator $L_\varepsilon+\lambda$ has a bounded inverse from $L_p(R_+;E)$ into $W_p^2(R_+;E(A),E)$.

Hence the estimate (3), (4) and the above relations implies the assertion.

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ASYMPTOTIC VALUE OF APPROXIMATION OF GENERALIZED DIFFERENTIABLE FUNCTIONS BY M-SINGULAR INTEGRALS

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In the paper we study asymptotic equalities on approximation of generalized differential functions by means of *m* singular integrals and the obtained results are applied to Valle – Pussen, Poisson *m* –singular integrals.

Let $L_{2\pi}^{p}$ be a space of 2π periodic functions $f(r)$ summable on $[-\pi,\pi]$ in *p*-th power with the norm

$$\|f(r)\|_{L_{2\pi}^{p}} = \left(\int_{-\pi}^{\pi} | f(r) dr \right)^{\frac{1}{p}} \quad (1 \le p < \infty)$$

$$\|f(r)\|_{L_{2\pi}^{\infty}} = \underset{|r| \le \pi}{ess \sup} \, | f(r) | .$$

For approximation of functions $f(r)\in L_{2\pi}^{p}$ we consider the m-singular integral

$$A_{\lambda}^{[m]}(f,r)=\int_{-\pi}^{\pi}\left[\sum_{k=1}^{m}(-1)^{k-1}\binom{m}{k}f(r+kt)K_{\lambda}(t)dt,\right]$$

where 2π -periodic function $K_{\lambda}(t)$ depends on the parameter λ and satisfies the conditions:

1^0. $K_{\lambda}(t)$ is an even function on $[-\pi,\pi]$

2^0. $\int_{-\pi}^{\pi}K_{\lambda}(t)dt=1.$

Theorem. Let $f(r)\in L_{2\pi}^{p}$, the non-negative function $K_{\lambda}(t)$ satisfy conditions $1^0,2^0$ and

$$\nu_{\lambda,\delta}=\int_{g}^{\pi}K_{\lambda}(t)dt=0\ \ [\gamma_{\lambda,0}^{(s)}(\varphi)]\ as\ \lambda\to\infty,\ \ for\,any\,\delta>0.$$

Then if there exists a function $d_{L_{2\pi}^{p}}^{(m,s)}f(r)$ such that

$$\lim_{t\to 0}\left\|\frac{1}{t^{s}}\varphi_{m}(f,r,t)-d_{L_{2\pi}^{p}}^{(m,s)}f(r)\right\|_{L_{2\pi}^{p}}=0,$$

then the following equality holds

$$\lim_{\lambda\to\infty}\left\|\frac{(-1)^{m+1}}{\gamma_{\lambda,0}^{(s)}(\varphi)}\left[A_\lambda^{[m]}(f,r)-f(r)\right]-d_{L_{2\pi}^p}^{(m,s)}f(r)\right\|_{L_{2\pi}^p}=0,$$

where

$$d_{L_{2\pi}^p}^{(m,s)}f(r)=\lim_{t\to0}\frac{\varphi_m(f,r,t)}{t^s},$$

$$\varphi_m(f,r,t)=[\Delta_t^m+\Delta_{-t}^m]f(r)\ (1\le s\le m+1)$$

$$\Delta_t^m f(r)=\sum_{k=0}^m(-1)^{m-k}\binom{m}{k}f(r+kt).$$

CHARACTERIZATION OF FRACTIONAL MAXIMAL OPERATOR ON ORLICZ SPACES IN THE DUNKL SETTING

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Harmonic analysis associated to the Dunkl transform and the Dunkl differential-difference operator gives rise to convolutions with a relevant generalized translation. On the R^d the Dunkl operators $\{D_{k,j}\}_{j=1}^d$ are the differential-difference operators associated with the reflection group Z_2^d on R^d.

Let $B(x,r):=\{y \in R^d : |x-y|<r\}$ denote the ball in R^d that centered in $x \in R^d$ and having radius $r>0$. Then having $|B(0,r)|_k = \int_{B(0,r)} h_k^2(x)dx$.

The *fractional maximal operator* $M_{\alpha,k}$, $0<\alpha<d+2\gamma_k$ associated with the Dunkl operator on R^d is given by

$$M_{\alpha,k} f(x) = \sup_{r>0} \left(|B(x,r)|_k\right)^{-1+\frac{\alpha}{d+2\gamma_k}} \int_{B(x,r)} |f(y)| h_k^2(y)dy, \ \forall x \in R^d.$$

It is well known that fractional maximal operators play an important role in harmonic analysis. In this work, in the framework of the Dunkl analysis in the setting R^d, we shall give a necessary and sufficient condition for the boundedness of fractional maximal operator $M_{\alpha,k}$ on Orlicz spaces $L_\Phi\left(R^d, h_k^2(x)dx\right)$ and weak Orlicz spaces $WL_\Phi\left(R^d, h_k^2(x)dx\right)$.

This contribution is based on recent joint work with Y.Y. Mammadov.

QUANTITATIVE GENERALIZATION OF THE HARTMAN-SARASON THEOREM

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Let T be a contraction on a Hilbert space H and assume that

$$\lim_{n\to\infty} \|T^n x\| = \lim_{n\to\infty} \|T^{*n} x\| = 0 \text{ for all } x \in H.$$

In addition if $\dim\left(I-TT^*\right)H = \left(I-T^*T\right)H = 1$, then by the Model Theorem of Nagy-Foiaş [1, Ch.6], T is unitary equivalent to its model operator $S_\theta = P_\theta S\big|_{H^2_\theta}$ acting on the model space $H^2_\theta = H^2 - \theta H^2$, where H^2 is the Hardy space, S is the unilateral shift operator on H^2, θ is an inner function, and P_θ is the orthogonal projection from H^2 onto H^2_θ. The Beurling-Helson theorem [2, Lecture I] says that the spaces H^2_θ are generic invariant subspaces for the backward shift operator S^*. Let θ be an inner function and let S_θ be the model operator on the model space H^2_θ. For an arbitrary $f \in H^\infty$, we can define the operator $f(S_\theta) = P_\theta f(S)\big|_{H^2_\theta}$ which is unitary equivalent to $f(T)$. The map $f \to f(S_\theta)$ is linear, multiplicative, and by the Nehari formula, $\|f(S_\theta)\| = \operatorname{dist}(\bar{\theta} f, H^\infty)$ [2, Lecture VIII]. Let us mention that the classical theorem of Hartman and Sarason [2, Lecture VIII] classifies compactness of the operator $f(S_\theta)$: The operator $f(S_\theta)$ is compact if and only if $\bar{\theta} f \in H^\infty + C(\Gamma)$. We have the following quantitative generalization of the Hartman-Sarason theorem.

Theorem. Let θ be an inner function and let S_θ be the model operator on the model space H^2_θ. Then, for an arbitrary $f \in H^\infty$, we have $\|f(S_\theta)\|_{ess} = \operatorname{dist}(\bar{\theta} f, H^\infty + C(\Gamma))$.

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BOUNDEDNESS OF WEIGHTED ITERATED HARDY-TYPE OPERATORS INVOLVING SUPREMA FROM WEIGHTED LEBESGUE SPACES INTO WEIGHTED CESARO FUNCTION SPACES

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In this work the boundedness of the weighted iterated Hardy-type operator involving suprema from weighted Lebesgue spaces into weighted Cesaro function spaces is characterized. As an application of obtained results, the norm of the fractional maximal operator between the weighted Lorentz spaces is calculated.

ON EMBEDDING THEOREMS IN SMALL SMALL SOBOLEV-MORREY SPACES

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In the abstract small small Sobolev-Morrey spaces were constructed. Differential and differential-difference properties of

functions from the constructed space were studied the integral representations method. More precisely,here by method integral representations we prove an Sobolev-type inequality in these spaces, and also prove that for $f \in W^{l}_{(p,(\chi,a,\alpha}(G)$, the generalized derivatives of $D^{\nu} f$ satisfy the Holder condition.

VISUALIZATION OF CONTROL OF ISOBUTENE DEHYDROGENATION PROCESS BASED ON PHASE TRAJECTORIES METHOD

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The paper proposes a set of differential equations relating to dynamics of thermal states of reaction-regeneration systems with fine-dispersed catalysts. Particular emphasis is placed on the existence of positive feedbacks between thermal and chemical processes through two interrelated channels – by temperature and degree of residual carburization of a catalyst after regeneration. It is demonstrated that the said set of model equations of the reaction-regeneration system, thermo-dynamics processes a plurality of stationary states in the space of the named variables.

Unlike the overwhelming majority of cracking processes, those of hydrocarbon dehydrogenation are conducted in a catalyst pseudo-fluidized bed, which makes it necessary to coke into account the third phase coordinate of a reactor-regenerator dynamic system [1]. This coordinate is coke content on a spent catalyst.

Our mathematical model takes into account the rate of coke formation in the fluidized bed of the catalyst. In this regard, the model is transformed into a system of three differential equations [2]. The study of such a model in a three-dimensional state space requires the development of special algorithms for

screen display of phase paths. The need for this development comes from the requirement of visualization for management.

In addition, the study required the solution of a number of problems in the field of constructing algorithms for controlling the state of the system.

Analysis of three-dimensional phase portraits of the system has been made, parameters have been studied. A subject of operating control based on visualization algorithms of state space structure has been considered.

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ON LATERAL VIBRATIONS OF EXPONENTIALLY INHOMOGENEOUS ANISOTROPIC RECTANGULAR PLATE LYING ON VISCOELASTIC FOUNDATION

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At present when constructing large engineering complexes and in many other fields of engineering the structural elements made of natural and artificial materials are widely used.

Rectangular plates are the most common ones among them.

In this paper we suppose that elastic characteristics and density depend on two coordinates (x, z).

The coordinate system is chosen in the following way: the axes X and Y are in the middle surface. The axis Z is perpendicular to them

$$a_{ij} = a_{ij}^0 f(z)\left(1 + \varepsilon e^{\bar{x}}\right); \quad \rho = \rho_0 \eta_1(z) \eta_2(x) \tag{1}$$

Here a_{ij}^0 and ρ_0 correspond to the homogeneous case $\bar{x} = xa^{-1}$.

It is supposed that the plate lies on a visco-elastic foundation whose reaction R is connected with deffection by the following relation

$$R = k_1 w + k_2 \frac{\partial^2 w}{\partial t^2}, \tag{2}$$

where k_1 and k_2 are the characteristics of the foundation, t is time.

Relation between the stress and strain tensor components are written in the form [12]

$$\sigma_{11} = \left(a_{11}^0 \varepsilon_{11} + a_{12}^0 \varepsilon_{22} + a_{13}^0 \varepsilon_{12}\right) f(z)\left(1 + \varepsilon e^{\bar{x}}\right)$$
$$\sigma_{22} = \left(a_{21}^0 \varepsilon_{11} + a_{12}^0 \varepsilon_{22} + a_{23}^0 \varepsilon_{12}\right) f(z)\left(1 + \varepsilon e^{\bar{x}}\right) \qquad (3)$$
$$\sigma_{12} = \left(a_{31}^0 \varepsilon_{11} + a_{32}^0 \varepsilon_{22} + a_{33}^0 \varepsilon_{12}\right) f(z)\left(1 + \varepsilon e^{\bar{x}}\right)$$

Itisacceptedthatforcontinuouslyinhomogeneousansitropic platetheKirchoff-Liav hypothesis is valid and we have

$$\varepsilon_{11} = e_{11} - z\chi_{11}$$
$$\varepsilon_{22} = e_{22} - z\chi_{22} \qquad (4)$$
$$\varepsilon_{12} = e_{12} - z\chi_{12}$$

Here e_{11}, e_{22}, e_{12} are small deformations and curvature of the central surface.

Aftersimplifications:

$$\int_{-h/2}^{+h/2} \sigma_{11} dz = 0; \quad \int_{-h/2}^{+h/2} \sigma_{22} dz = 0; \quad \int_{-h/2}^{+h/2} \sigma_{12} dz = 0\,. \qquad (5)$$

Substituting the value $\sigma_{11}, \sigma_{22}, \sigma_{12}$ in (7) we get

$$a_{11}^0 e_{11} + a_{12}^0 e_{22} + a_{13}^0 e_{12} = A_2 \cdot A_1^{-1}\left(a_{21}^0 e_{11} + a_{22}^0 e_{22} + a_{23}^0 e_{12}\right)$$
$$a_{21}^0 e_{11} + a_{22}^0 e_{22} + a_{23}^0 e_{12} = A_2 \cdot A_1^{-1}\left(a_{21}^0 e_{11} + a_{22}^0 e_{22} + a_{23}^0 e_{12}\right) \quad (6)$$
$$a_{31}^0 e_{11} + a_{32}^0 e_{22} + a_{33}^0 e_{12} = A_2 \cdot A_1^{-1}\left(a_{31}^0 e_{11} + a_{32}^0 e_{22} + a_{33}^0 e_{12}\right)$$

Here $A_1 = \int_{-h/2}^{+h/2} f(z) dz; \; A_2 = \int_{-h/2}^{+h/2} z(z) dz.$

Using (6) for $\varepsilon \neq 0$ we can determine the expression for the moments through W and substituting in the motin of equation and introducing the denotation, we get:

$$L_1(w) + L_2(w) + L_3(w) + k_1 w + \left(\bar{\rho}\psi(x) + k_2\right)\frac{\partial^2 w}{\partial t^2} = 0 \qquad (7)$$

When solving (7) we will apply the combined method. In the first step we use the method of separation of variables, in the second step
theBunov –Galerkin method, and we choose deflection in the form:

$$W(x, y, t) = V(x, y)e^{iwt} \qquad (8)$$

here $V(x, y)$ satisfies homogeneous boundary conditions w is frequency t is time.

Hence for calculations ω^2 we get the following formula

$$\omega^2 = \frac{\int_0^a\int_0^b \left(\overline{L}_1(\varphi,\psi) + \overline{L}_2(\varphi,\psi) + \overline{L}_3(\varphi,\psi) + k_1\varphi_1 \cdot \psi_1\right)\varphi_1\psi_1 dxdy}{\int_0^a\int_0^b \left(k_2 + \overline{\rho}\varphi(x)\right)\varphi_1^2(x)\psi_1^2(x)dxdy} \qquad (9)$$

UNIFORM CONVERGENCE OF FOURIER SERIES EXPANSIONS IN THE SYSTEM OF ROOT FUNCTIONS OF A SPECTRAL PROBLEM CORRESPONDING TO BENDING VIBRATIONS OF A HOMOGENEOUS ROD

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We consider the following spectral problem

$$y^{(4)}(x) - (q(x)y'(x)' = \lambda y(x),\ 0 < x < 1, \ (1)$$

$$y''(0) = y''(1) = Ty(0) - a\lambda y(0) = Ty(1) - c\lambda y(1) = 0, (2)$$

where $\lambda \in C$ is a spectral parameter, $Ty \equiv y'' - qy'$, $q(x)$ is positive absolutely continuous function on [0, 1], a, c are real constants such that $a > 0$ and $c \neq 0$.

Problem (1), (2) describes the bending vibrations of a homogeneous rod, in the cross sections of which a longitudinal force acts, the left end of which is rigidly fixed, and an elastically fixed load is concentrated on the right end.

From the results of [2, 3] it follows that the eigenvalues of problem (1), (2) are real, simple, with the exception of at least one, and form an infinitely increasing sequence $\{\lambda_k\}_{k=1}^{\infty}$ such that $\lambda_k > 0,\ k = 4, 5, \ldots$. Moreover, if r and l ($r \neq l,\ r, l \geq 4$) are arbitrary fixed natural numbers that have different parities, then the system of root functions $\{y_k(x)\}_{k=1,\,k\neq r,l}^{\infty}$ of this problem forms a basis in the space $L_p(0,1),\ 1 < p < \infty$ (even a Riesz basis for $p = 2$).

Let $f(x) \in C[0,1]$ and

$$\Delta^0_{f,r,l} = \begin{vmatrix} y_r(0) & y_l(0) \\ y_r(x) & y_l(x) \end{vmatrix},\ \Delta^1_{f,r,l} = \begin{vmatrix} y_r(x) & y_l(x) \\ y_r(1) & y_l(1) \end{vmatrix}.$$

The following theorem holds.

Theorem. *Let r and l ($r \neq l,\ r, l \geq 4$) are arbitrary fixed positive integers that have different parity and the Fourier series expansion of function $f(x) \in C[0,1]$ in the system $\{\sqrt{2}\sin k\pi x\}_{k=1}^{\infty}$ uniformly converges on the interval $[0, 1]$. Then the Fourier series expansion of function $f(x)$ in the system $\{y_k(x)\}_{k=1,\,k\neq r,l}^{\infty}$ uniformly converges on each interval $[b, d]$ $(0 < b < d < 1)$, if $\Delta^0_{f,r,l}\Delta^1_{f,r,l} \neq 0$, uniformly converges on each interval $[0, d]$, if $\Delta^0_{f,r,l} \neq 0,\ \Delta^1_{f,r,l} = 0$, and uniformly converges on each interval $[b, 1]$, if $\Delta^0_{f,r,l} = 0,\ \Delta^1_{f,r,l} \neq 0$. Moreover, the Fourier series expansion of $f(x)$ uniformly converges on the interval $[0, 1]$ if and only if $\Delta^0_{f,r,l}\Delta^1_{f,r,l} = 0$.*

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GLOBAL BIFURCATION OF SOLUTIONS OF NONLINEAR STURM-LIOUVILLE PROBLEMS WITH INDEFINITE WEIGHT

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We consider the nonlinear Sturm-Liouville problem

$$-(py')' - qy = \lambda r(x)y + h(x, y, y', \lambda),\ x \in (0, 1), \tag{1}$$

$$U_1(y) \equiv \alpha_0 y(0) - \beta_0 y'(0) = 0, \tag{2}$$

where $\lambda \in R$ is a spectral parameter, $p(x) \in C^1[0,1]$, $q(x) \in C[0,1]$, $r(x) \in C[0,1]$, $p(x) > 0$, $q(x) \geq 0, x \in [0,1]$, $meas\{x \in [0,1]: \sigma r(x) > 0\} > 0$ for each $\sigma \in \{+,-\}$, α_0, β_0, α_1, β_1 are real constants such that $|\alpha_i| + |\beta_i| > 0$, and $\alpha_i \beta_{i1} \geq 0$, $i = 0,1$. The nonlinear term has a representation $h = f + g$, where $f, g \in C\left([0,1] \times R^3\right)$ and satisfy the following conditions:

$$u\, f(x,u,s,\lambda) \leq 0,\ u\, g(x,u,s,\lambda),\ (x,u,s,\lambda) \in [0,\,1] \times R^3;$$

there exists $M > 0$ such that

$$\left|\frac{f(x,u,s,\lambda)}{u}\right| \le M,\ (x,u,s,\lambda)\in[0,\,1]\times R^3;$$

for any bounded interval $\Lambda \subset R$

$$g(x,u,s,\lambda) = o(|u|+|s|) \text{ as } |u|+|s|\to 0,$$

uniformly for $x\in[0,\,1]$ and $\lambda\in\Lambda$.

Global bifurcation of problem (1)-(3) for $f\equiv 0$ was investigated in [1].

Let $E = C^1[0,\,\pi]\bigcap\{u: U_1(u)=U_2(u)=0\}$ be a Banach space with the usual norm. For each $k\in\mathrm{N}$, each $\sigma\in\{+\,,-\}$ and each $\nu\in\{+\,,-\}$ let $S_k^{\sigma,\nu}$ be the set of $u\in E$ which have exactly $k-1$ simple zeros in $(0,\,1)$, $\sigma\int\limits_0^1 r(x)u^2(x)\,dx>0$ and $\nu u(x)$ is positive for small $x>0$. These set are open subsets of E and $\partial S_k^{\sigma,\nu} = \{u\in E$: either u must have a double zero in $[0,\,1]$ or $\int\limits_0^1 r(x)u^2(x)\,dx=0$.

It follows from [2] that the eigenvalues of problem (1)-(3) with $h\equiv 0$ are all real, simple and form a two sequences $-\infty\leftarrow\lambda_k^- > \ldots > \lambda_2^- > \lambda_1^- > 0$ and $0<\lambda_1^+<\lambda_2^+<\ldots<\lambda_k^+\to+\infty$. Moreover, for each $k\in\mathrm{N}$ and each $\sigma\in\{+\,,-\}$ the eigenfunction $y_k^\sigma(x)$, corresponding to the eigenvalue λ_k^σ, has exactly $k-1$ simple zeros in $(0,1)$ and $\sigma\int\limits_0^1 r(x)\,(u_k^\sigma(x))^2\,dx>0$ (more precisely, $y_k^\sigma\in S_k^{\sigma,\nu}$).

Theorem 1. *For each* $k\in\mathrm{N}$, *each* $\sigma\in\{+\,,-\}$ *and each* $\nu\in\{+\,,-\}$ *there exists a connected component* $D_k^{\sigma,\nu}$ *of the set of solutions of problem (1)-(3) that contains* $I_k^\sigma\times\{0\}$, *lies in*

$(R^{\sigma} \times S_k^{\sigma,\nu}) \cup (I_k^{\sigma} \times \{0\})$ *and is unbounded in* $R \times E$, *where* $I_k^{+} = [\lambda_k^{+}, \lambda_k^{+} + d^{+}]$, $I_k^{-} = [\lambda_k^{-} - d^{-}, \lambda_k^{-},]$ *and*

$$d^{\sigma} = \sigma \int_0^1 r(x)(u_k^{\sigma}(x))^2\, dx \Big/ \int_0^1 (u_k^{\sigma}(x))^2\, dx .$$

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STABILITY FOR AN KLEIN-GORDON EQUATION TYPE WITH A BOUNDARY DISSIPATION OF FRACTIONAL DERIVATIVE TYPE

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This paper deals with the stability for an Klein-Gordon equation with a boundary dissipation of fractional derivative type. We have proved well posedness and polynomial stability using the semigroup theory and a sharp result provided by Borichev and Tomilov.

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GLOBAL REGULARITY IN ORLICZ-MORREY SPACES OF SOLUTIONS TO PARABOLIC EQUATIONS WITH VMO COEFFICIENTS

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We show continuity in generalized parabolic Orlicz-Morrey spaces $M_{\Phi,\varphi}$ of sublinear integral operators generated by parabolic Calderon-Zygmund operator and their commutators with BMO functions. As a consequence, we obtain a global $M_{\Phi,\varphi}$-regularity result for the Cauchy-Dirichlet problem for linear uniformly parabolic equations with vanishing mean oscillation (VMO) coefficients (see [1]).

This contribution is based on recent joint work with V.S. Guliyev, A. Ahmadli, L. Softova.

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ESTIMATIONS OF THE NORM OF FUNCTIONS SPACES TYPE $S^l_{p,\varphi,\beta}W(G)$ REDUCED BY POLYNOMIALS

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In this paper we use the method of integral representations to prove inequalities of Poincare type for functions f from the spaces type Sobolev Morrey with dominant mixed derivatives. A space of the form $S^l_{p,\varphi,\beta}W(G)$ is defined as

$$\|f\|_{S^l_{p,\varphi,\beta}W(G)} = \sum_{e\subseteq e_n} \left\| D^{l^e} f \right\|_{p,\varphi,\beta;G},$$

where

$$\|f\|_{L_{p,\varphi,\beta}(G)} = \|f\|_{p,\varphi,\beta;G} = \sup_{\substack{x\in G,\\ t_j>0}} \left(\left|\varphi([t]_1)\right|^{-\beta} \|f\|_{p,G_\varphi(x)} \right),$$

$$G_{\varphi(t)}(x) = G \cap I_{\varphi(t)}(x) = G \cap \left\{ y : \left| y_j - x_j \right| < \frac{1}{2} \varphi_j(t_j), \left(j \in e_n \right) \right\}$$

here $G \subset R^n$; $e_n = \{1,2,\ldots,n\}$, $e \subseteq e_n$, $1 \le p < \infty$; $l = (l_1,\ldots,l_n)$, $j_j > 0$ are integers, $l^e = (l_1^e,\ldots,l_n^e)$, $l_j^e = l_j > 0$ for $j \in e$; $l_j^e = 0$ for $j \in e_n \setminus e = e'$; $\varphi(t) = (\varphi_1(t\ ),\ldots,\varphi_n(t_n))$, $\varphi_j(t_j) > 0, (t > 0)$- is a Lebesgue measurable functions; $\lim\limits_{t_j \to +0} \varphi_j(t_j) = 0$, and

$\lim\limits_{t_j \to +\infty} \varphi_j(t_j) = \infty$, $\lim\limits_{t \to +0} \varphi_j(t) = 0$, $\lim\limits_{t \to +\infty} \varphi_j(t) = \infty$,

$| \varphi([t]_1) |^{-\beta} = \prod\limits_{j=1}^{n} \varphi_j([t]_1)^{-\beta_j}$, $[t]_1 = \min\{1,t\}$.

The following theorem is proved.

Theorem. Let $G \subset R^n$ satisfy the condition of flexible φ-horn , $1 \le p \le q \le \infty$, $\nu = (\nu_1,\nu_2,..,\nu_n)$, $\nu_j \ge 0$ integer $j = 1,2,\ldots,n$;

$$\mu_j = l_j - \nu_j - (1 - \beta_j p)\left(\frac{1}{p} - \frac{1}{q} \right), j \in e_n$$

i.e. for $f \in S^l_{p,\varphi,\beta} W(G)$ there exists a generalized derivative $D^\nu f$ in G and the following inequalities are true

$$\left\| D^\nu (f - P_{l-1}) \right\|_{q,G} \le C \| f \|_{s^l_{p,\varphi,\beta} w(G)}$$

where

$$\| f \|_{s^l_{p,\varphi,\beta} w(G)} = \sum_{\varnothing \ne e \subseteq e_n} \left\| D^{l^e} f \right\|_{p,\varphi,\beta;G},$$

C is constant independent of f .

C*-TERNARY HOMOMORPHISM-DERIVATION AND FUNCTIONAL INEQUALITIES

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In this talk, we introduce and solve the following additive-additive (s,t)-functional inequality

$$\left|\|g(x+y+z)-g(x)-g(y)-g(z)\|\right|$$
$$+\left|\left|3\,h\left(\frac{x+y+z}{3}\right)+h(x-2y+z)+h(x+y-2z)-3h(x)\right|\right|$$
$$\leq \|s(\,3\,g\left(\frac{x+y+z}{3}\right)-g(x)-g(y)-g(z)\|$$
$$+\|t\,(h(x+y+z)+\ h(x-2y+z)+h(x+y-2z)-$$
$$3\,h(x))\|$$

,

where s and t are fixed nonzero complex numbers with |s| <1 and |t| <1.

Furthermore, we investigate C*-ternary derivations and C*-ternary homomorphisms in C*-ternary algebras, associated to the above additive-additive (s,t)-functional inequality and the following functional inequality

$$\|g([x,y,z])-\ [g(x),y,z]-[x,g(y),z]-[x,y,g(z)]\|$$
$$+\|h([x,y,z])-\ [h(x),h(y),h(z)]\|\leq \varphi(x,y,z).$$

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EVALUATION OF THE RESERVOIR TECHNOLOGICAL PARAMETERS OF IN-SITU GENERATED CO_2 GAS-LIQUID SLUG

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The main technological advantage of this method of bed stimulation is the ability to control the volumetric and pressure characteristics of the generated gas-liquid slug while minimizing capital and operating costs for planning and implementing field operations. The gas-liquid in-situ generated CO_2 slug is a displacing agent in the process of bed impact by a complex multicomponent system. The process of gas-liquid slug generation is resulted by the three phases - water, oil and gas, consisting of k components. Assuming that the first component is carbon dioxide CO_2, assume that (k - 3) are hydrocarbon components, (k - 1) are an aqueous component, and k is an additional chemical agent to regulate foaming process. From the condition of thermodynamic equilibrium, we assume that carbon dioxide can be in all three phases, hydrocarbon components – in oil and gas, and the aqueous component and chemical agent – only in the aqueous phase. We assume that the adsorption process of the chemical agent is equilibrium. Then the conservation equation for hydrocarbon components and carbon dioxide, water and chemical agent is written as follows:

$$\nabla\cdot\left[m\sum_{\alpha=1}^{3}s_\alpha D_{i\alpha}\nabla(\rho_\alpha c_{i\alpha})-\sum_{\alpha-1}^{3}\vec{V_\alpha}\rho_\alpha c_{i\alpha}\right]+\sum_{\alpha=1}^{3}\rho_\alpha c_{i\alpha}q_\alpha\delta=\frac{\partial}{\partial t}\left[m\sum_{\alpha=1}^{3}\rho_\alpha s_\alpha c_{i\alpha}\right],$$

$$-\nabla[\vec{V_1}\rho_1(1-c_{11}-c_{k1})]+\rho_1(1-c_{11}-c_{k1})q_1\delta(R-R')=\frac{\partial}{\partial t}[m\rho_1 s_1(1-c_{11}-c_{k1})],$$

$$\nabla\cdot[ms_1D_{k1}\nabla(\rho_1c_{k1})-\vec{V_1}\rho_1c_{k1}]+\rho_1c_{k1}q_1(R-R')=\frac{\partial}{\partial t}m\rho_1s_1c_{k1}+\alpha_{k1}$$

$i=\overrightarrow{1,k-2};\nabla\equiv div;\qquad \nabla\equiv grad$

m – porosity; $s_\alpha,\rho_\alpha,\overrightarrow{V_\alpha}$ and q_α, respectively, saturation, density, velocity filtering and the intensity of the source phase α; $D_{i\alpha}$and $c_{i\alpha}$, respectively, the diffusion coefficient and the mass concentration of the *i*-th component in the phase α; δ - Delta function Dirac; R'- coordinates of the source; α_{ki} – number chemical foaming agent; and *t* is time. Indexes α = 1 – aqueous phase, 2 – oil 3 – gas phase.The motion of each phase are subject of the generalized Darcy's law$\vec{V}=\frac{KK_\alpha}{\mu}(\nabla p_\alpha+\rho_\alpha g\nabla z),\alpha=\overline{1,3}$

where *K* and K_α respectively the absolute and relative permeability, μ_α - viscosity, p_α - pressure, g is the modulus of the gravitational acceleration, *z* is the vertical coordinate. The pressures in the phases are related by known relations $p_\alpha=p_1-p_2,p_{k2}=p_3-p_2,$, where p_k is the capillary pressure. Equilibrium conditions of components in phases: $c_{12}=\lambda_0c_{11},\quad c_{is}=\lambda_ic_{i2},\quad i=\overline{1,k-1}$ and the obvious equality $s_1+s_2+s_3=1,\sum_{i=1}^{k}c_{i2}=1,\sum_{i=1}^{k}c_{is}=1,$ where λ_0– the distribution coefficient of carbon dioxide between the oil and aqueous phases; $\lambda_1$is the distribution coefficient of the *i*-th component between the gas and oil phases. The system of equations describing particular cases of mixing displacement of oil formed in the reservoir rims of the gas-liquid system are the equations of two-phase three-component filtration. The following finite difference operators and notation are used here

$$\Delta^n = \Delta t^{-1}(\varphi^{n+1} - \varphi^n)_{i,j,k}$$

$$\Delta\varphi\Delta\psi = (\Delta\varphi\Delta\psi)_x + (\Delta\varphi\Delta\psi)_y + (\Delta\varphi\Delta\psi)_z;$$

where

$$(\Delta\varphi\Delta\psi)_y = \Delta y^{-2}\left[\varphi_{i,j+\frac{1}{2},k}\left(\psi_{i,j+\frac{1}{2},k} - \psi_{i,j,k}\right) - \varphi_{i,j-\frac{1}{2},k}\left(\psi_{i,j,k} - \psi_{i,j-\frac{1}{2},k}\right)\right]$$

By performing linearization using Newtonian iterations by the method of nonlinear expressions on the (n+1) layer in difference equations, the resulting system of linear algebraic equations can be written in vector form.At relatively weak dependence of viscosity and density of phases on pressure and concentration of CO_2 an efficient algorithm with separate determination of pressure, saturation and concentrations of chemical agents at each time step is chosen.

CLAY SWELLING CHARACTERISTICS IN ELECTROLYTE SOLUTIONS

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The efficiency of oil fields exploitation is largely determined by the properties of productive formations, and last but not least, by the presence of clay minerals in their composition. The latter, as a consequence, characterize the reservoir with the ability to swell, permeability, plasticity in the wet state, etc. and affect the flow of saturating fluids [1]. The fluids flow in a clay-containing porous medium is a flow through the pores and capillaries, which is accompanied by certain transformations of clay minerals and leads to a decrease in the reservoirs permeability associated with the clay swelling and a decrease in the rate of oil recovery [2, 3]. In minerals with a fixed crystal structure, chemical adsorption of metal cations from the injected water to the surface of the mineral occurs with the substitution of potential-determining cations. Clay minerals can be considered as solid solutions (ionites) capable of replacing cations in the lattice sites with electrolyte cations. In the present work some effects observed at clays swelling in electrolytes of various concentration are considered. Aqueous solutions and alkalis of Na_2CO_3 and NaCl salts were used as electrolytes prepared both in fresh and distilled water. The change in the volume of test samples as a result of contact with aqueous solutions was studied on bentonite clay samples using a linear swelling meter LSM-2100 (Fann manufacturer).

Studies were conducted with electrolyte solutions on unsaturated samples of bentonite clay and clay samples pre-saturated with water (swollen clay).
It was found that with increasing electrolyte concentration, the swelling rate of unsaturated clay samples is nonlinear (Fig. 1). The rate of swelling of pre-swollen clay as a result of contact with 1% NaCl solution is lower compared to the rate of swelling in fresh water (Fig. 2).

The results obtained can serve as a basis for the development of methods for control of aqueous solutions flow in clay-bearing sandstone under oil field flooding process. Pre-saturation of the porous medium with electrolyte solutions will significantly reduce the clay swelling of the productive layers and thereby increase oil recovery.

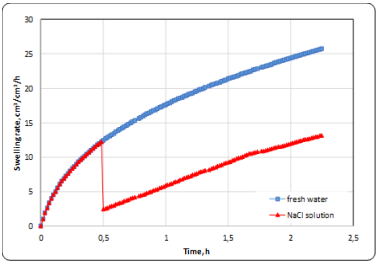

Fig. 1. Change in the swelling rate of electrolyte solutions NaCl and Na_2CO_3 in fresh water over time at different concentrations

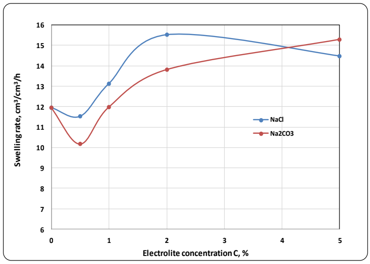

Fig. 2. Kinetics of clay swelling in fresh water and clay pre-swollen in NaCl solutions

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ON THE ELECTROSTATIC FIELD IN EXPANSION DYNAMICS OF GAS BUBBLES

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Dynamics of the gas bubbles formation in liquids and viscoelastic materials has been studied for quite a long time [1]. Bubbling and its expansion dynamics is a major component of a wide range of chemical, environmental processes and fluid, especially, oil mechanic, bubbles generally have broad size distributions and complex property variations over time too [2,3,4].Bubbling has been the theme of numerous studies [5,6,7].

Taking into account the studies of the gas bubbles dynamics through various physical parameters, regulation of process of bubble formation through the potential difference parameter was considered in present work.

The work is devoted to the study of the dynamics of the formation of bubbles in a gas-liquid system taking into account the potential difference. The electrical conductivity of the fluid is determined depending on the concentration of the electrolyte and, accordingly, the electrostatic field resulting from the flow of the fluid. The effect of the electrostatic field on the bubble formation dynamics has shown that the radius of the gas bubbles and the dynamics of its expansion, formed by the pressure difference, can be regulated by the potential difference parameter.

Depending on the electrolytic concentration, the electric conductivity of the liquid and, accordingly, the electrostatic field arising from friction in fluid is determined. The effect of the electrostatic field on the dynamics of the bubble formation showed that the radius of gas bubbles and its expansion dynamics formed by the pressure drop can be regulated by the potential difference parameter. It is shown that one of the main factors affecting the flow of two-phase fluids is the nature of the liquid phase and the concentration of electrolyte added. The results of regulation of the bubble formation dynamics in the gas-liquid system via the electrostatic field and a number of physical parameters can be applied in the oil and gas industry, chemical processes, biomechanics.

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CENTRAL LIMIT THEOREM FOR A FAMILY OF THE FIRST PASSAGE TIMES OF THE LEVEL BY A RANDOM WALKDESCRIBED BY THE AUTOREGRESSION PROCESS OF ORDER ONE $(AR(1))$

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Let $\xi_n;\ n \geq 1$ be a sequence of independent identically distributed real random variables, determined on some probability space $(\Omega, \mathcal{F}, P)$.

As is known ([4,5]) the sequence of real random variables $X_n, n \geq 0$ determined on $(\Omega, \mathcal{F}, P)$ is called autoregression process of order one $(AR(1))$ if the following recurrent relation is fulfilled

$$X_n = \beta_0 X_{n-1} + \xi_n, \tag{1}$$

for some fixed β_0 and the initial value of the process X_0 is independent of innovation $\{\xi_n\}$.

At present paper we prove the central limit theorem for a family ofstopping times $\tau_a, a \geq 0$ of the following form:

$$\tau_a = \inf(n \geq 1 : \theta_n > a), \tag{2}$$

where $\theta_n = n\beta_n, \beta_n = \dfrac{T_n}{S_n}$.

Here for the correct definition of the variable τ_a we assume $\inf\{\emptyset\} = \infty$.

We can write the family of stop moments (2) in theform:

$$\tau_a = \inf\left\{n \geq 1 : ng\left(\frac{T_n}{n}, \frac{S_n}{n}\right) > a\right\},$$

where $g(x,y)=\dfrac{x}{y}$ and $g\left(\dfrac{T_n}{n},\dfrac{S_n}{n}\right)=\beta_n$.

Note that, the families of the stop moments of type τ_a play important role on the theory of Markov renewal and in applied fields of random processes and in sequential analysis ([1-3]).

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WOVEN FUSION FRAMES AND IT'S APPLICATIONS

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A new notion in frame theory has been introduced recently under the name woven frames. Woven and weaving frames are powerful tools for pre-processing signals and distributed data processing.The purpose of introducing fusion frame or frame of subspace is to first construct local components and then build a global frame from these. This type of frame behaves as a generalization of frames. Motivating by the concepts of fusion and weaving frames,we investigate the notion woven-weaving fusion frames and present some of their features.

Introduction. Frames are generalizations of orthonormal bases in Hilbert spaces. A frame, as well as an orthonormal basis, allows each element in Hilbert space to be written as an infinite linear combination of the frame elements so that unlike the bases conditions, the coefficients might not be unique. A countable family of elements$\{f_i\}_{i\in I}$ in *H* is a frame for *H*, if there exist constants *A*, *B*>0 such that

$$A\|f\|^2 \leq \sum_{i\in I} |\langle f,f_i\rangle|^2 \leq B\|f\|^2, \quad \forall f \in H.$$

The numbers A and $B$are celled the lower and upper frame bounds, respectively. The frame $\{f_i\}_{i\in I}$is called tight frame, if *B*=*A*and is called Parseval frame, if*B*=*A*=1. Also the sequence $\{f_i\}_{i\in I}$is called Bessel sequence, if satisfy only the upper inequality.

In the early 21'th century, new type of frames were presented to the scientific community, with name offrame of subspaces, which are nowknown as fusion frames.A family of closed subspaces $\{W_i\}_{i\in I}$ of Hilbert space H is called fusion frame with respect to weights $\{v_i\}_{i\in I}$, if there exist constants *C*, *D*>*0* such that

$$C\|f\|^2 \leq \sum_{i\in I} v_i^2 \left\|P_{W_i}(f)\right\|^2 \leq D\|f\|^2, \quad \forall f \in H.$$

Fusion frames is a generalization of frames which were introduced by Cassaza and Kutyniok in 2003 .The significance of fusion frame is the construction of global frames from local frames in Hilbert space, so the characteristic fusion frame is special suiting for application such as distributed sensing, parallel processing, and packet encoding and so on.

In recent years, Bemrose et.al. introduced weaving frames. From the point of view of its introducers, weaving frames are powerful tools for pre-processing signals and distributed data processing.

Improving and extending the notions of fusion and woven (weaving) frames, we investigate the new notion under the name woven (weaving) fusion frames. A finite family of fusion frames $\{W_{ij}\}_{i\in I, j\in[m]}$with respect to weights $\{\boldsymbol{v_{ij}}\}_{\boldsymbol{i\in I, j\in[m]}}$is said woven fusion frames if there are universal constant*A*, *B*>*0*such that for every partition$\{\sigma_j\}_{j\in[m]}$of I, the family $\{W_{ij}\}_{i\in\sigma_j}$ for every $j \in [m]$is a fusion frame for H with frame bounds A and B. Each family $\{W_{ij}\}_{i\in\sigma_j}$ for every $j \in [m]$ is called weaving fusion frame, $where [m] \coloneqq \{0,1,2,\dots,m\}$.

Theorem. Assume that$\{W_i\}_{i\in I}$ and $\{V_i\}_{i\in I}$are fusion frames with weights$\{\mu_i\}_{i\in I}$ and $\{\vartheta_i\}_{i\in I}$respectively. Also, if $\{W_i\}_{i\in I}$ and $\{V_i\}_{i\in I}$ are woven fusion frame and E is a self-adjoint and invertible operator on H, such that $E^*E(W)$ is the subset of every closed subspace W of H. Then for every subset σ of I, the sequence $\{EW_i\}_{i\in\sigma} \cup \{EV_i\}_{i\in\sigma^c}$ is a fusion frame with frame operator $ES_\sigma E^{-1}$ where S_σ is frame operator of $\{W_i\}_{i\in\sigma} \cup \{V_i\}_{i\in\sigma^c}$, i. e. $\{EW_i\}_{i\in I}$ and $\{EV_i\}_{i\in I}$ constitute woven fusion frame.

Example: Suppose $\{e_k\}_{k=1}^{\infty}$ be an orthonormal basis of $H$and $H = l^2(N)$. In this example the indexing set I=N is natural numbers set. For $i \in N, H_i = \overline{span}\{e_k\}_{k=i}^{\infty}$ and $\{e_{ij}\}_{j=1}^{\infty} = \{e_{i+j-1}\}_{j=1}^{\infty}$is an orthonormal basis for H_i.

1) Let $\{P_i\}_{i=1}^{\infty}$ and $\{P_i^{'}\}_{i=1}^{\infty}$be the family of orthogonal projections $P_i: H \to \overline{span}\{e_i\}$ and $P_i^{'}: H \to \overline{span}\{e_i, e_{i+1}\}$ for each $i \in N$. Also let $f_{i,j} = P_i(e_{i,j})$and$g_{i,j} = P_i^{'}(e_{i,j})$. Then $\{f_{i,j}\}_{i,j=1}^{\infty}$is a tight frame with bound $A = B = 1$ and $\{g_{i,j}\}_{i,j=1}^{\infty}$ is a frame with bounds $A = 1$and $B = 2$, such that these frames constitute woven frames.
2) Now if $\{P_i\}_{i=1}^{\infty}$ and $\{P_i^{'}\}_{i=2}^{\infty}$ are same as above, except $P_i^{'}$. If we get $\sigma = N\backslash\{1\}$, then $\{f_{i,j}\}_{i,j=1}^{\infty}$ and $\{g_{i,j}\}_{i,j=1}^{\infty}$don't constitute woven frames.

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CONTINUOUS DEPENDENCE OF THE SOLUTION TO THE FIRST GENERALIZED DIRICHLET PROBLEM ON THE RIGHT-HAND SIDE OF THE SCHRÖDINGER MAGNETIC EQUATION

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Let G be an arbitrary domain in R^n. Consider in G Schrödinger magnetic expression

$$H_{a,V} = \sum_{k=1}^{n} \left(\frac{1}{i} \frac{\partial}{\partial x_k} + a_k(x) \right)^2 + V(x),$$

where $a(x) = (a_1(x), a_2(x), \ldots, a_n(x))$ isthe real magnetic potential, $V(x)$ the real electric potential, $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, and i the imaginary unit.

Considerinthedomain G the first generalized Dirichlet problem:

Let $f(x) \in \overset{0}{W}{}_2^{\prime,1}(G)$, where $\overset{0}{W}{}_2^{\prime,1}(G)$ is theadjoint space to the first-order Sobolev space $\overset{0}{W}{}_2^1(G)$. Itisrequiredtofind such a function $u(x)$ from the class $\overset{0}{W}{}_2^1(G)$ that satisfies the equation

$$\sum_{k=1}^{n}\left(\frac{1}{i}\frac{\partial}{\partial x_k}+a_k(x)\right)^2 u(x)+V(x)u(x)-\lambda u(x)=f(x)$$

inthe sense of the theory of generalized functions.

In the present work, to investigate the existence and uniqueness of a solution to the first generalized Dirichlet problem, we introduce the first-order magnetic Sobolev space and study its topological structure.

Notethatrecently, the properties of magnetic Sobolev spaces of any orderhave been intensively studied (see [1-4]). We prove the existence and uniqueness of the solution to the first generalized Dirichlet problem in the first-order Sobolev space. We establish the continuous dependence of the solution to the first generalized Dirichlet problem on the right-hand side of the Schrödinger magnetic equation in the space $L_2(G)$.

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INTUITION AS A MOTIVATOR OF ATHEMATICAL CREATIVITY(HISTORICAL AND METHODOLOGICAL ANALYSIS).

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One of the most characteristic features of mathematics is its "ability" to foresee the most "economical" way of the formation of abstract human knowledge and to anticipate its structure to a certain extent (1; 2; 3). Such anticipation, as a rule, is realized by the advancement of an intuitive guess based mainly in the field of the unconscious (4).

The regulatory role of intuition during the course of mathematical activity is expressed in the following inherent characteristics (5, 6, 7 etc.):

1. Intuition as goal setting.
2. Intuition as meaning formation.
3. Intuition as an emotionally colored process.

The history of the development of mathematical science shows that the discovery of certain laws can be based on intuitions of various kinds (8; 9, etc.). The “intuition of a formula” —the prediction of the results of a complex transformation — and the “intuition of a logical nature” —the “vision” of the structure of complex logical constructions — are varieties of conceptual intuition. Geometric intuition, which allows to represent these objects in a visual-figurative form, refers to the so-called eidetic intuition.

A special role in anticipating those or other essential characteristics of the developed mathematical theory is played by a generalization, which is understood here as an extension of the scope of a mathematical object (concept, theorem, algorithm, etc.) (10;11, etc.).

Initiation of the choice of the direction of creative scientific research can also be carried out by discovering hidden analogies (called contact analogies) between patterns that were previously considered separately and were considered unrelated, marks "one of the most pleasant moments" of mathematical creativity (11, p.48).

The report presents examples of the implementation of the described methods.

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CHARACTERISTICS AND FUNCTIONS OF MATHEMATICAL EDUCATION

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At present, mathematical education is a promising field.Thus, mathematics should be taught in the given linear structure; kindergarten, elementary school, school, university. Structure of mathematical education should be appropriate with the modern global mathematical education. Because mathematics is a global subject.

The mission of the school contains deep study of mathematics to format and develop mathematical thinking on students, to enrich mathematical outlook and develop mathematical creativity. Mathematical education is based on the science of mathematics and its features. Its peculiarities teach the students stringency, honesty, confidence too.

Acquaintance of students with mathematics and its methods helps them understand nature,technical and economical processes,to learn modern military equipment and computer technology. It should be remembered that harmonic development of students demands not only mathematics but also to learn and coordinate other subjects with mathematics.

To work out syllabus of the course in mathematics is of great importance in the implementation of mathematical education.

For example, in the compiling of the syllabus of I-IV courses,the following principles should be taken into account:

1. While compiling the content of program,it is important to base on the mathematical abilities of students. Primary school teachers and relevant professionals should be involved in the program analysis.
2. While compiling the content of the program, age,knowledge level,physical,psychological conditions of studentshould be considered.
3. Content of mathematical education of I-IV formsshould be coordinated with students' upbringing. Interdisciplinary communication,historicity and other features should be considered.
4. All primary school teachers should attend in the experiment of the program.

POINCARE TYPE INEQUALITY IN BESOV-MORREY TYPE SPACES WITH DOMINANT MIXED DERIVATIVES

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In thispaper we use the method of integral representations to prove inequalities of Poincare type for functions f from the spaces type Besov-Morreytype spaces with dominant mixed derivatives with the norm $\left(m_j > l_j > k_j \geq 0, j \in e_n\right)$:

$$\|f\|_{S^l_{p,\theta,\varphi,\beta}B(G_\varphi)} =$$

$$= \sum_{e\subseteq e_n} \left\{ \int_{0^e}^{h_{01}^e} \cdots \int_{0^e}^{h_{0n}^e} \left[\frac{\left\| \Delta^{m^e}\left(\varphi(h), G_{\varphi(h)}\right) D^{k^e} f \right\|_{p,\varphi,\beta}}{\prod_{j\in e} \left(\varphi_j\left(h_j\right)\right)^{\left(l_j - k_j\right)}} \right]^\theta \prod_{j\in e} \frac{dh_j}{h_j} \right\}^{\frac{1}{\theta}}$$

where

$$\|f\|_{p,\varphi,\beta;G} = \|f\|_{L_{p,\varphi,\beta}(G)} = \sup_{\substack{x\in G,\\ t>0}} \left(\prod_{e\subseteq e_n} \left|\varphi_j\left([t_j]_1\right)\right|^{-\beta_j} \|f\|_{p,G_{\varphi(t)}(x)} \right)$$

and $G \subset R^n$, $l \in (0,\infty)^n$, $l^e = \left(l_1^e, l_2^e, \ldots, l_n^e\right), l_j^e = l_j, (j \in e); l_j^e = 0\left(j \in e_n \setminus e\right), e_n = \{1,2,\ldots,n\}; e \subseteq e_n$; $m_j \in N$, $k_j \in N_0$, $j \in e_n$; $1 \le p < \infty$, $1 \le \theta \le \infty$, $\beta_j \in [0,1], (j = 1,\ldots,n)$ and $\varphi(t) = (\varphi_1(t_1),\ldots,\varphi_n(t_n))$, $\varphi_j(t_j) > 0$, $(t_j > 0)$ is continuously differentiable functions; $\lim\limits_{t\to+0} \varphi_j(t_j) = 0$, $\lim\limits_{t\to+\infty} \varphi_j(t_j) = P_j \le \infty$;

$$G_{\varphi(t)}(x) = G \cap I_{\varphi(t)}(x) = G \cap \left\{ y : \left|y_j - x_j\right| < \frac{1}{2}\varphi_j(t_j), (j = 1,2,\ldots,n) \right\}$$

The following theorem is proved.

Theorem. Let $G \subset R^n$ satisfy the condition of flexible φ-horn, $1 \le p \le q \le \infty$, $\nu = (\nu_1,\ldots,\nu_n)$, $\nu_j \ge 0$ integer $j = 1,2,\ldots,n$

$$Q_T^e = \int_{0^e}^{1^e} \prod_{j\in e} \left(\varphi_j\left(t_j\right)\right)^{-\nu_j - \left(1-\beta_j p\right)\left(\frac{1}{p}-\frac{1}{q}\right)} \prod_{j\in e} \frac{\varphi_j'\left(t_j\right)}{\left(\varphi_j\left(t_j\right)\right)^{1-l_j}} \prod_{j\in e} dt_j < \infty ,$$

i.e. for $f \in S^l_{p,\theta,\varphi,\beta}B\left(G_\varphi\right)$ there exists a generalized derivative $D^\nu f$ in G and the following inequalities are true

$$\left\| D^\nu \left(f - P_{l-1} \right) \right\|_{q,G} \le C \| f \|_{s^l_{p,\theta,\varphi,\beta} b\left(G_\varphi\right)}$$

C are the constants independent of f .

ASSESSMENT OF SOIL FERTILITY BASED ON THE THEORY OF RELIABILITY

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In scientific literature for assessment of the quality, productivity, ecological and reclamation state of soils different criteria and appropriate methods are used [1,2].

Location of Azerbaijan in dry climate zone, water scarcity, unequal distribution of water resources proneness of soils to salinization requires to use modern methods for assessment their ameliorative and ecological state.

In connection with transition to market economy, reformations in the agrarian field and creation of new forms of ownership on the soil, in addition to creation of regulatory legal framework the improvement of its scientific support for providing new development of agriculture in the country is the main goal.

Practice shows that the soil is a complicated and complex object of production.The goal in this field is not only to cultivate the soil, immediate repayment of funds, but at the same time, to prevent negative ecological social dangers when interfering with the soil.

Investigation of complex indicators of irrigated soils used in our Republic, shows that in a great majority of cultivation areas secondary salinization and swamp of sown occur.

Excessive deviation of the quality parameters of the soil reduces to decrease or completely lost of their quality and productivity.

To Assess the existing state of soils, to get information about their quality and productivity, the use of the reliability and mathematical statistics methods enable to get more efficient result.

In this work by using the methods of reliability theory, new results in the assessment of fertilityof soils were obtained.

BIFURCATION OF SOLUTIONS FROM INFINITY OF LINEARIZABLE ONE DIMENSIONAL DIRAC PROBLEMS

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We consider the following nonlinear one dimensional Dirac problem

$$Bw'(x) - P(x)w(x) = \lambda w(x),\ 0 < x < \pi, \tag{1}$$

$$U_1(w) \equiv (\sin\alpha,\ \cos\alpha)\, w(0) = \vartheta(0)\cos\alpha + u(0)\sin\alpha = 0, \tag{2}$$

$$U_2(w) \equiv (\sin\beta,\ \cos\beta)\, w(\pi) = \vartheta(\pi)\cos\beta + u(\pi)\sin\beta = 0, \tag{3}$$

where $B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $P(x) = \begin{pmatrix} p(x) & 0 \\ 0 & r(x) \end{pmatrix}$, $w(x) = \begin{pmatrix} u(x) \\ \vartheta(x) \end{pmatrix}$,

$\lambda \in R$ is a spectral parameter, $p(x), r(x) \in C([0,\pi];R)$, α and β are real constants such that $0 \le \alpha, \beta < \pi$. The nonlinear term $g = (g_1\ g_2)^t \in C([0,\pi] \times R^2 \times R; R^2)$ satisfies the condition:

$$g(x, w, \lambda) = o(|w|) \text{ as} |w| \to \infty$$

uniformly in $x \in [0, \pi]$ and $\lambda \in \Lambda$ for every bounded interval $\Lambda \subset R$ (here $|\cdot|$ denotes a norm in R^2).

Let $E = C([0, \pi]: R^2) \cap \{w: U(w) = (U_1(w), U_2(w))^t = (0,0)^t\}$ to be the Banach space with the usual norm $\| w \| = \max_{x\in[0,\pi]} |u(x)| + \max_{x\in[0,\pi]} |\vartheta(x)|$. Let S be the subset of E given by

$$S = \{w \in E : |u(x)| + |\vartheta(x)\} > 0,\ x \in [0,\ \pi]\}$$

with metric inherited from E.

For each $w = (u, \vartheta)^t \in S$ we define $\theta(w, x)$ to be continuous function on $[0, \pi]$ satisfying the conditions: $\cot\theta(w, x) = u(x)/\vartheta(x)$, $\theta(w, 0) = -\alpha$.

We denote by $S_k^\nu, k \in \mathrm{Z}, \nu \in \{+, -\}$, the set of functions $w \in S$ which satisfies the following conditions: (i) $\cot\theta(w, \pi) = -\beta + k\pi$; (ii) if $k > 0$ or $k = 0, \alpha \geq \beta$, then for fixed w, as x increases from 0 to π, the function θ cannot tend to a multiple of $\pi/2$ from above, and as x decreases, the function θ cannot tend to a multiple of $\pi/2$ from below; if $k < 0$ or $k = 0, \alpha < \beta$, then for fixed w, as x increases from 0 to π, the function θ cannot tend to a multiple of $\pi/2$ from below, and as x decreases, the function θ cannot tend to a multiple of $\pi/2$ from above; (iii) the function $\nu u(x) > 0$ is positive in a deleted neighborhood of $x = 0$ (see [1]).

Theorem 1. *For each for each $k \in \mathrm{Z}$ and each $\nu \in \{+, -\}$ there exists a continua C_k^ν of solutions of problem (1)-(3) that meets (λ_k, ∞) and has the following properties: (i) there exists a neighborhood U_k of (λ_k, ∞) in $R \times E$ such that*

$(U_k \cap C_k^\nu) \subset R \times S_k^\nu$; *(ii) either* C_k^ν *meets* (λ_k', ∞) *respect to the set* $R \times S_{k'}^{\nu'}$ *for some* $(k',\nu') \neq (k,\nu)$ *or* C_k^ν *meets* $(\lambda',0)$ *for some* $\lambda \in R$ *or* C_k^ν *has an unbounded projection on* R, *where* λ_k *is a* k *-th eigenvalue of problem (1)-(3) with* $g \equiv (0,0)^t$.

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ON THE ASYMPTOTICS OF THE SOLUTION OF A BOUNDARY VALUE PROBLEM FOR ONE CHARACTERISTIC DIFFERENTIAL EQUATION DEGENERATED INTO ELLIPTIC ONE

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In an infinite strip we consider the following boundary value problem with a small parameter $\varepsilon > 0$

$$\varepsilon \frac{\partial}{\partial x}(\Delta u) - \Delta u + au = f(x,y), \tag{1}$$

$$u\big|_{x=0} = u\big|_{x=1} = 0, \quad \frac{\partial u}{\partial x}\Big|_{x=1} = 0, \tag{2}$$

$$\lim_{|y|\to+\infty} u = 0, \tag{3}$$

where $\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $a = const > 0$, $f(x,y)$ is a given smooth function.

The goal of the paper is to construct complete asymptotics of the solution of problem (1)-(3) with respect to a small parameter.
We prove the following theorem.

Theorem. Let the function $f(x,y)$ have continuous derivatives with respect to the variable x to the ($(n+2)$-th order inclusively, with respect to the variable y be infinitely differentiable, and for any pair of negative numbers l,k satisfy the inequality of the form

$$\sup_y \left(1+|y|^l\right)\left|\frac{\partial^k f(x,y)}{\partial x^{k_1}\partial y^{k_2}}\right| = c_{l_{k_1k_2}} < +\infty,$$

where $c_{l_{k_1k_2}} > 0$ are constants, $k = k_1 + k_2$, moreover $k_1 \le n+2$, k_2 is arbitrary. Then for the solution of the problem (1)-(3) we have the asymptotic representation of the form

$$u = \sum_{i=0}^{n} \varepsilon^i w_i + \sum_{j=0}^{n} \varepsilon^{1+j} v_j + \varepsilon^n z,$$

where the functions w_i are determined in the first iterative process as the solution of the following boundary value problems

$$-\Delta w_0 + a w_0 = f(x,y);\ \ w_0\Big|_{x=0} = w_0\Big|_{x=1} = 0,\ \lim_{|y|\to+\infty} w_0 = 0,$$

$$-\Delta w_s + a w_0 = -\frac{\partial}{\partial x}(\Delta w_{s-1});\ \ w_s\Big|_{x=0} = w_s\Big|_{x=1} = 0,\ \lim_{|y|\to+\infty} w_s = 0;$$

$$s = 1,2,\dots,n,$$

v_j are boundary layer type functions near the boundary $x=1$, determined by the second iterative process, $\varepsilon^n z$ is a residual term and for z the estimation

$$\|z\|^2_{w_2^1(\Pi)} \le c\varepsilon$$

is valid.
Here $c>0$ is a constant independent of ε.

ASYMPTOTICS OF THE SOLUTION OF A BOUNDARY VALUE PROBLEM IN AN INFINITE STRIP FOR AN ARBITRARY TYPE SINGULARLY PERTURBED QUASILINEAR NON-CLASSIC TYPE DIFFERENTIAL EQUATION

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In an infinite strip $\Pi=\{(t,x)|0\le t\le 1, -\infty<x<+\infty\}$ we consider the following boundary value problem:

$$(-1)^m \varepsilon^{2m}\frac{\partial^{2m+1}u}{\partial t^{2m+1}}-\varepsilon^p\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)^p-$$

$$-\varepsilon\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}+au-f(t,x)=0, \qquad (1)$$

$$u\big|_{t=0}=\frac{\partial u}{\partial t}\Big|_{t=0}=\ldots=\frac{\partial^m u}{\partial t^m}\Big|_{t=0}=0, \quad (-\infty<x<+\infty), \qquad (2)$$

$$\frac{\partial^{m+1}u}{\partial t^{m+1}}\Big|_{t=1}=\frac{\partial^{m+2}u}{\partial t^{m+2}}\Big|_{t=1}=\ldots=\frac{\partial^{2m}u}{\partial t^{2m}}\Big|_{t=1}=0, \ (-\infty<x<+\infty), \qquad (3)$$

$$\lim_{|x|\to+\infty}u=0, \qquad (4)$$

where $\varepsilon>0$ is a small parameter $p=2k+1$, k and m are arbitrary natural numbers, $a>0$ is a constant, $f(t,x)$ is a given function. It is assumed that $f(t,x)$ has continuous derivatives with respect to t to the $(2m+2)$-th order inclusively, with respect to

x is infinitely differentiable and for each t from [0,1] satisfies the condition

$$\sup_x \left(1+|x|^l\right)\left|\frac{\partial^k f(t,x)}{\partial t^{k_1}\partial x^{k_2}}\right| = C_{l_{k_1k_2}} < +\infty, \qquad (5)$$

where l is an arbitrary non-negative number, $k = k_1 + k_2$, $k_1 \le 2m+2$, k_2 is any non-negative integer.

We prove the following theorem.

Theorem. Let the function $f(t,x)$ be a given smooth function satisfying condition (5). Then for solving the boundary value problem (1)-(4) it is valid the asymptotic representation in the form

$$u = \sum_{i=0}^{n} \varepsilon^i w_i + \sum_{j=0}^{n+m-1} \varepsilon^{1+j} v_i + \sum_{j=0}^{n+m-1} \varepsilon^{1+m+j} \eta_j + z,$$

where the functions w_i are determined by the first iterative process, v_j, η_j are boundary layer type functions near the boundaries $t=0$ and $t=1$, that are also determined by the corresponding iterative processes, z is a residual term and the following estimation is valid for it

$$\varepsilon^{2m} \int_{-\infty}^{+\infty} \left(\frac{\partial^m z}{\partial t^m}\Big|_{t=1} \right)^2 dx + \varepsilon^p \iint_{\Pi} \left(\frac{\partial z}{\partial x} \right)^{p+1} dtdx + \varepsilon \iint_{\Pi} \left(\frac{\partial z}{\partial x} \right)^2 dtdx +$$
$$+ c_1 \iint_{\Pi} z^2 dtdx \le c_2 \varepsilon^{2(n+1)}.$$

Here $c_1 > 0,\ c_2 > 0$ are constants independent of ε .

ON THE SOLVABILITY OF THE NONHOMOGENEOUS RIEMANN PROBLEM IN THE WEIGHTED SMIRNOV CLASSES

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Let $G(\xi)=|G(\xi)|e^{i\theta(\xi)}$ be complex-valued functions on the curve Γ. We make the following basic assumptions on the coefficient $G(\cdot)$ of the considered boundary value problem and Γ :

(i) $|G(\cdot)|^{\pm 1}\in L_{\infty}(\Gamma)$; (ii) $\theta(\cdot)$ *is piecewise continuous on* Γ, *and* $\left\{\xi_k, k=\overline{1,r}\right\}\subset\Gamma$ *are discontinuity points of the function* $\theta(\cdot)$*:*

We impose the following condition on the curve Γ.

(iii) Γ *is either Lyapunov or Radon curve with no cusps. Direction along* Γ *will be considered as positive, i.e. when moving along this direction the domain* D *stays on the left side. Let* $a\in\Gamma$ *be an initial (and also a final) point of the curve* Γ.

Consider the nonhomogeneous Riemann problem

$$F^{+}(z(s))-G(z(s))F^{-}(z(s))=g(z(s)),\ s\in(0,S),\qquad (1)$$

where $g\in L_{p,\rho}(\Gamma)$ is a given function. By the solution of the problem (1) we mean a pair of functions

$$\left(F^{+}(z);F^{-}(z)\right)\in E_{p,\rho}\left(D^{+}\right)\times_m E_{p,\rho}\left(D^{-}\right),$$

whose boundary values $F^{\pm}$ on Γ a.e. satisfy (1).

Theorem. *Let the coefficient* $G(z(s))$, $0 \le s \le S$, *of nonhomogeneous problem (1) satisfy the conditions (i),(ii),* $\arg G(\cdot)$ *be a Hölder function on* Γ, *and the curve* $\Gamma = z([0,S])$ *satisfy the condition (iii). Assume that the weight function* $\nu(\cdot)$ *defined by* $\nu(s) =: \sigma^p(s)\rho(z(s))$, $s \in (0,S)$, *belongs to the Muckenhoupt class* $A_p(\Gamma), 1 < p < +\infty$, *and the condition* $\int_\Gamma \rho^{-q/p}(z(s))|dz(s)| < +\infty$ *holds, i.e.* $\rho^{-q/p} \in L_1(\Gamma)$. *Then the following assertions are true with regard to the solvability of the problem (1) in the classes* $E_{p;\rho}(D^+) \times_m E_{p;\rho}(D^-)$:

1) *for* $m \ge -1$, *the problem (1) has a general solution of the form*

$$F(z) = Z_\theta(z)P_m(z) + F_1(z),$$

where $Z_\theta(\cdot)$ *is a canonical solution of corresponding homogeneous problem,* $P_m(z)$ *is an arbitrary polynomial of degree* $k \le m$ *(for* $m = -1$ *we assume* $P_m(z) \equiv 0$*), and* $F_1(\cdot)$ *is a particular solution of nonhomogeneous problem (1) of the form*

$$F_1(z) \equiv \frac{Z_\theta(z)}{2\pi i}\int_\Gamma \frac{g(\xi)d\xi}{Z_\theta^+(\xi)(\xi - z)} \qquad (2);$$

2) *for* $m < -1$, *the nonhomogeneous problem (1) is solvable only*

3) *when the right-hand side* $g(\cdot)$ *satisfies the following orthogonality conditions* $\int_\Gamma \frac{g(\xi)}{Z_\theta^+(\xi)}\xi^{k-1}d\xi = 0$, $k = \overline{1,-m-1}$, *and the unique solution* $F(z) = F_1(z)$ *is defined by (2).*

ON THE SUBDIFFERENTIAL OF THE SECOND ORDER AND THE OPTIMALITY CONDITION

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In this work we study the property of a second order subdifferential and obtain second order optimality conditions for the minimization problem.

Subdifferentsial of the second order is studied by different authors, but at the present time there is no unequiwcally a capted definition of the subdifferential. It is known that determination of differential of the second order is the analog of differential of the first order. But there are no definitions of such analog second order subdifferential in general case. If we consider F. Clarke subdifferential, then different analogues of second order subdifferential can be obtained.

Let X - Banach space and $f: X \to R$.

In the works [1] and [2] considered definition of the sub-differential of arbitrary order. In particular from this it follows, that

$$f^{\{2\}}(x_0; x_1, x_2) = \sup_{z_1, z_2 \in X} \overline{\lim_{\lambda_1 \downarrow 0, \lambda_2 \downarrow 0}} \frac{1}{\lambda_1 \lambda_2} (f(x_0 + \lambda_1 z_1 + \lambda_2 z_2 +$$
$$\lambda_1 x_1 + \lambda_2 x_2) - f(x_0 + \lambda_1 z_1 + \lambda_2 z_2 + \lambda_1 x_1) -$$
$$- f(x_0 + \lambda_1 z_1 + \lambda_2 z_2 + \lambda_2 x_2) + f(x_0 + \lambda_1 z_1 + \lambda_2 z_2))$$

at $(x_1, x_2) \in X \times X$ and

$$\partial_{\{2\}} f(x_0) = \{b \in \overline{B}(X^2; R) : f^{\{2\}}(x_0; x_1, x_2) \geq b(x_1, x_2) \text{ at } (x_1, x_2) \in X^2\},$$

where the set of all continuous symmetric bilinear functions from $X \times X$ in R is denoted by $\overline{B}(X^2; R)$.

In the work is studied a number of properties of the subdifferential $\partial_{\{2\}} f(x_0)$.

In the works [1] and [2] also considered another definition of the subdifferential of second order. In work the number of their properties is studied. Let's put

$$f^{\{2\}+}(x_0; x) = \sup_{z \in X} \overline{\lim_{\lambda \downarrow 0}} \frac{1}{\lambda^2} (f(x_0 + \lambda z + 2\lambda x) - 2f(x_0 + \lambda z + \lambda x) + f(x_0 + \lambda z)),$$

$$f^{\{2\}-}(x_0; x) = \inf_{z \in X} \underline{\lim_{\lambda \downarrow 0}} \frac{1}{\lambda^2} (f(x_0 + \lambda z + 2\lambda x) - 2f(x_0 + \lambda z + \lambda x) + f(x_0 + \lambda z))$$

at $x \in X$ and

$$D_2 f(x_0) = \{Q \in B_0(X) : f^{\{2\}-}(x_0; x) \geq Q(x) \leq f^{\{2\}+}(x_0; x) \ \text{ at } \ x \in X\},$$

where $B_0(X)$ the set of all continuous quadratic functionals.

In the work is studied a number of properties of the subdifferential $D_2 f(x_0)$.

In the work the optimality conditions of the second order are obtained.

In a smooth analysis the distance function plays an important role. The author in works [1] and [2] showed that at the research of the subdifferential and condition of the optimality of high order also an important role is played the degree of the distance function.

Note that the article with the proof is available in [3].

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ON BOUNDEDNESS OF THE ADJOINT MULTIDIMENSIONAL HAUSDORFF OPERATOR IN WEIGHTED LEBESGUE SPACES

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Let R^n n-dimensional Euclidean space of points $x=(x_1,\dots,x_n)$ and let $A:=(a_{ij})$ be an $n\times n$-matrix whose entries $a_{ij}:R^n\to R$ are Lebesgue measurable functions, $i,j=1,\dots,n.$ Suppose $f:R^n\to R$ is a Lebesgue measurable function. For a fixed kernel function $\phi\in L_1^{loc}(R^n)$ the multidimensional Hausdorff operator is defined in the integral form by

$$Hf(x)=\int_{R^n}\phi(y)\,f\big(xA(y)\big)dy.$$

Let $A:=(a_{ij})$ is a non-singular matrix, so it is invertible. The corresponding multidimensional adjont Hausdorff operator is defined as follows (see [1])

$$H^*f(x)=\int_{R^n}\phi(y)\left|\det A^{-1}(y)\right|f\big(xA^{-1}(y)\big)dy.$$

In this abstract the boundedness of adjoint multidimensional Hausdorff operator on variable Lebesgue spaces is studied.

In the case of onedimensional Hausdorff operator we sent detail to [2].

This is jointly work with Rovshan Bandaliyev.

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ON HOLOMORPHIC PSEUDO-RIEMANNIAN MANIFOLDS

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The authors believe that geometry of holomorphic manifolds over commutative algebras is a very fruitful research domain and povides many new problems in the study of modern differential geometry. However, in spite of its importance, pseudo-Riemannian holomorphic manifolds over commutative algebras and related topics are not as yet so well-known. This was the motivation for writing the present paper.

Our goal is to study pseudo-Riemannian holomorphic manifolds over commutative algebras by using the Tachibana operator. The study of Tachibana operators was started in the early 1960s by

Tachibana [4] and Sato [2]. Shirokov [3] and Kruchkovich [1] developed the theory of Tachibana operators associated with a commutative algebraic structure.The main purpose of this paper is to investigate the holomorphic pure pseudo-Riemannian metrics with respect to the commutative algebraic structure. We show that the manifold admitting these metrics is a Kähler B-manifold. Some examples are also included.

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ON BASICITY OF PERTURBED EXPONENTIAL SYSTEMINGRANG-SOBOLEV SPACES

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In this work, one subspace of the grand- Sobolev space is introduced and basicity of the perturbed system of exponentials in this subspace is studied.

For $1 < p < +\infty$ a space measurable on $(a,b) \subset R$ functions f such that

$$\|f\|_{p)} = \sup_{0<\varepsilon<p-1} \left(\frac{\varepsilon}{|b-a|} \int_a^b |f(t)|^{p-\varepsilon} dt \right)^{\frac{1}{p-\varepsilon}} < +\infty, \quad (1)$$

is called $L_{p)}(a,b)$ grand-Lebesgue space [1].

Similarly, the Grand-Sobolev space $W_{p)}(a,b) = \{ f : f, f' \in L_{p)}(a,b) \}$ [2] is introduced with a norm

$$\|f\|_{W_{p)}} = \|f\|_{L_{p)}} . + \|f'\|_{L_{p)}} . \quad (2)$$

It is known that this is not separable Banach space. Denote by $\tilde{G}W_{p)}(a,b)$ the set of all functions from $W_{p)}(a,b)$ for which $\left\| \hat{f}'(\cdot+\delta) - \hat{f}'(\cdot) \right\| \to 0$ as $\delta \to 0$, where

$$\hat{f}(t) = \begin{cases} f(t), t \in (a,b), \\ 0, t \notin (a,b). \end{cases}$$

It is clear that $\tilde{G}W_{p)}(a,b)$ is a manifold in $\tilde{G}W_{p)}(a,b)$. Let $GW_{p)}(a,b)$ be a closure of $\tilde{G}W_{p)}(a,b)$ with respect to the norm (2).

Lemma. *Operator* $A(u,\lambda) = \lambda + \int_a^t u(\tau) d\tau$ *forms an isomorphism between spaces* $L_{p)}(a,b)$ *and* $GW_{p)}^1(a,b)$ *, where* $C -$ *is complex plane and* $p > 1$.

Theorem. *System* $t \cup \left\{ e^{i(n-\beta \operatorname{sign} n)t} \right\}_{n\in Z}$ *forms a basis in* $GW_{p)}(-\pi,\pi)$, *if* $-\frac{1}{2p} < \beta < \frac{1}{2q}$.

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A MIXED PROBLEM FOR A PARTIAL QUASILINEAR EQUATION

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Let us consider the problem

$$\frac{\partial^2 u}{\partial t^2} - A(t)\frac{\partial^3 u}{\partial t \partial x^2} - B(t)\frac{\partial^2 u}{\partial x^2} = f(u)\frac{\partial u}{\partial t} + g(u) \quad (1)$$

$$u(x,0) = u_0(x), u_t'(x,0) = u_1(x), \quad 0 \le x \le l \quad (2)$$

$$u(0,t) = u(l,t) = 0, \quad 0 \le t \le T; \quad (3)$$

where $A(t)$ and $B(t)$ are continuous functions.
Equation (1) is reduced to an abstract parabolic equation [1].

Problems (1)-(3) were studied by many authors. We will research the finite difference method for solving this problem.

In the rectangle $[0,T]\times[0,l]$ we construct a uniform grid

$$\omega_{\tau n}=\omega_\tau\times\omega_n=\{t_k=k\tau,\ k=\overline{0,n},\ x_j=jn,\ \overline{j=0,m}$$

with the steps $\tau=\frac{T}{n}, h=\frac{l}{m}$.

The value of the grid function y of determined on the grid $\omega_{\tau h}$ at the point (t_k,x_j) will be denoted by y_k^j.

To the problem (1)-(3) we associate the difference scheme

$$\frac{y_{k+1}^j-2y_k^j+y_{k-1}^j}{\tau^2}-A(t_{k+1})\frac{y_{k+1}^{j+1}-y_k^{j+1}-2y_{k+1}^j+2y_k^j+y_{k+1}^{j-1}-y_k^{j-1}}{\tau h^2}-B(t_k)\frac{y_k^{j+1}-2y_k^j+y_k^{j-1}}{h^2}=$$
$$=f(y_k^j)\frac{y_k^j-y_{k-1}^j}{\tau}+g(y_k^j),\ k=\overline{1,n-1},\ j=\overline{1,m-1} \tag{4}$$

$$y_k^0=y_k^m=0,\ k=\overline{0,n} \tag{5}$$

$$y_0^j=u_0(x_j)-\tau u_1(x_j)+O_1(1),\ y_1^j=u_0(x_j)+O_2(1),\ j=\overline{0,m} \tag{6}$$

where $O_2(1)-O_1(1)=O(\tau)$.

Theorem. *Let the following conditions be fulfilled***.**

1) $A(t)\in C^1[0,T],\ B(t)\in C[0,T], A(t)\ge a^2>0,\ B(t)\ge 0$

2) $f(u), g(u)-$ are continuously-differentiable on a real axis.

3) There exist constants $c_i, a_i b_i,\ i=1,2$ such that $f(u)\le c_1\ ug(u)\le c_2$

$$\int_{z_0}^{z} sf(s)ds\le a_1z^2+b_1,\ \int_{z_0}^{z} g(s)ds\le a_2z^2+b_2,$$

where a_i, b_i is dependent on z_0.

4) $u_0(x)\in C_0^2[0,l],\ u_1(x)\in C_0^2[0,l],$

Then the scheme (4)-(6) is well –posed and converges, i.e. it is uniquely solvable and we have the estimation

$$\sum_{j=1}^{m-1}\left|\frac{y_{k+1}^{j+1}-2y_{k+1}^{j}+y_{k+1}^{j-1}}{h^2}-\frac{z_{k+1}^{j+1}-2z_{k+1}^{j}+z_{k+1}^{j-1}}{h^2}\right|^2 h+\sum_{j=1}^{m-1}\left|\frac{y_{k+1}^{j+1}-y_{k}^{j+1}-y_{k+1}^{j}+y_{k}^{j}}{\tau h}-\frac{z_{k+1}^{j+1}-z_{k}^{j+1}+z_{k+1}^{j}+z_{k}^{j}}{h^2}\right|^2 h\le$$

$$\le C(R)\left[\sum_{j=1}^{m-1}\left|\frac{y_{0}^{j+1}-2y_{0}^{j}+y_{0}^{j-1}}{h^2}-\frac{z_{0}^{j+1}-2z_{0}^{j}+z_{0}^{j-1}}{h^2}\right|^2 h+\sum_{j=1}^{m-1}\left|\frac{y_{1}^{j+1}-y_{0}^{j+1}-y_{1}^{j}+y_{0}^{j}}{\tau h}-\frac{z_{1}^{j+1}-z_{0}^{j+1}+z_{1}^{j}+z_{0}^{j}}{h^2}\right|^2 h\right].$$

where y and z are two any solutions of the system (4) responding to initial data (y_0, y_1) and (z_0, z_1) such that

$$\sum_{j=1}^{m-1}\left|\frac{y_{0}^{j+1}-2y_{0}^{j}+y_{0}^{j-1}}{h^2}\right|^2 h\le R,\quad \sum_{j=1}^{m-1}\left|\frac{z_{0}^{j+1}-2z_{0}^{j}+z_{0}^{j-1}}{h^2}\right|^2 h\le R$$

$$\sum_{j=1}^{m-1}\left|\frac{y_{1}^{j+1}-y_{0}^{j+1}-y_{1}^{j}+y_{0}^{j}}{\tau h}\right|^2 h\le R,\quad \sum_{j=1}^{m-1}\left|\frac{z_{1}^{j+1}-z_{0}^{j+1}-z_{1}^{j}+z_{0}^{j}}{\tau h}\right|^2 h\le R$$

and the relation

$$\max_{k=\overline{1,n-1}}\sum_{j=1}^{m-1}\left|\frac{y_{k+1}^{j+1}-2y_{k+1}^{j}+y_{k+1}^{j-1}}{h^2}-\frac{u(t_{k+1},x_{j+1})-2u(t_{k+1},x_{j})+u(t_{k+1},x_{j-1})}{h^2}\right|^2 h+$$

$$\sum_{j=1}^{m-1}\left|\frac{y_{k+1}^{j+1}-y_{k}^{j+1}-y_{k+1}^{j}+y_{k}^{j}}{\tau h}-\frac{u(t_{k+1},x_{j+1})-u(t_{k},x_{j+1})-u(t_{k+1},x_{j})+u(t_{k},x_{j})}{\tau h}\right|^2 h\to 0$$

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THE DIRICHLET PROBLEM OF SEMILINEAR ELLIPTIC EQUATIONS

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Let E_n-n dimensional Euclidian space of the points $x=(x_1,x_2,...,x_n)$-is a bounded domain in E_n with the boundary $\partial\Omega \in C^2$.

Consider in Ω the following Dirichlet problem

$$\sum_{i,j=1}^{n} a_{ij}(x)u_{x_i x_j} + g(x, u_x) = f(x), \quad x\in\Omega \qquad (1)$$

$$u_{\partial\Omega} = 0. \qquad (2)$$

Assume, that the coefficients $a_{ij}(x)$ are measurable bounded functions satisfying the following conditions:

$$\gamma|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \le \gamma^{-1}|\xi|^2, \qquad (3)$$

$$\forall x\in\Omega, \forall x\in E_n, \xi\in(0,1) \text{ -const},$$

$$ess\sup_{x\in\Omega} \frac{\sum_{i,j=1}^{n} a^2_{ij}(x)(x)}{\left[\sum_{i=1}^{n} a_i(x)\right]^2} \le \frac{1}{n-1} - \delta, \qquad (4)$$

Where $\delta\in(0,1)$ is some number and $g(x,u_x)$ is a Caratheodory function and in addition to this, satisfying and condition:

$$\left|g(x,u_{x})\right| \le b_0 \left|u_x\right|^q \ , \ b_0 \succ 0 \qquad (5)$$

We denote by $\dot{W}_2^2(\Omega)$ the closure of the class of functions $u \in C^{\infty}(\overline{\Omega}) \bigcap C(\overline{\Omega})$ $, u\big|_{\partial\Omega} = 0$ with respect to the norm

$$\|u\|_{\dot{W}_2^2(\Omega)} = \left[\Omega\int_{\Omega}\left(|u|^2 + \sum_{i=1}^{n}\left|u_{x_i}\right|^2 + \sum_{i,j=1}^{n}\left|u_{x_i x_j}\right|^2\right)dx\right]^{1/2}.$$

The following theorem is proved.

Theorem: Let (3)-(5) be fulfilled and $n \succ 2$ $(n=1)$ $1 \le q \prec \frac{n}{n-2}$, $(1 \prec q \prec \infty)$ $\partial\Omega \in C^2$.Then there exists a sufficiently small positive constant $C_1 = C_1(n,\gamma,\delta,q,b_0,\Omega)$ $(C_2 = C_2(n,\gamma,\delta,q,b_0,\Omega))$ such that problem (1),(2) has at least one solution from $\dot{W}_2^2(\Omega)$ any $f(x) \in L_2(\Omega)$ satisfying the condition

$$\|f\|_{L_2(\Omega)} \le C_1(mes_n\Omega)^{\frac{-n+(n-2)q}{2n(q-1)}} \left(\|f\|_{L_2(\Omega)} \le C_3(mes_n\Omega)^{-\frac{1}{2(q-1)}}\right)$$

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SPECTRAL PROBLEMS FOR ELLIPTIC OPERATORS WITH SINGULAR COEFFICIENTS

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This work is concerned to some inverse scattering and inverse spectral problems for the bi-harmonic operator.
The operator is perturbed by the first and zero order perturbations, which are complex-valued (in general) and singular. We show the existence of the scattering solutions in the Sobolev spaces. One of the main result is the analogue of Saito's formula (in different form compare with known before), which can be used to prove a uniqueness theorem for the inverse scattering problem. Another main result is the estimates for the kernel of the resolvent of the direct operator in the Sobolev spaces that allow to obtain the reconstruction of the unknown coefficients for this perturbation. Another problem is concerned to the inverse boundary (spectral) value problem for this operator. It is proved that the Borg-Levinson data uniquely determine the unknown singular coefficients of the bi-harmonic operator.

CONVERGENCE OF SPECTRAL EXPANSION IN EIGEN-FUNCTIONS OF OOD ORDER DIFFERENTIAL OPERATOR

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Consider the ood order differential operator

$$Lu = u^{(n)} + P_1(x)u^{(n-1)} + \ldots + P_1(x)u$$

on the interval $G=(0,1)$, where $n=2m+1,\ m=1,2,\ldots,$ $P_1(x)\in L_2(G),\ P_l(x)\in L_1(G),\ l=\overline{2,n}.$

Denote by D_n a class of functions absolutely continuous on $\overline{G}=[0,1]$ together with their drivatives up to $(n-1)$ order.

By the eigenfunction of the operator L corresponding to the eigenvalue λ we understand any indentically nonzero function $u(x)\in D_n(G)$ satisfying the equation $Lu+\lambda u=0$ almost everywhere in G (see [1]).

Assume that the system $\{u_k(x)\}_{k=1}^{\infty}$ is a complete system of orthonormal eigenfunctions of the operator L in the space $L_2(G)$, and $\{\lambda_k\}$ is an appropriate system of eigenvalues, and $\operatorname{Re}\lambda_k=0, k=1,2,\ldots$. Denoting

$$\mu_k=\begin{cases}(-i\lambda_k), & \operatorname{Im}\lambda_k\ge 0,\\ (i\lambda_k), & \operatorname{Im}\lambda_k<0,\end{cases}$$

we define patial sum

$$\sigma_\nu(x,f)=\sum_{\mu_k\le\nu} f_k u_k(x),\ \nu>0,$$

of spectral expansion of function $f(x)\in W_2^1(G)$ in the system $\{u_k(x)\}_{k=1}^{\infty}$, where the Fourier coefficients f_k are defined by the formula $f_k=(f,u_k)=\int_G f(x)\overline{u_k(x)}dx.$

Denote $R_\nu(x,f)=f(x)-\sigma_\nu(x,f).$

Theorem. Let the conditions

$$\left|f(1)\overline{u_k^{(2m)}}(1)-f(0)\overline{u^{(2m)}}(0)\right|\le C_1(f)\mu_k^{\alpha}\|u_k\|_\infty,\ \mu_k\ge 1,$$

$0\le\alpha<2m,$ be satisfied for the function $f(x)\in W_2^1(G)$ and he system $\{u_k(x)\}_{k=1}^{\infty}$ absolutely and uniformly converges on $\overline{G}=[0,1]$ and the estimate

$$\|R_\nu(\cdot,f)\|_{C[0,1]} \le const\Big\{ C_1(f)\nu^{\alpha-2m} + \nu^{-\frac{1}{2}}\left(\| fP_2\|_2 + \| f'\|_2\right) +$$
$$+\nu^{-1}\| f\|_\infty \sum_{l=2}^{2m+1} \nu^{2-l}\|P_l\|_1 \Big\}, \nu \ge 2,$$

is valid. Here $\|P_l\|_2 = \|P\|_{L_2(G)}$, const is independent of $f(x)$.

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MAXIMAL IDEALS AND COMPACT WEIGHTED ENDOMORPHISMS OF FUNCTIONAL ALGEBRAS

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Let X be a compact Hausdorff topological space and S is a topolical ring. By $C(X,S)$ we denote the topological space of all continuous S-valued functions defined on X with compact-open topology. In this work we will investigate structures of maximal ideals of topological rings $C(X,S)$. Further, we will consider the case when S is a uniform algebra defined on this compact and we will investigate compactness of weighted composition operators on the subspaces (or compactness of weighted endomorphisms on the subalgebras) of

$C(X,A)$, where $A = A(X)$ is a uniform algebra defined on the compact X. Another words, we will investigate the operators of the forms , $(Tf)(x) = (u*(f \circ \varphi))(x), f \in C(X,A)$ where $u \in C(X,A)$ is a fixed A-valued function and $\varphi : X \to X$ is a self-mapping of X, which is continuous on the set X_u of points $x \in X$ such that $u(x)$ is not identically zero function (where, the symbol $\circ$ denote a composition of mappings and the symbol $*$ denote a multiplication operation in algebras). We assume that the topological space X a completely regular with respect to topological algebras A (in Tikhonov`s sense), i.e., for any closed subset K of X and for any point $x \in X \setminus K$ there exists S -valued functions $f, g \in C(X, A)$, such that $f(K)=0,\ f(x)=1$, and $g(K)=1,\ g(x)=0$. In particular, when the topological ring S is a commutative Banach algeba

First of at all, we investigate compactness of composition operator T on the closed subspaces (compactness of weighted endomorphisms on the subalgebras) of $C(X,A)$. Therefore, we introduce the notions “a peak set”, “a peak point” with respect to subspaces of $C(X,A)$, “a compactly connected component” of the set X_u, etc., and by use these notions we investigate the compactness of weighted composition operators on the closed subspaces of $C(X, A)$

In the first stage we receive next general necessary condition.

Theorem 1: Let S be a closed subspace of $C(X, A)$ and the operator of weighted composition $T : A \to C(X, A)$: $(Tf)(x) = (u*(f \circ \varphi))(x)$, $f \in C(X, A)$ is a compact operator. Then for any compactly connected component $Y \subset X_u$ and for any peak set E with respect to S, we have: either $\varphi(Y) \subseteq E,$ or $\varphi(Y) \cap E = \varnothing$

Further we give some applications of this general necessary condition for many special subalgebras S of $C(X, A)$ and we have differently transparently verifiable compactness criterion for weighted endomorphisms of subalgebras of $C(X, A)$, when the subalgebras S have differently boundaries (analogous Shilov boundaries). F or example, if X is a connected compact Hausdorff space, then we have next theorem.

Theorem 2: Endomorphisms on $C(X, A)$ of the form $T : C(X, A) \to C(X, A):\quad (Tf)(x) = (f \circ \varphi)(x), \quad f \in C(X, A)$ is compact if, and only if, when the self- mapping of X, $\varphi : X \to X$ is a constant.

RESTORATION OF SOURCE FUNCTION IN SYSTEM OF PARABOLIC EQUATIONS

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This thesis researches the approximate solution of the inverse problem of finding the right-hand side of space variation in a class of system of parabolic equations. $\{u_k(x,t), f_k(x), k = \overline{1,m}\}$. The following is the inverse problem of finding pairs of functions:

$$u_{kt} - u_{kxx} = f_k(x) g_k(x,t); (x,t) \in D = (0,l) \times [0,T] \tag{1}$$

$$u_k(x,0) = \varphi_k(x), x \in [0,l] \tag{2}$$

$$u_k(0,t)=\psi_k\left(t,\bar{u}_k(0,t)\right), u_k(l,t)=\phi_k\left(t,\bar{u}_k(l,t)\right), t\in[0,T] \quad (3)$$

$$\int_0^T u_k(x,t)dt = H_k(x), t\in[0,T] \quad (4)$$

Where $g_k(x,t), \varphi_k(x), \psi_k\left(t,\bar{u}_k\right), \phi_k\left(t,\bar{u}_k\right), H_k(x), k=\overline{1,m}$ are given functions with certain smoothness conditions, $\bar{u}_k=\left(u_1,\ldots,u_{k-1},u_{k+1},\ldots,u_m\right), T>0.$

(1) - (4) is a non-correct problem in the Adamar sense. In terms of contribution, the correctness in the sense of Tikhonov of problems related to (1)-(4) has been analyzed in $[1]$ and other cases.

If there is a classic solution $u_k(x,t)\in C^{2,1}\left(\bar{D}\right), f_k(x)\in C[0,l], k=\overline{1,m}$ of (1)-(4), then it can be shown, the problem (1)-(4) is equivalent to (1)-(3) and

$$f(x)=\left[u_k(x,T)-\varphi_k(x)-H_{kxx}(x)\right]\Big/\int_0^T g_k(x,t)dt \quad (5)$$

We can write a discrete analogue of (1)-(3), (5) by applying a two-step, finite-difference scheme as follows $[2]$:

$$\frac{u_k^{i,j+1}-u_k^{i,j}}{\tau}-\sigma\frac{u_k^{i+1,j+1}-2u_k^{i,j+1}+u_k^{i-1,j+1}}{h^2}- -(1-\sigma)\frac{u_k^{i+1,j}-2u_k^{i,j}+u_k^{i-1,j}}{h^2}=f_k^{i,j}g_k^{i,j} \quad (6)$$

$$u_k^{i,0}=\varphi_k^i, i=\overline{0,n} \quad (7)$$

$$u_k^{0,j+1}=\psi_k\left((j+1)\tau,u_k^{\overline{0,j}}\right), u_k^{n,j+1}=\phi_k\left((j+1)\tau,u_k^{\overline{n,j}}\right), j=\overline{0,p} \quad (8)$$

$$f_k^{i,j} = \left[u_k^{i,j} - \varphi_k^i - \frac{H_k^{i+1} - 2H_k^i + H_k^{i-1}}{h^2} \right] \Big/ q_k^i \qquad (9)$$

For given $h = \frac{l}{n}, \tau = \frac{T}{p}, u_k^{i,j} = u_k(ih, j\tau), i = \overline{0,n}, j = \overline{0,p}.$

The convergence to exact solution of (1)-(4) problem of (6) - (9) weighted scheme of the difference was investigated and the acceleration rate was evaluated.

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ON NEW METHOD TO OBTAIN PROBABILITY DENSITY FUNCTION OF SOLUTION OF STOCHASTIC DIFFERENTIAL EQUATIONS

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In this study, we have developed a new method to obtain approximate solution of Fokker-Planck-Kolmogorov (FPK) equation by using Generalized Entropy Optimization Methods **(**GEOM). The probability density function according to random variable which is solution of stochastic differential equations (SDEs) at a fixed time is the solution of FPK equation at this time. For this reason, the probability density function of approximate random variable of solution of SDEs obtained by using Euler-Maruyama's (EM) method or other methods can be considered as approximate solution of FPK equation. Mentioned approximate solution of FPK equation can be obtained by GEOM. We illustrated the use of this new method to describe SDE model fitting on given statistical data and corresponding approximate solution of FPK equation.

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INTEGRABLE DISSIPATIVE DYNAMICAL SYSTEMS: BACKGROUNDS, METHODS, AND APPLICATIONS

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We study nonconservative systems for which the usualmethods of the study, e.g., Hamiltonian systems, are inapplicable.Thus, for such systems, we must “directly” integrate the mainequation of dynamics. We generalize previously known cases andobtain new cases of the complete integrability in transcendentalfunctions of the equation of dynamics of a rigid body of differentdimensions in a nonconservative force field.

We obtain a series of complete integrable nonconservative dynamicalsystems with nontrivial symmetries. Moreover, in almost all cases,all first integrals are expressed through finite combinations ofelementary functions; these first integrals are transcendentalfunctions of their variables. In this case, the transcendence isunderstood in the sense of complex analysis, when the analyticcontinuation of a function into the complex plane has essentiallysingular points. This fact is caused by the existence of attractingand repelling limit sets in the system (for example, attracting andrepelling focuses). We detect new integrable cases of the motion ofa rigid body, including the classical problem of the motion of amulti-dimensional spherical pendulum in a flowing medium.

This activity is devoted to general aspects of the integrability ofdynamical systems with variable dissipation. First, we propose adescriptive characteristic of such systems. The term “variabledissipation” refers to the possibility of alternation of its signrather than to the value of the dissipation coefficient (therefore,it is more reasonable to use the term “sign-alternating”) [1, 2].

We introduce a class of autonomous dynamical systems with oneperiodic phase coordinate possessing certain symmetries that aretypical for pendulum-type systems. We show that this class ofsystems can be naturally embedded in the class of systems withvariable dissipation with zero mean, i.e., on the average for theperiod with respect to the periodic coordinate, the dissipation inthe system is equal to zero, although in various domains of thephase space, either energy pumping or dissipation can occur, butthey balance to each other in a certain sense. We present someexamples of pendulum-type systems on lower-dimension manifolds fromdynamics of a rigid body in a nonconservative field [2, 3].

Then we study certain general conditions of the integrability inelementary functions for systems on the two-dimensional plane andthe tangent bundles of a one-dimensional sphere (i.e., thetwo-dimensional cylinder) and a two-dimensional sphere (afour-dimensional manifold). Therefore, we propose an interestingexample of a three-dimensional phase portrait of a pendulum-likesystem which describes the motion of a spherical pendulum in aflowing medium (see also [1, 4, 5]).

The assertions obtained in the work for variable dissipation systemare a continuation of the Poincare–Bendixon theory for systems onclosed two-dimensional manifolds and the topological classificationof such systems.

The problems considered in the work stimulate the development ofqualitative tools of studying, and, therefore, in a natural way,there arises a qualitative variable dissipation system theory.

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ON FRAME PROPERTIES OF ITERATES OF A MULTIPLICATION OPERATOR

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Dynamical sampling that is a relatively new research topic in applied harmonic analysis has attracted considerable attention in recent years (see, for example, [1] and the bibliography therein). One of the central problems in dynamical sampling is investigation of frame properties for families of elements obtained by iterates of operators.

This note is dedicated to the study of frame properties of iterates of a multiplication operator $T_\varphi f(t) = \varphi(t) \cdot f(t), \; f \in L_2(a,b)$.

The following theorem is obtained in [1]:

Theorem 1. *Let* $\varphi(t)$ *be any measurable function and* $f(t)$ *any square summable function on* (a,b). *The system* $\left\{T_\varphi^n f\right\}_{n=0}^{\infty}$ *cannot be a frame in* $L_2(a,b)$.

This fact shows in particular that a system of the form $\left\{\varphi^n(t)\right\}_{n=0}^{\infty}$ cannot be a frame in $L_2(a,b)$ for any measurable function $\varphi(t)$.

The classical exponential system shows that the situation changes drastically when one considers systems of the form $\left\{\varphi^n(t)\right\}_{n=-\infty}^{\infty}$ instead of $\left\{\varphi^n(t)\right\}_{n=0}^{\infty}$.

To our knowledge, the characterization of all frames of the form $\left\{\varphi^n(t)\right\}_{n=-\infty}^{\infty}$ in $L_2(a,b)$ remains unanswered in the general statement.

In this note we give a partial answer to this problem by the following statement:

Theorem 2. *Suppose that a function $\alpha(t)$ defined on $[a,b]$ is an invertible function, inverse $\xi:[p,q]\to[a,b]$ of which satisfies the following conditions:*

1) $\xi(t)$ *is absolutely continuous, strictly increasing function on* $[p,q]$; $\xi(p)=a$ *and* $\xi(q)=b$;

2) $[p,q]\subset[0,2\pi]$ *and there are constants* $A,B>0$ *such that* $A\le\xi'(t)\le B$ *for all* $t\in[p,q]$.

Then the system $\left\{e^{in\alpha(t)}\right\}_{n=-\infty}^{\infty}$ *is a frame in* $L_2(a,b)$.

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THE SHEAVES REPRESENTATION OF HAUSDORFF SPECTRA

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The study which was carried out in [1]–[2] of the derivatives of the projective limit functor acting from the category of countable inverse spectra with values in the category of locally convex spaces made it possibleto resolve universally homomorphism questions about a given mapping in terms of the exactness of a certain complex in the abelian category of vector spaces. Later in [3] a broad generalization of the concepts of direct and inverse spectra of objects of an additive semiabelian category *G* was introduced: the concept of a Hausdorff spectrum, analogous to the σs-operationin descriptive set theory. This idea is fruitful even for algebraic topology,general algebra, category theory and the theory of generalized functions.

On the other hand, the issues related to different types of sheaves, including bundles, still attracts great interest and primarily in solving problems ofalgebraic geometry (see, for example, [4]). And it should be recognized that the concept of the spectrum ofa refxive sheaf of rank two on P^3, introduced in [5] for the field of any characteristic, turned out to be very useful for such purposes. Such spectrum have a numerical nature.

We present an essentially new approach to the study of sheaves by means of non-numerical spectra. This approach based on the notion of Hausdorff spectrum associated with the presheaf.

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ENERGY OF AN AC ELECTRIC FIELD IN A SEMI-INFINITE ELECTRON PLASMA WITH MIRROR BOUNDARY CONDITIONS

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The character of electric field screening near the surface of a conductor is critically important for different problems of surface physics [1, 2], in particular, the problem of propagation of plasma oscillations [3, 4]. Surface Raman scattering related to the specific behavior of the electric field near the surface is of great interest [5]. The properties of a screened electric field are important for description of electromagnetic field interaction with surfaces of conducting media and thin films [6–9].

In [10], we have obtained an analytical solution to the problem on the behavior of a semi-infinite plasma with an arbitrary degree of electron gas degeneracy in an external ac electric field perpendicular to the plasma surface. Such a situation takes place, e.g., when analyzing a solid-state semiconductor plasma. We use the Vlasov–Boltzmann kinetic equation with the Bhatnagar–Gross–Krook (BGK) collision integral for the electron distribution function and Maxwell's equation for the electric field.

The use of a classical kinetic equation (namely, the Vlasov−Boltzmann equation) imposes restrictions on the parameters of the problem under consideration. For instance, in the case of a solid-state semiconductor plasma, the frequency ω should satisfy the condition $\omega < E_0/\hbar$, where E_0 is the band gap. Here, we imposed no limitations on the degree of electron gas degeneracy; however, the electron density was limited by the requirement of the applicability of the classical kinetic analysis. For example, such an approach is inapplicable for electron densities typical of white dwarfs. A detailed analysis of limitations of the classical kinetic analysis as a function of the electron gas density can be found in [11, 12]. When classical analysis is insufficient, a more general approach based on the quantum kinetic equations should be used. For instance, the Wigner function is used instead of the classical distribution function [9, 12]. Note that the Wigner approach is used mainly in the collisionless case [12]. The capabilities of the energy-dissipation analysis in this approach are limited.

First, it is demonstrated that the Drude mode describing volumetric conductivity exists at all values of the problem parameters. Second, the Debye mode describing electric field screening exists at external field frequencies below a certain critical frequency lying near the plasma resonance. Finally, we show that the van Kempen modes, representing a chaotic mixture of eigenfunctions of the Vlasov−Boltzmann equation, also exist at all values of the problem parameters. It is demonstrated that an analytical solution to the boundary-value problem can be represented in the form of expansion in the eigenmodes corresponding to the above-mentioned spectra.

The obtained analytical results allow one to reveal the character of electromagnetic energy dissipation in plasma. They make it possible to separate contributions to dissipation stemming from the processes occurring in the plasma volume and near the plasma surface, i.e., to separate energy absorption into the volume and surface components. Surface absorption is analyzed in detail. A nontrivial character of the dependence of surface absorption on the ratio between the volumetric electron collision frequency and the frequency of the external electric field is demonstrated.

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ON WIMAN-VALIRION-TYPE ESTIMATIONS FOR THE SOLUTIONS OF PARABOLIC EQUATIONS

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In Hilbert space we consider a differential equation of the form

$$u'(t) + A(t)u(t) = 0, \tag{1}$$

where $A(t)$ is a positive self-adjoint operator with a dicrete spactrum. It is assumed that some conditions of asymptotic type on the distribution function $N(\lambda; A)$ eigen-values of the operator $A(t)$ are fulfilled, and for the solution of equation (1) the Viman-Valirion-type theorem is proved. By this theorem the following estimation is valid :

$$\|u(t)\| \le t^{-\alpha} \mu(t)\psi\left(t^{-\beta} \log \mu(t)\right),$$

where $\alpha, \beta \ge 0; \mu(t) = \max_k |(u(t), \varphi_k(t))|$, $\{\varphi_k(t)\}$ is a complete orthonormed system of eigen-functions of the operator $A(t)$;

$\psi(y)$ is a positive function incresing for $y>0$ such the that integral of the form

$$\int^{\infty}\left(\int^{y}\psi(t)dt\right)^{-\gamma}dy,\ \ \gamma>0.$$

is finite.

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ON VARIATIONAL STATEMENT OF INVERSE PROBLEM ON DETERMINING LOWER COEFFICIENT OF A PARABOLIC EQUATION

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Let Ω be bounded domain in R^n with smooth boundary $S=S'\cup S'', T>0$ is a given number, $Q_T=\Omega\times(0,T)$ is a cylinder, $S_T=S\times(0,T)$ is a lateral surface of the cylinder Q_T, $S'_T=S'\times(0,T), S''_T=S''\times(0,T)$.

We minimize the functional

$$J(v)=\int_{\Omega}\left|\int_0^T\omega(t)u(x,t;v)dt-\alpha(x)\right|^2dx \quad (1)$$

under conditions

$$u_t - \sum_{i,j=1}^{n} a_{ij}(x,t) u_{x_j})_{x_i} + v(x)u = f(x,t), (x,t) \in Q_T \qquad (2)$$

$$u\big|_{t=0} = \varphi(x), x \in \Omega, \qquad (3)$$

$$u\big|_{S'_T} = 0,\ \frac{\partial U}{\partial N}\bigg|_{S''_T} = \int_{\Omega} K(x,y,t) u(y,t) dy\big|_{S''_T}, \qquad (4)$$

$$v = v(x) \in V = \{v(x) : v(x) \in L_2(\Omega), |v(x)| \le d, x \in \Omega\}\ , \qquad (5)$$

here $\omega(t), \alpha(x), a_{ij}(x,t), i; j = \overline{1,n}, f(x,t), \varphi(x), K(x,y,t) -$ are known functions, $v = v(x) -$ is a desired coefficient or control, $d > 0 -$ is a given number, $u = u(x,t) = u(x,t;v) -$ is a generalized solution to the boundary value problem (2)-(4) from $V_2^{1,0}(Q_T) -$ corresponding to the control v.

Assume that the input data of the problem (1)-(5) satisfy the following conditions:

$$a_{ij}(x,t) = a_{ji}(x,t), i, j = 1,n\ ,$$

$$\upsilon \xi^2 \le \sum_{i,j=1}^{n} a_{ij}(x,t) \xi_j \xi_i \le \mu \xi^2, (x,t) \in Q_T\ ,$$

$$|a_{ijt}(x,t)| \le \mu_1, i, j = \overline{1,n}, (x,t) \in Q_T\ ,$$

$$\varphi(x) \in W_{2,0}^1(\Omega), f(x,t) \in L_2(Q_T)\ ,$$

$$\omega(t) \in L_2(0,T), \alpha(x) \in L_2(\Omega)\ ,$$

$$|K(x,y,t)| \le \mu_2, |K_t(x,y,t)| \le \mu_3, (x,y,t) \in S'' \times \Omega \times (0,T)\ ,$$

where $\upsilon, \mu, \mu_1, \mu_2, \mu_3 > 0$ are given numbers.

Problem (1)-(5) is a variational formulation of the inverse problem of determining the functions $\{u(x,t), v(x)\}$ satisfying conditions (2)-(5) and the additional condition

$$\int_0^T \omega(t)u(x,t;v)dt = \alpha(x),\ x \in \Omega . \qquad (6)$$

The functional (1) is a residual functional in $L_2(\Omega)$ corresponding to the condition (6).

In this work, the correctness of the problem (1)-(5) is investigated and the necessary optimality condition is established in the form of a variational inequality.

THE SOLUTION ALGORITHM FOR FINDING THE COEFFICIENT OF HYDRAULIC RESISTANCE IN THE PROCESS OF GAS LIFT

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Introduction.

As we know that the finding of the coefficient of hydraulic resistance is one of the key issues for the creation of an automated control system during oil extraction [4]. The paraffins in the fluid gas mixture adheres to the pipe and creates a surface with certain thickness and after a certain time it hinders the operation of relevant regime [1]. From this point of view, it always helpful to define the coefficient of hydraulic resistance so that the regime can be corrected at any time. Therefore, the solution of equations of motion described by hyperbolic equations are given by sequence [3].

Keywords:gas-lift,gas-liquid mixture,hydraulic resistance coefficient,sequence method.

Statement of the problem.

It is known that the mathematical model of the gas lift event is brought to the question of the boundary problem set for the system of the first-order hyperbolic of differential equations with particular derivative [2].

$$\begin{cases} -F\dfrac{\partial \overline{P}(x,t)}{\partial x} = \dfrac{\partial \overline{Q}(x,t)}{\partial t} + 2a\overline{Q}(x,t), \\ -F\dfrac{\partial \overline{P}(x,t)}{\partial t} = c\dfrac{\partial \overline{Q}(x,t)}{\partial x}, \qquad x \in (l,2l),\ t \in (0,T) \end{cases} \tag{1}$$

The boundary conditions as following:

$$\begin{cases} \overline{P}(0,t) = \overline{P}_0(t), \\ \overline{Q}(0,t) = \overline{Q}_0(t), \qquad t \in [0,T]. \end{cases} \tag{2}$$

Where F,a,c and l are the given constants, $\overline{P}_0(t)$and $\overline{Q}_0(t)$ are the given continuous functions, $\overline{P}(x,t)$ and $\overline{Q}(x,t)$ are the functions to be found.

Solution algorithm:

Seeking a solution to this question [3] as the line.

$$\begin{cases} \overline{P}(x,t) = \sum\limits_{k=0}^{\infty} \overline{P}_k(t)\dfrac{x^k}{k!}, \\ \overline{Q}(x,t) = \sum\limits_{k=0}^{\infty} \overline{Q}_k(t)\dfrac{x^k}{k!},\ x \in (0,l) \cup (l,2l),\ t \in (0,T) \end{cases} \quad 3)$$

Helps to find some unknown $P_k(t)$ and $Q_k(t)$ coefficients $P_0(t)$and $Q_0(t)$ are given in the boundary condition) that are as follows. Depending on whether the coefficients are even or odd, they are identified as follows:

$$\begin{cases} \overline{P}_{2k}(t) = \left(\dfrac{d}{dt} + 2a_2\right)^k \dfrac{\overline{P}_0^{(k)}(t)}{c_2^k}, \\ \overline{Q}_{2k}(t) = \left(\dfrac{d}{dt} + 2a_2\right)^k \dfrac{\overline{Q}_0^{(k)}(t)}{c_2^k}, \quad k \geq 0, \end{cases} \tag{4}$$

$$\begin{cases} \overline{P}_{2k+1}(t) = -\left(\dfrac{d}{dt} + 2a_2\right)^{k+1} \dfrac{\overline{Q}_0^{(k)}(t)}{F_2 c_2^k}, \\ \overline{Q}_{2k+1}(t) = -\left(\dfrac{d}{dt} + 2a_2\right)^{k} \dfrac{F_2 \overline{P}_0^{(k+1)}(t)}{c_2^{k+1}}, \qquad k \geq 0, \end{cases} \tag{5}$$

Let's mark these solutions as $\overline{P}_2(x,t,a_2)$ and $\overline{Q}_2(x,t,a_2)$and look at functional as follows:

$$I(a_2) = \int_0^T \left\{ \left[\overline{P}_2(2l,t,a_2) - \tilde{P}_2(2l,t)\right]^2 + \left[\overline{Q}_2(2l,t,a_2) - \tilde{Q}_2(2l,t)\right]^2 \right\} dt \tag{6}$$

Where $\tilde{P}_2(2l,t)$ and $\tilde{Q}_2(2l,t)$ are the given functions. For finding a_2, from the functional (6) we have obtained the following equalities.

$$\int_0^T \left\{ \left[\sum_{k=0}^{\infty} \overline{P}_k(t,a_2) \cdot \frac{(2l)^k}{k!} - \tilde{P}_2(2l,t) \right] \cdot \sum_{m=0}^{\infty} \frac{\partial \overline{P}_m(t,a_2)}{\partial a_2} \cdot \frac{(2l)^m}{m!} + \right.$$
$$\left. + \left[\sum_{k=0}^{\infty} \overline{Q}_k(t,a_2) \cdot \frac{(2l)^k}{k!} - \tilde{Q}_2(2l,t) \right] \cdot \sum_{m=0}^{\infty} \frac{\partial \overline{Q}_m(t,a_2)}{\partial a_2} \cdot \frac{(2l)^m}{m!} \right\} dt = 0$$

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THE METHOD OF SOLUTION OF GENERAL QUASISTATIC PROBLEM OF THE LINEAR VISCOELASTICITY THEORY AND ITS APPLICATION

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The method which allows reducing the general quasistatic problems of the linear isotropic viscoelasticity theory to Volterra integral equation of second kind is offered. Entering into the determining relations, the two creep (relaxation) functions corresponding to shift and volumetric expansion states are accepted to be independent, i.e. the viscoelastic Poisson’s ratio is considered to be time-dependent. Unlike many existing methods, here traditional application of integral transformations is excluded. The approbation of the method is carried out on the example of applied problems: 1) the problem of the rotating round thick disk, 2) the problem of action of the concentrated force in an infinite body (problem of Kelvin).

The method applied here may be used in solving otherconcretely stated of series problems of the theory of linear isotropic viscoelasticity. Therewith other elastic solution should be represented for stresses and displacements in the form of linear dependence from $Sf_k(\upsilon, G, a_1, a_2, ..a_m)$, where $k = 1,2,\ldots,n, S = S(t)$ are the given external force quantities of type of force or displacements; υ, G are elastic constants, $a_1, a_2, ..a_m$ are geometric parameters of the body under investigation. Such a representation of the elastic problem is possible in view of linearity of this problem. It is important the function f_k must not contain current coordinates of body points as an argument. In order to simplify the further operations, it is undesirable to include the quantities $a_1, a_2, ..a_m$ into the argument of the functions f_k, if they may be separated. For obtaining the solution of the viscoelastic problem we must make substitution of the quantities Sf_k by the functions $A_k(t, a_1, a_2, ..a_m)$. The conditions of satisfaction of relations of the isotropic viscoelastic problem by the represented solutions will compose a system of equations for defining the functions $A_k(t, a_1, a_2, ..a_m)$. These equations will not contain coordinates of body points,since differential equations of equilibrium, strain compatibility conditions and Cauchy geometric relations that are satisfied by the solution of the elastic problem, are also contained in viscoelastic problem statement. The obtained system of equations, as in the case of Boussenesque viscoelastic problem considered here, by means of the method of successive exclusion of unknowns may be reduced to second kind Volterra integral equation that has a unique solution. If the external action quantities in the viscoelastic problem are given in the form $S = S(x, t)$ where x are the body point coordinates, then for the application of the method, it is required that S allow separation of variables: $S = S_1(x) \cdot S_2(t)$. In this case, instead of Sf_k it should be used $S_2 f_k$.

This method is applied to the solutions of new boundary value problems of the theory of viscoelasticity about a rotating circular thick disk and about the action of a concentrated force in an infinite body.

HOMOGENIZATION OF A COMPRESSIBLE CAVITATION MODEL WITH MOVING ROUGH SURFACE

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In this paper we consider mathematical modeling of thin film flow between two rough surfaces which are in relative motion. For example such flows take place in different kinds of bearings and gears when a lubricant is used to reduce friction and wear. Usually the pressure, p in the fluid is computed by solving the Reynolds equation. We study a compressible Reynolds equation and take into account cavitation for the case where both surfaces are rough and moving with velocity V_i (i=1,2). Moreover, we use two-scale convergence to homogenize the cavitation model.

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PARAMETER IDENTIFICATION OF CONVECTION-DIFFUSION TRANSPORT MODELS IN THE PRESENCE OF NOISY MEASUREMENTS

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In recent decades there has been an increased interest in inverse problems for partial differential equations in various areas of science and technology [1]. Consider the convection-diffusion transport model with mixed boundary conditions described by the following equations:

$$\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}=\alpha\frac{\partial^2 c}{\partial x^2},\ a<x<b,\ 0<t<+\infty, \quad (1)$$

$$c(x,0)=\varphi(x),\ a\le x\le b, \quad (2)$$

$$c(a,t)=f(t),\ \frac{\partial c(b,t)}{\partial x}=-\lambda[c(b,t)-g(t)],\ 0<t<+\infty, \quad (3)$$

where $c(x,t)$ is the function of interest, x is the spatial coordinate, t is the time. Suppose that there is a need to determine coefficients v and α in equation (1) or functions $f(t)$ and $g(t)$ in boundary conditions (3) but only the noisy measurements of $c(x,t)$ at some points are available.

Using the discretization process let us move from the model (1)–(3) to a discrete linear stochastic model, whose equations generally have the following form:

$$\begin{cases} c_k = F(\theta)c_{k-1} + B(\theta)u_{k-1} + w_{k-1}, \\ z_k = H(\theta)c_k + \xi_k, \qquad k = 1,2,..., \end{cases} \quad (4)$$

where $c_k \in \mathbf{R}^n$ is the system state vector, $u_k \in \mathbf{R}^r$ is the input vector, $z_k \in \mathbf{R}^m$ is the measurements vector, noises $w_k \in \mathbf{R}^n$ and $\xi_k \in \mathbf{R}^m$ form independent normally distributed sequences with zero mean and covariance matrices $Q(\theta) \geq 0$ and $R(\theta) > 0$ respectively, matrices $F(\theta)$, $B(\theta)$, $H(\theta)$, $Q(\theta)$, $R(\theta)$ can depend on an unknown parameter $\theta \in \mathbf{R}^p$.

Now, to identify the unknown parameters of the model (1)–(3) we may use methods of parameter identification developed for discrete linear stochastic system models (see [2], [3] for details).

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PARAMETER ESTIMATION FOR GEOMETRIC PROCESS WITH POWER ISHITA DISTRIBUTION

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This paper considers the problem of parameter estimation for the geometric process (GP) when the first occurrence time of an event is assumed to follow power Ishita distribution. The maximum likelihood (ML) method is used to derive the estimators of the parameters in GP. The performances of the ML estimators are also compared with the corresponding non-parametric modified moment (MM) estimators in terms of bias and mean squared error through an extensive simulation study. The results of the simulation study indicate that the ML estimators perform better than the MM estimators.

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PARAMETERIZATION OF STABILIZING REGULATORS OVER OF PART OF THE PHASE COORDINATES IN THE DISCRETE CASE

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To solve a linear-quadratic Gaussian problem of optimal synthesis in discrete-time output applying the frequency parameterization method, proposed computational algorithm. The proposed algorithm uses procedures of discrete factorization and separation with respect to the unit circle, MFD representation, and solution Diophantine equation.

Key words: parameterization, matrix fraction decomposition, Diophantine equation, factorization, Hermitian matrix polynomials.

ON THE DYNAMIC STABILITY OF DEFORMABLE ELEMENTS OF AEROELASTIC STRUCTURES

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The dynamics and stability of deformable structural elements that are in interaction with a gas or liquid flow are investigated.Mathematical models of the dynamics of elements (elastic plates, rods) are considered, which are components of vibration devices, protective screens, pressure sensors, piping systems, etc. Mathematical models are initial-edge tasks for related systems of differential equations with private derivatives for hydrodynamic functions and elastic deformations (see, for example, [1-6]).

The dynamics and stability of the elastic elements of mechanical systems are studied in the external and internal flow of a liquid (gas). The subsonic and supersonic flow regimes are considered. To describe the dynamics of elastic elements, both linear and nonlinear models of a solid deformable body are used. Aerohydrodynamic effects on structures are determined from asymptotic or exact equations of gas dynamics in a model of an ideal or viscous medium. An analytical study of the dynamics and stability of deformable elements is carried out based on the construction of functionals of the Lyapunov type for differential and integro-differential equations with partial derivatives. The numerical-analytical solution is based on the Bubnov-Galerkin method. A numerical experiment was carried out to determine the nature of the oscillations.

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ON THE SOLUTION OF AN OPTIMAL CONTROL PROBLEM FOR A THIRD ORDER EQUATION

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1. Statement of the problem. Let us consider the process described by the problem

$$\beta z_{tt} + z_t - \varepsilon z_{xxt} - z_{xx} = u(x,t), \quad (x,t) \in Q = \{0 < x < 1, 0 < t < t_1\}, \tag{1}$$

$$z(x,0) = 0, \quad z_t(x,0) = \upsilon(x), \quad 0 \le x \le 1, \tag{2}$$

$$z(0,t) = z(1,t) = 0, \ 0 \le t \le t_1, \tag{3}$$

where β, ε are given sufficiently small positive constants, t_1 is fixed moment of time.

It is required to find such a distributed control $u = u(x,t) \in L_2(Q)$ and start control $\upsilon = \upsilon(x)$, for which the corresponding solution $z(x,t)$ of problem (1)-(3) satisfies condition $z(x,t_1) = \varphi(x)$ functional

$$J(u,v) = \iint_Q u^2(x,t)dxdt + \alpha \int_0^l v^2(x)dx \tag{4}$$

takes the smallest value, where α is a positive number.

We seek the solution of this problem (1)-(3) in the form $z(x,t) = y(x,t) + \omega(x,t)$, where $y(x,t)$ and $\omega(x,t)$ are the solutions of the following problems, respectively

$$\begin{cases} \beta y_{tt} + y_t - \varepsilon y_{xxt} - y_{xx} = 0, \\ y(x,0) = 0, \quad y_t(x,0) = \upsilon(x), \\ y(0,t) = y(1,t) = 0; \end{cases} \text{(I)} \quad \begin{cases} \beta \omega_{tt} + \omega_t - \varepsilon \omega_{xxt} - \omega_{xx} = u(x,t), \\ \omega(x,0) = 0, \quad \omega_t(x,0) = 0, \\ \omega(0,t) = \omega(1,t) = 0; \end{cases} \text{(I I)}$$

We seek the solution of problem (I) by the method of separation of variables, i.e. we seek in the form

$y(x,t)=X(x)\cdot T(t)$. Then we obtain an eigenvalue problem with respect to $X(x)$

$$X''(x)+\lambda X(x)=0, \qquad X(0)=X(1)=0.$$

It is known that the eigenvalues and orthonormal eigenfunctions of this problem are $\lambda_k=\pi^2k^2, \quad X_k(x)=\sqrt{2}\sin \pi kx,\ k=1,2,\ldots.$

Taking this into account with respect to $T(t)$, we obtain the following Cauchy problem

$$\beta\ddot{T}+\left(1+\varepsilon\pi^2k^2\right)\dot{T} \ +\pi^2k^2=0, \ \ T(0)=0, \ \dot{T}(0)=\upsilon_k. \ (5)$$

For $y(x,t)=X(x)\cdot T(t)$, we obtain the following representation

$$y(x,t)=\sum_{k=1}^{\infty}\frac{1}{l_2(k)-l_1(k)}\upsilon_k\cdot\left(e^{l_2(k)t}-e^{l_1(k)t}\right)\sin \pi kx,$$

where $l_1(k), l_2(k)$ are the roots of the characteristic equation corresponding to (5) .The solution of problem (II) in the form

$$\omega(x,t)=\sum_{k=1}^{\infty}\frac{1}{\beta(l_2(k)-l_1(k))}\int_0^t\left(e^{l_2(k)(t-s)}-e^{l_1(k)(t-s)}\right)u_k(s)ds\sin \pi kx.$$

Then the formal solution of problem (1)-(3) is

$$z(x,t)=\sum_{k=1}^{\infty}\frac{1}{(l_2(k)-l_1(k))}\left[\upsilon_k\cdot\left(e^{l_2(k)t}-e^{l_1(k)t}\right)+\frac{1}{\beta}\int_0^t\left(e^{l_2(k)(t-s)}-e^{l_1(k)(t-s)}\right)u_k(s)ds\right]\cdot\sin \pi kx.$$

Is proved

Theorem 1. For any $u(x,t)\in L_2(Q), \upsilon(x)\in L_2(0,1)$ problem (1)-(3) has a unique solution.

After the controlling (1)-(4), redused to the problem of finding minimum of functional

$$J(x,\upsilon)=\sum_{k=1}^{\infty}\left[\int_0^{t_1} u_k^2(t)dt+\alpha(\upsilon_k)^2\right]. \tag{6}$$

with conditional

$$2\int_0^{t_1} A_k(t)u_k(t)dt+\beta C_k\upsilon_k = 2\beta(l_2(k)-l_1(k))\varphi_k,\ k=1,2,..., \tag{7}$$

Where $A_k(t)$, $C_k(t)$ known functions we compose the Lagrange functional of this problem and obtain.

$$L(u_k,\upsilon_k,\lambda_0,\lambda_1)=\lambda_0\left[u_k^2(t)+\alpha\upsilon_k^2\right]+\lambda_1\left[2A_k(t)u_k(t)+\beta C_k\upsilon_k^2\right].$$

To define $u_k(t),\upsilon_k$, we obtain the system

$$\lambda_0 u_k(t)+\lambda_1 A_k(t)=0,\quad \lambda_0\alpha\upsilon_k+\tilde{\lambda}_1\beta C_k=0.$$

Assuming $\lambda_0=1$, after defining λ_1 from (11), we obtain

$$u_k(t)=\frac{\alpha\beta(l_2(k)-l_1(k))\varphi_k A_k(t)}{D_k},\quad \upsilon_k=\frac{\beta^2(l_2(k)-l_1(k))\varphi_k C_k}{D_k}, \tag{8}$$

where

$$D_k=2\alpha\int_0^{t_1} A_k^2(t)dt+\beta_k^2 C_k^2.$$

On the basis of these calculations the following theorem is proved:

Theorem 2. For any $u(x,t)\in L_2(Q),\ \upsilon(x)\in L_2(0,1)$ the problem of the minimum of the functional (5) under the condition (4) has a solution $\tilde{u}_k(x,t),\ \tilde{\upsilon}_k(x)$ that is representable in the form

$$\tilde{u}(x,t)=\sum_{k=1}^{\infty}\tilde{u}_k(t)\sin\pi kx\ ,\ \tilde{\upsilon}(x)=\sum_{k=1}^{\infty}\tilde{\upsilon}_k\sin\pi kx,$$

where $\tilde{u}_k(t)$, $\tilde{\upsilon}_k$ are represented by formulas (8).

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SECOND ORDER ELLIPTIC DIFFERENTIAL-OPERATOR PROBLEMS WITH THE SPECTRAL PARAMETER AND THE OPERATOR IN BOUNDARY CONDITIONS

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First, we give some review on general results for elliptic differential-operator problems in Hilbert and UMD Banach spaces and then we present non-coercive solvability for a second order problem, when the spectral parameter and an abstract operator enter into the boundary conditions. Some non-classical phenomena are observed. Finally, we give some application of the obtained abstract results to elliptic partial differential problems.

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GLOBAL REGULARITY IN ORLICZ-MORREY SPACES OF SOLUTIONS HIGHER-ORDER ELLIPTIC EQUATIONS WITH VMO COEFFICIENTS

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We study higher-order uniformly elliptic equations

$$\sum_{|\alpha|,|\beta|\leq 2m} a_{\alpha,\beta}(x)D^{\alpha}D^{\beta}u = f \text{ in}\,\Omega \subset R^{n}$$

İn Orlicz-Morrrey space with the Dirichlet boundary condition. Ω tubenon-smooth domain. The coefficients $a_{\alpha,\beta}(x)$ are discontinuously. The right hand of equation f belonging to Orlicz-Morrey spaces. We global regularity of generalized solution in Orlicz-Morrey spacesis proved.

OPTIMIZATION OF OPERATION OF SYSTEM ON THE BASIS OF THE INTEGRATED MODERN MODULE OF THE GAS LİFT PROCESSES

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It iscreated improved adequate mathematical models taking into account change of structure and phase for description and optimal control of multiphase time-dependent processes which happen in oil wells exploited by gas lift method, expressed by means of a system of differential equations and taking into account using of their stochastic analogues.It has been got a solution by analytical method of a motion process under influence of different phase speeds of the two-phase mixed liquid and the nonlinear changing temperature field. It has been shown applied ways of the solution to learning of gas lift processes. A mathematical model and control algorithm of the problem of distribution of the working substance among gas lift wells have been developed. Taking into account a stochastic character of the forces influencing to the system during the vertical motion of gas-liquid mixture through the oil-well tubing an optimal control system of gas lift complex has been created. A more perfect and adequate mathematical deterministic and stochastic model of motion of gas-liquid mixture through the vertical oil-well tubing inside the well is offered.A more perfect and adequate mathematical deterministic and stochastic model of motion of gas-liquid mixture through the vertical oil-well tubing inside the well. Algorithms and software which will be used in the practice have been created. Fractal models simplify the analysis of a whirl of liquid or gas, and also course process that is important for industrial technologies of technology of development of oil fields and gas. In particular, intense large-scale fractal structures arise at downloading in layer of water, gas and other agents supporting sheeted pressure. Existence of fractal structures can be connected with pollution of priskvazhinny zones of layer. The purpose of the work is to develop mathematical models that provide effective and optimal working conditions for oil extraction through the mines, develop the theoretical basis of the productive layers of labor parameters based on these models and theoretical basis of their management systems.

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INVERSE BOUNDARY VALUE PROBLEM FOR ONE THIRD-ORDER PARTIAL DIFFERENTIAL EQUATION

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Let $D_T=\{(x,t):0<x<1, 0<t<T\}$, and let $f(x,t)$, $\varphi_i(x,t)$, $(x,t)\in\overline{D}_T$, $i=0,1,2$, and $h(t), t\in[0,T]$, are given functions.

In this note we consider the following inverse boundary value problem: find the functions $u(x,t)$ and $a(t)$ connected in D_T by the equation

$$u_{ttt}(x,t)+u_{xx}(x,t)=a(t)u(x,t)+f(x,t) \qquad (1)$$

under the following boundary and initial conditions as well as the condition of overriding for the function $u(x,t)$:

$$u(x,0)=\varphi_0(x),\ u_t(x,0)=\varphi_1(x), u_{tt}(x,T)=\varphi_2(x),\ 0\le x\le 1, \quad (2)$$

$$u(0,t)=u_x(1,t)=0,\ \ 0\le t\le T, \ (3)\ \ u(1,t)=h(t),\ 0\le t\le T. \quad (4)$$

Similar problems for partial differential equation of a third-order were investigated in [6] (see also therein references).

Under certain assumptions on the data, the existence and uniqueness of the classical solution of the inverse boundary value problem (1)-(4) is proved.

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A CHANGE OF SCALE FORMULA FOR WIENER INTEGRALS ABOUT THE FIRST VARIATION ON THE PRODUCT ABSTRACT WIENER SPACE

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We prove the existence of the analytic Feynman integrals of the first variation for certain functionals on the product abstract Wiener space and we obtain some relationships between the analytic Wiener integrals and the analytic Feynman integral and the abstract Wiener integral. And we obtain the change of scale formula for abstract Wiener integrals of the first variation on the product abstract Wiener space.

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EXISTENCE OF A SADDLE-POINT IN DIFFERENTIAL GAME WITH NONLOCAL CONDITIONS

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It is known that many problems of the economy are brought to the solution of differential games. In this thesis, we consider a differential game involving two players. Let some process be described by the system of differential equations

$$\dot{x}(t) = f(t, x(t), u(t), v(t)),\ t \in [t_0, T], \quad (1)$$

$$Ax(t_0) + Bx(T) = C \quad (2)$$

where $x \in R^n, u \in R^r, v \in R^q; f : [0,T] \times R^n \times R^r \times R^q \to R^n$ is given function.

It is assumed that U and V are the given sets in the Hilbert spaces $L_2^r[t_0,T]$ and $L_2^q[t_0,T]$, respectively. On the totality $(u,v) \in U \times V$ let's consider the functional

$$J(u,v)=\varphi(x(T)), \qquad (3)$$

where the vector-function $x(t)=x(t;u,v), t\in[t_0,T]$ is determined as a solution of system (1) under condition (2), corresponding to the control $(u,v)\in U\times V$.

Let the controls $(u,v)\in U\times V$. be selected at each moment of time from the domain of determination depending on the state of the process x to optimize quality criterion (3).If u and$v$ belong to two sides whose interests in the sense of selected criterion are opposite, then such a controlled process is called adifferential game, optimized criterion charge.

Definition. The admissible controls and we call optimal strategies if for them

$$J(u_\bullet,v)\le J(u_\bullet,v^\bullet)\le J(u,v^\bullet)$$

are fulfilled.

The trajectory $x_\bullet(t)$ generated by the optimal strategies $u_\bullet(t),v^\bullet(t)$ is called an optimal trajectory.

Under some conditions on the initial data of the problem, sufficient conditions for the existence and uniqueness solutions of boundary value problem (1),(2), also existence of a saddle point in a differential game (1)-(3) are obtained.

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